NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 6

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

NEBRASKA DEPARTMENT OF EDUCATION

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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 6
Number

6.N.1 Numeric Relationships

6.N.1.a

Determine common factors and common multiples.

Extended: Identify the common factors of 4, 6, 8, 9, 10, 12, 15, and 20, given the factors of both numbers in an array or a multiplication sentence.

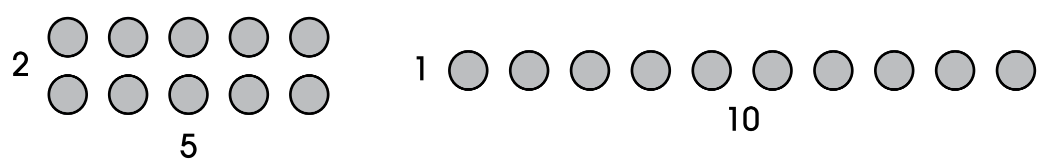
Scaffolding Activities for the Extended Indicator

❑ **Identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20 when given an array or a multiplication sentence.**

- Use a multiplication sentence to demonstrate how to identify the factors of a number. For example, present the multiplication sentence $3 \times 2 = 6$. Explain that factors are numbers that are multiplied to get a product. Therefore, 3 and 2 are factors of 6. Another multiplication sentence with a product of 6 is $1 \times 6 = 6$. Explain that 1 and 6 are also factors of 6. The factors of 6 are 1, 2, 3, and 6.

  Repeat the process of using multiplication sentences to identify all the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

- Use an array to demonstrate finding the factors of 10. Present an array of 2 rows of 5. Explain that the number of rows and the number of columns represent factors of 10. Therefore, 2 and 5 are factors of 10. Present an array of 1 row of 10 to demonstrate that 1 and 10 are also factors of 10. The factors of 10 are 1, 2, 5, and 10.

  Repeat the process of using arrays to identify all the factors of 4, 6, 8, 9, 12, 15, and 20.

- Ask students to identify the factors when given a multiplication sentence with a product of 4, 6, 8, 9, 10, 12, 15, or 20.

- Ask students to identify the factors when given an array for 4, 6, 8, 9, 10, 12, 15, or 20.

- ❑ **Identify the common factors of 4, 6, 8, 9, 10, 12, 15, and 20 when given the factors of both numbers.**

  - Use multiplication sentences to demonstrate how to identify common factors of 4 and 6. Present the multiplication sentences with a product of 4 and a product of 6, as shown. Model making a list of the factors for 4 and a list of the factors for 6. Explain that the common factors of 4 and 6 are the numbers that are in both lists of factors. Identify 1 and 2 as the common factors of 4 and 6.

    $1 \times 4 = 4$ $\quad\quad$ $1 \times 6 = 6$

    $2 \times 2 = 4$ $\quad\quad$ $2 \times 3 = 6$

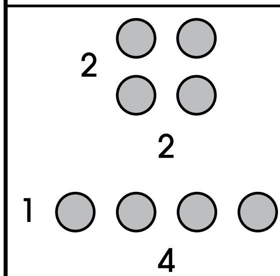
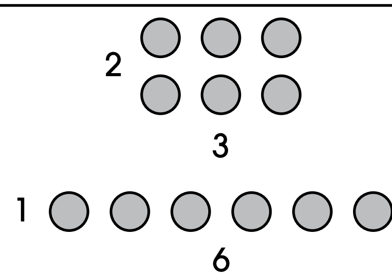
    factors of 4: 1, 2, 4

    factors of 6: 1, 2, 3, 6

    common factors: 1 and 2

    Repeat the process to identify the common factors between two multiplication sentences using the numbers 4, 6, 8, 9, 10, 12, 15, or 20.

  - Use arrays to demonstrate how to identify the common factors of 4 and 6. Present the arrays for 4 and 6 as shown. Model making a list of factors for each number and then identifying the common factors as the numbers that appear in both lists.

Arrays of 4	Arrays of 6
	
Factors	**Factors**
1, 2, 4	1, 2, 3, 6

    Repeat the process to identify the common factors between two arrays using the numbers 4, 6, 8, 9, 10, 12, 15, or 20.

  - Ask students to identify the common factors of two numbers when given a list of factors for each number, using the numbers 4, 6, 8, 9, 10, 12, 15, or 20.

  - Ask students to identify the common factors of two numbers using arrays or multiplication sentences for each number. Use numbers 4, 6, 8, 9, 10, 12, 15, or 20.

Prerequisite Extended Indicator

MAE 4.N.4.b—Identify numbers 1–20 as odd or even, and identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

Key Terms

array, column, common, common factor, factor, multiplication sentence, product, row

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1590

http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html?from=search.html?qt=factor

(Note: Java required for website. Most recent version recommended, but not needed.)

6.N.1.c

Model integers using drawings, words, manipulatives, number lines, models, and symbols.

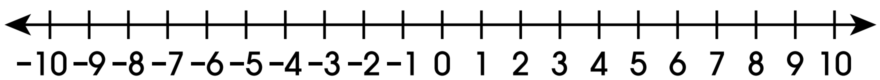
Extended: Identify models of integers from –10 to 10 using drawings, words, manipulatives, number lines, and symbols.

Scaffolding Activities for the Extended Indicator

❑ **Understand the meaning of the word “integer.”**

- Use a number line to define whole numbers. Start by presenting a number line from 0 to 10, and indicate the whole numbers shown on the number line. Next, extend the number line to the right. For example, show the number line from 0 to 20. Explain that when the number line is extended to the right, greater whole numbers are shown.

- Demonstrate that a number line can be extended in either direction to introduce integers. Present a number line from –10 to 10, and emphasize that now the number line has been extended to the left. When the number line is extended to the left from 0, negative numbers are shown.



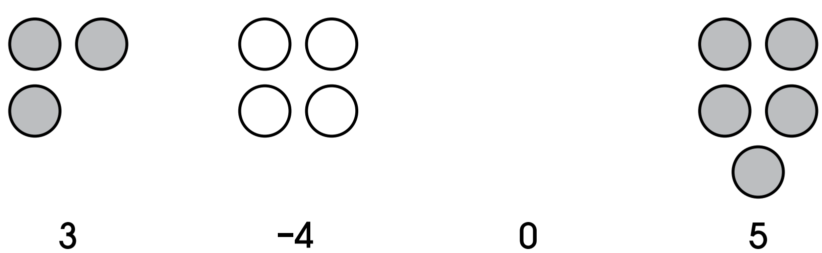
Negative numbers are the opposite of positive numbers, and each whole number starting with 1 has an opposite that is the same distance from 0 on the left side of the number line. So the opposite of 1 is –1, the opposite of 2 is –2, and so on. This category of numbers contains "integers," which are whole numbers and their opposites.

- Ask students to identify opposite integers from a number line or list of positive and negative numbers.

❑ **Use manipulatives to represent integers.**

- Use tokens of two different colors (e.g., blue and red) to represent positive integers and negative integers. For this example, gray tokens represent positive integers and white tokens represent negative integers.

Represent a variety of integers using the tokens as shown.

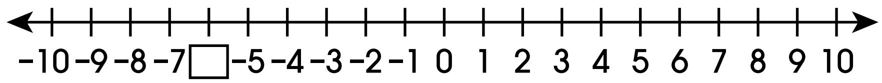


Show how to represent the integers from –10 to 10 by using the tokens.

- Ask students to use manipulatives to represent integers from –10 to 10.

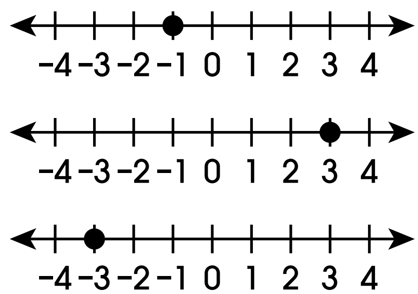
❑ **Identify an integer on a number line.**

- Use a number line to model identifying missing integers.



In the number line shown, there is a missing integer. Demonstrate how to identify the missing integer. Some strategies include finding the opposite (i.e., 6) or counting down from 0.

- Ask students to identify an integer on a number line. For example, present the following three number lines.



Then ask students to identify which number line has a point at the integer –3. Students should identify the last number line as the correct choice.

Prerequisite Extended Indicators

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Key Terms

integer, negative, negative integer, number line, opposite, positive, positive integer

Additional Resources or Links

https://www.mathlearningcenter.org/apps/number-line

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB4SUP-A11_NumOpNegNum-201304.pdf

6.N.1.d

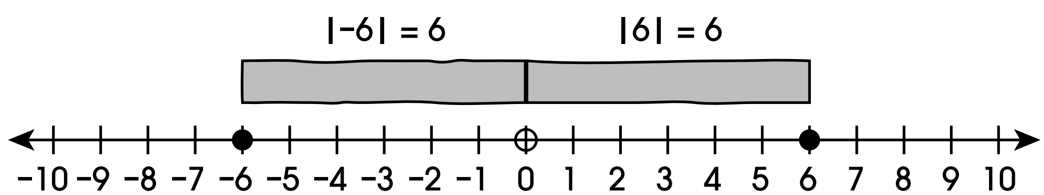
Determine absolute value of rational numbers.

Extended: Identify the absolute value of an integer between –10 and 10.

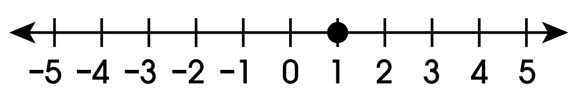
Scaffolding Activities for the Extended Indicator

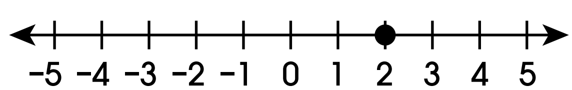
❑ **Use a horizontal number line to recognize the absolute value of an integer from –10 to 10.**

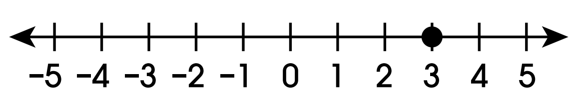
- Explain that the absolute value of a number is its distance from 0. Using the number line, explain that the absolute value of –6 equals 6 because –6 is 6 units away from 0. Similarly, explain that the absolute value of 6 equals 6 because 6 is 6 units away from 0. Cut a strip of paper that matches the distance from 0 to –6 on the number line. Place one end of the strip of paper on the number line at –6 and the other end at 0. To demonstrate that both 6 and –6 are the same distance from 0, move the strip of paper on the number line so that one end is at 6 and the other end remains at 0. Show that absolute value is indicated with two vertical lines; |–6| = 6 can be read aloud as "the absolute value of negative six equals six."



- Demonstrate recognizing number lines with a point plotted at a given absolute value. For example, present the three number lines shown and three questions. Which number line has a point that represents the absolute value of 1? Which number line has a point that represents the absolute value of 2? Which number line has a point that represents the absolute value of 3? Model answering each question by indicating the correct number line.

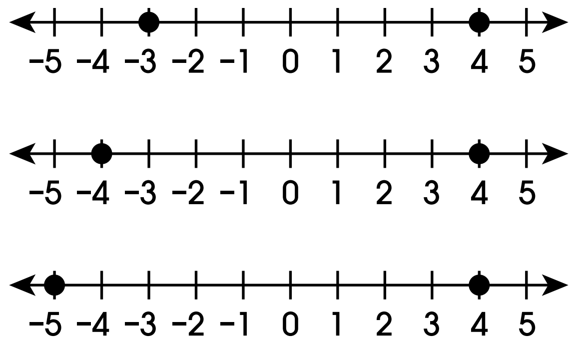






- Repeat the process with number lines that show points plotted at negative integers. Then, repeat the process with number lines that show points plotted at both positive and negative integers from –10 to 10.

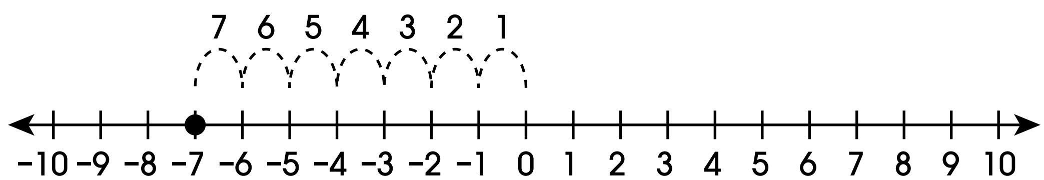
- Demonstrate recognizing pairs of numbers that have the same absolute value. For example, present three number lines as shown and indicate the number line showing two points that represent the same absolute value.



- Ask students to recognize number lines with points plotted that represent a given absolute value.

❑ **Use a horizontal number line to identify the absolute value of an integer from –10 to 10.**

- Present a number line with a point plotted at –7. Demonstrate identifying the absolute value of –7 by counting the intervals from 0 to –7. The absolute value of –7 is 7.



  As appropriate, progress to identifying the absolute value of an integer from –10 to 10 without using a number line.

- Ask students to identify the absolute value of integers from –10 to 10 when plotted on a number line and without being plotted on a number line.

Prerequisite Extended Indicators

MAE 6.N.1.c—Identify models of integers from –10 to 10 using drawings, words, manipulatives, number lines, and symbols.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

absolute value, distance, integer, number line

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-11/file/41656

https://curriculum.illustrativemathematics.org/MS/students/1/7/6/index.html

6.N.1.e

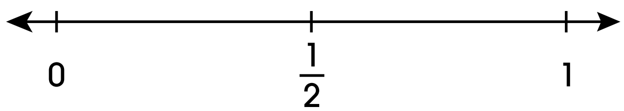
Compare and order numbers including non-negative fractions and decimals, integers, and absolute values and locate them on the number line.

Extended: Compare and order halves with halves, quarters with quarters, and tenths with tenths from 0 to 1 on a number line and compare and order integers from –10 to 10 on a number line.

Scaffolding Activities for the Extended Indicator

❑ **Identify halves, quarters, and tenths on a number line.**

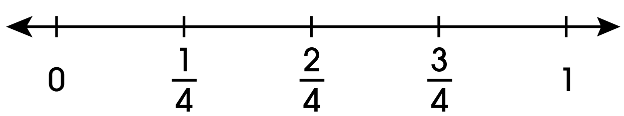
- Use a number line to show the location of $\frac{1}{2}$. Explain that $\frac{1}{2}$ is between 0 and 1. Emphasize that the whole number 1 is equal to the fraction $\frac{2}{2}$.



- Ask students to identify $\frac{1}{2}$ and $\frac{2}{2}$ on a number line.

- Ask students to place the fractions $\frac{1}{2}$ and $\frac{2}{2}$ on a number line from 0 to 1 with tick marks at 0, $\frac{1}{2}$, and 1 and the 0 and 1 tick marks labeled.

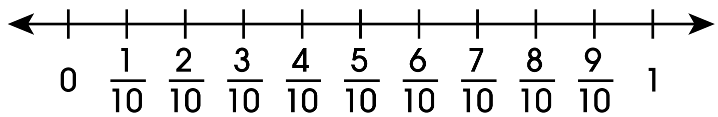
- Use a number line to show the location of quarters. Another term for quarters is fourths. Fourths divide the distance from 0 to 1 on a number line into 4 equal-size parts, each part representing $\frac{1}{4}$ of the whole. As the number line goes from left to right, the number of fourths increases: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$. Emphasize that $\frac{4}{4}$ is equal to 1 whole.



- Ask students to identify $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ on a number line.

- Ask students to place the fractions $\frac{1}{4}$ and $\frac{4}{4}$ on a number line from 0 to 1 with tick marks at each quarter interval and the 0, $\frac{2}{4}$, $\frac{3}{4}$, and 1 tick marks labeled.

- Use a number line to show the location of tenths. Tenths divide the distance from 0 to 1 on a number line into 10 equal parts, each part representing $\frac{1}{10}$ the whole. As the number line goes from left to right, the number of tenths increases: $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$. Emphasize that $\frac{10}{10}$ is equal to 1 whole.



- Ask students to identify $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$, and $\frac{10}{10}$ on a number line.

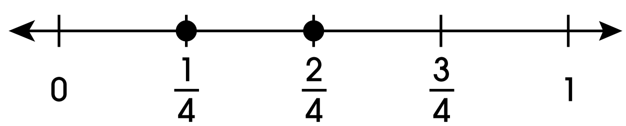
- Ask students to place the fractions $\frac{3}{10}$ and $\frac{8}{10}$ on a number line from 0 to 1 in which the other tenths are labeled.

❑ **Compare halves with halves, quarters with quarters, and tenths with tenths on a number line.**

- Use symbols to show comparisons of numbers. For example, the symbol > means "greater than" and can be used in the number sentence 5 > 1 and the symbol < means "less than" and can be used in the number sentence 2 < 7. It may be helpful to demonstrate that the opening of the symbols always goes toward the greater number, like a wide-open mouth that is hungry for the greater amount.

- Ask students to use the symbols > and < to compare whole numbers 1 through 9.

- Use a number line to demonstrate how to compare quarters, or fourths. For example, the points on this number line are at $\frac{2}{4}$ and $\frac{1}{4}$. Explain that numbers on a number line go from left to right and that the farther you go to the right on a number line, the greater the numbers are. So, in this example, $\frac{2}{4}$ is greater than $\frac{1}{4}$, which can be written as $\frac{2}{4} > \frac{1}{4}$.

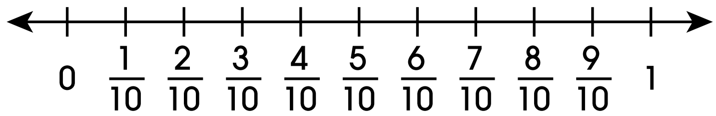


- Repeat the process to demonstrate comparing halves and tenths on the respective number lines and using the symbols > and < to write expressions.

- Ask students to select the correct symbol to compare halves with halves, quarters with quarters, and tenths with tenths on the respective number lines. For example, ask students to select the symbol > or < to fill in the box. Students should select the < symbol to place in the box.

$$\frac{3}{10} \square \frac{7}{10}$$

- Ask students to compare halves, quarters, and tenths on a number line. For example, give students the following figure and ask, “Which statement is true?”

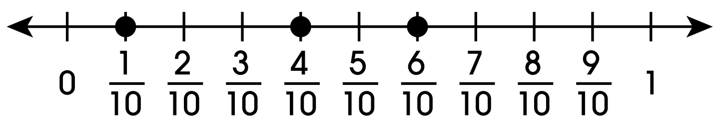


$\frac{7}{10} < \frac{9}{10}$ $\frac{7}{10} = \frac{9}{10}$ $\frac{7}{10} > \frac{9}{10}$

Students should choose $\frac{7}{10} < \frac{9}{10}$.

❑ **Order halves with halves, quarters with quarters, and tenths with tenths on a number line.**

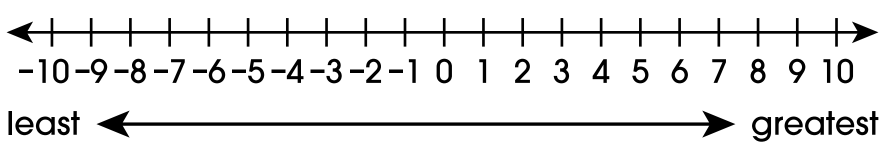
- Use a number line to demonstrate ordering halves, quarters, and tenths from least to greatest. For example, present the number line shown. Explain that the fractions should be read from left to right to place them in order from least to greatest. The correct order from least to greatest is $\frac{1}{10}$, $\frac{4}{10}$, $\frac{6}{10}$.



- Repeat the process to demonstrate putting halves and tenths in order from least to greatest using the respective number lines.
- Ask students to order halves, quarters, and tenths when given the respective number line and three points plotted.
- Ask students to order halves, quarters, and tenths when presented with a number line containing fractions. Direct students to examine three selected fractions on the number line and identify the correct order of the fractions from least to greatest.

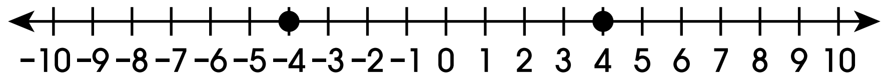
❑ **Compare integers –10 to 10 on a number line.**

- Discuss relevant real-world examples of values less than zero to introduce the concept of negative integers (e.g., very cold temperatures, the score of a game that involves negative numbers, a debt).
- Use a number line from –10 to 10 to show how to compare integers. Indicate that numbers on the right are greater than numbers on the left.



For example, 7 is greater than 6 and 0 is greater than –1. Comparing negative integers can be confusing. For example, –2 is greater than –10. It might be helpful to use number line slider boards to reinforce the concept that numbers on the right are greater than numbers on the left.

- Demonstrate comparing two integers on a number line. Present the number line as shown and explain that 4 is greater than –4 because it is farther to the right on the number line.

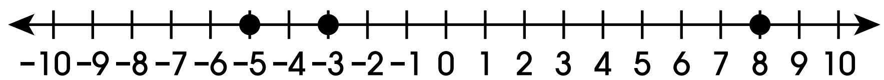


  Be sure to provide examples comparing two positive integers, two negative integers, and a positive and a negative integer, as well as examples comparing other integers to zero.

- Ask students to compare two positive integers on a number line.
- Ask students to compare a positive integer and a negative integer on a number line.

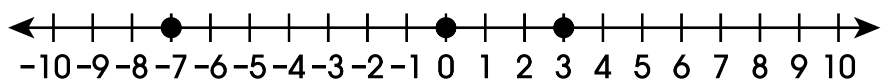
❑ **Order integers –10 to 10 on a number line.**

- Use a number line from –10 to 10 to show how to order integers from least to greatest. Present a number line with points plotted at –5, 8, and –3.



  Move from left to right to list the integers from least to greatest: –5, –3, 8. Demonstrate with a variety of examples including positive numbers, negative numbers, and zero.

- Ask students to identify the correct order of integers. For example, present the number line as shown and three possible choices for the order from least to greatest.



0, 3, –7

–7, 0, 3

3, –7, 0

  Students should choose –7, 0, 3 as the correct order for the integers given.

Prerequisite Extended Indicators

MAE 6.N.1.c—Identify models of integers –10 to 10 using drawings, words, manipulatives, number lines, and symbols.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

compare, greatest, integer, least, negative, number line, order, positive

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-lesson-1

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-7

6.N.2 Operations

6.N.2.a

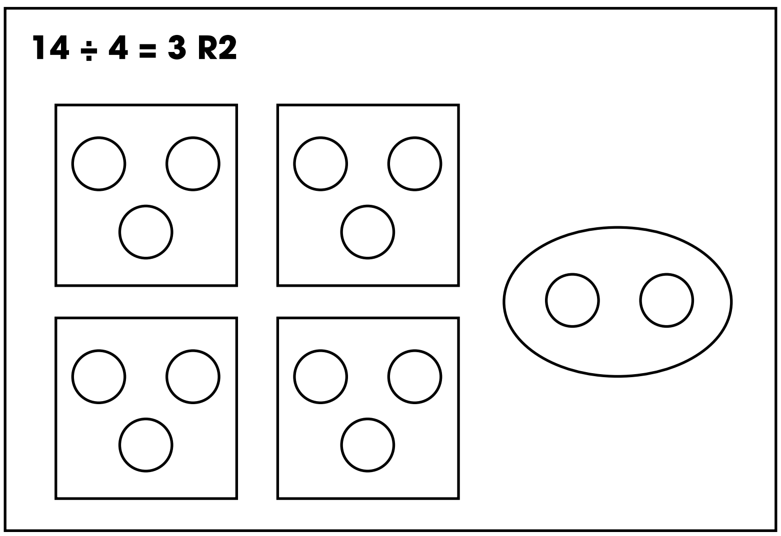
Divide multi-digit whole numbers and decimals using an algorithm.

Extended: Divide a two-digit number by a one-digit number with a remainder.

Scaffolding Activities for the Extended Indicator

❑ **Use manipulatives (chips, popsicle sticks, blocks, dried beans, stickers, etc.) and a counting mat to divide a two-digit number by a one-digit number with a remainder.**

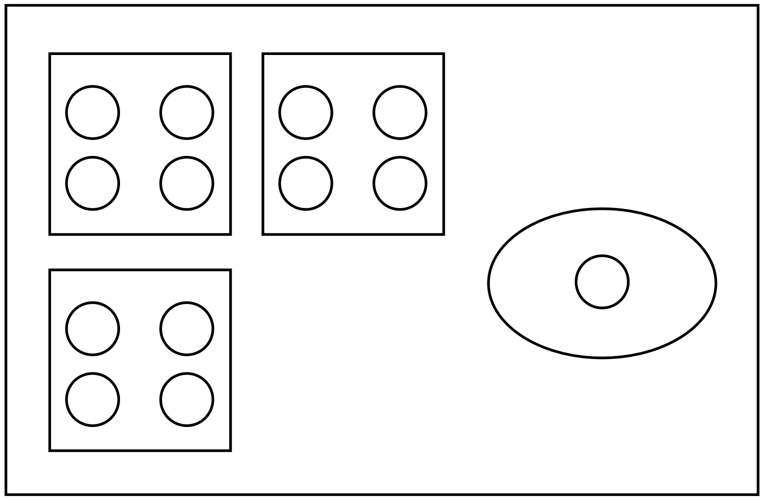
- Use manipulatives to demonstrate that when a set of objects is divided into smaller groups of an equal amount, sometimes there are objects left over that are called remainders. For example, use 14 counters and a divide-by-4 counting mat, as shown.



  Demonstrate how counters can be placed one at a time in each of the four squares on the mat. After the first four are placed, point out that there are enough counters to place another one in each box. Repeat. After the third placement, point out that there aren't enough counters left to put one in each box, so those two counters go into the remainder oval. Write the number sentence 14 ÷ 4 = 3 R2. Indicate that 14 is the total number of counters, 4 is the number of squares, 3 is the number of counters in each square, and 2 is the number of counters that remain. Therefore, 14 divided into 4 groups equals 3 in each group with 2 left over.

- Ask students to follow the process modeled with the counting mat using a division problem with a two-digit number divided by a one-digit number with a remainder. Present the student with the appropriate counting mat for the divisor and the appropriate number of counters or manipulatives. Ask students to determine the number of counters in each group and the number remaining to indicate the solution to the division problem.

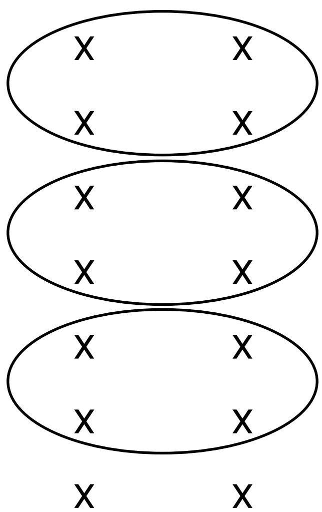
- Ask students to write or select the division problem or the answer to the division problem when given a counting mat with counters already placed on the mat to represent a division problem. For example, provide the following division mat for a student.



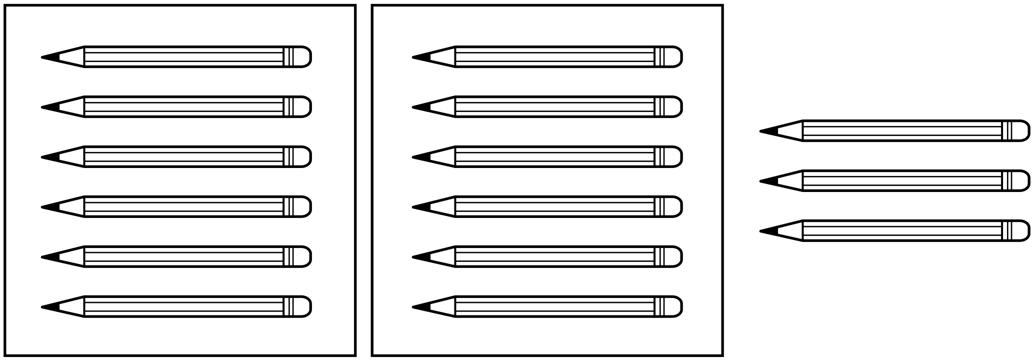
The division problem shown is 13 ÷ 3 = 4 R1.

❑ **Use an array to divide a two-digit number by a one-digit number with a remainder.**

- Use the array shown to demonstrate 14 ÷ 4. Explain that the array shows the original whole of size 14. Indicate that the division by 4 is represented by circling groups of size 4.



Circle all the groups of size 4 and indicate that there are 3 groups of 4 with 2 remaining. Write the number sentence 14 ÷ 4 = 3 R2 and indicate that 14 divided into groups of 4 equals 3 groups with 2 left over. Demonstrate this same activity using manipulatives. For example, use a box of 15 pencils to demonstrate 15 divided by 6.



Write the number sentence 15 ÷ 6 = 2 R3.

- Ask students to use an array to divide a two-digit number by a one-digit number with a remainder.
- Ask students to use real-world objects to divide a two-digit number by a one-digit number with a remainder.

Prerequisite Extended Indicators

MAE 5.A.1.b—Divide a two-digit whole number by a single-digit whole number, limited to quotients with no remainders.

MAE 4.A.1.c—Identify division equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent division without a remainder, limited to groups up to 20.

Key Terms

divide, equal, groups, left over, remainder

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-e-lesson-15/file/34136

https://www.insidemathematics.org/sites/default/files/materials/diminishing%20return.pdf

6.N.2.b

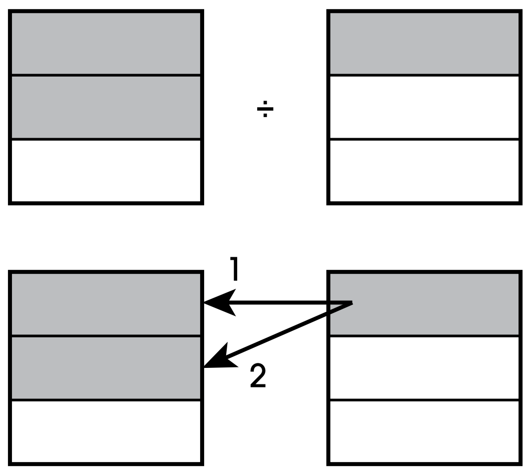
Divide non-negative fractions and mixed numbers.

Extended: Use models to divide positive fractions with like denominators, limited to halves, fourths, thirds, and tenths.

Scaffolding Activities for the Extended Indicator

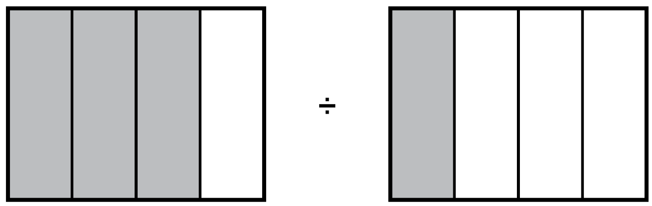
- ❑ **Use area models to represent division problems with positive fractions with like denominators (halves, fourths, thirds, and tenths).**
  - Recognize area models that represent division problems with positive fractions and like denominators. Identify division problems represented by area models. Present a fraction problem and an area model that represents the problem as shown.

$$\frac{2}{3} \div \frac{1}{3}$$



Explain that the shaded part of the rectangle on the left represents the fraction $\frac{2}{3}$ because it is partitioned into three equal parts with two parts shaded. The shaded part of the square on the right represents the fraction $\frac{1}{3}$ because it is partitioned into three equal parts with one part shaded. Emphasize that for both fractions the whole is the same size. The problem $\frac{2}{3} \div \frac{1}{3}$ can be thought of as "how many $\frac{1}{3}$s can fit into the $\frac{2}{3}$?" Model pointing from the shaded part of $\frac{1}{3}$ to the first shaded part in the $\frac{2}{3}$ model and counting 1, then pointing from the shaded $\frac{1}{3}$ to the second shaded part of $\frac{2}{3}$ and counting 2. Explain that it took 2 of the $\frac{1}{3}$s to cover the shaded areas in the $\frac{2}{3}$ model, so the answer is 2. $\frac{2}{3} \div \frac{1}{3} = 2$.

- Ask students to recognize a division problem with positive fractions with like denominators using halves, fourths, thirds, and tenths. Present an area model and three division problems as shown and ask students to select the division problem that is represented by the area model.



A. $\frac{2}{3} \div \frac{1}{3}$

B. $\frac{3}{4} \div \frac{1}{4}$

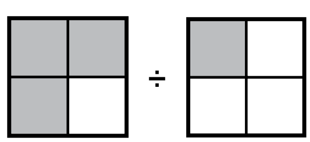
C. $\frac{4}{4} \div \frac{2}{4}$

- Ask students to identify the area model that represents a division problem with positive fractions and like denominators, such as $\frac{6}{10} \div \frac{2}{10}$. Present a division problem and three area models and ask students to select the area model that represents the division problem.

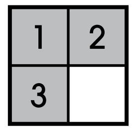
❑ **Use area models to solve division problems with positive fractions with like denominators (halves, fourths, thirds, and tenths).**

- Use area models to solve division problems with positive fractions. Start with division problems in which the divisor is a unit fraction, such as $\frac{1}{4}$. Present a fraction problem and an area model that represents the problem as shown.

$$\frac{3}{4} \div \frac{1}{4} = 3$$



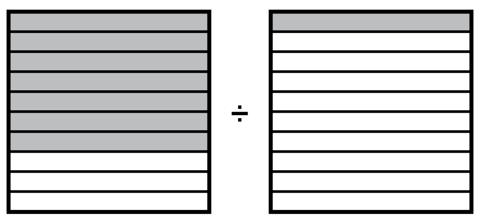
Explain that the shaded part of the square on the left represents the fraction $\frac{3}{4}$ because it is partitioned into four equal parts with three parts shaded. The shaded part of the square on the right represents the fraction $\frac{1}{4}$ because it is partitioned into four equal parts with one part shaded.The problem $\frac{3}{4} \div \frac{1}{4}$ can be thought of as “how many parts of size $\frac{1}{4}$ are in a part of size $\frac{3}{4}$?”



Explain that there are 3 parts of size $\frac{1}{4}$ in the part of size $\frac{3}{4}$. Therefore, $\frac{3}{4} \div \frac{1}{4} = 3$.

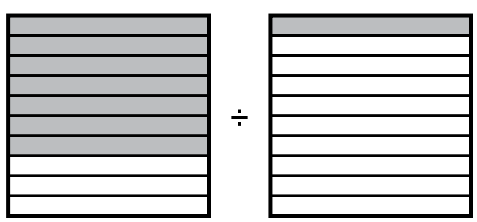
- Ask students to identify the quotient or solution to a division problem with positive fractions such as $\frac{7}{10} \div \frac{1}{10}$ when given a completed area model and the division equation as shown.

$$\frac{7}{10} \div \frac{1}{10} = 7$$



- Ask students to solve a division problem with positive fractions such as $\frac{7}{10} \div \frac{1}{10}$ when given a completed area model as shown.

$$\frac{7}{10} \div \frac{1}{10} =$$



When appropriate, progress to modeling division problems that have non unit fractions as the divisor, the same denominators, and one of the numerators a multiple of the other numerator, such as $\frac{9}{10} \div \frac{3}{10} = 3$.

Prerequisite Extended Indicators

MAE 5.N.3.c—Use a visual model to divide a whole number by $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ (e.g., 3 divided by $\frac{1}{2}$).

MAE 5.N.3.e—Use a visual model to add and subtract fractions with like denominators of halves, thirds, fourths, and fifths, limited to minuends and sums with a maximum of 1 whole.

Key Terms

area model, denominator, divide, fourth, fraction, half, numerator, tenth, third

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-4-topic-e-overview

https://www.engageny.org/resource/grade-5-mathematics-module-4-topic-g-overview

6.N.2.c

Evaluate numerical expressions including absolute value and/or positive exponents with respect to order of operations.

Extended: Evaluate numerical expressions involving addition, subtraction, and multiplication with respect to order of operations.

Scaffolding Activities for the Extended Indicator

❑ **Identify the order in which operations are evaluated.**

- Show 4 + 7 – 2 as an example of a numerical expression. Explain that there are special rules to follow when evaluating numerical expressions. When a numerical expression has addition and subtraction, start by evaluating on the left. For example, in the expression 4 + 7 – 2, 4 + 7 is evaluated first.

6 + 3 – 5	9 – 3 + 4	4 – 2 + 7	5 + 5 – 3

- Ask students to circle, underline, highlight, etc. the part of the expression that needs to be evaluated first. For example, in the expression 6 + 3 – 5, students should identify 6 + 3 as the part of the expression to evaluate first.

- Explain that when multiplication is included in a numerical expression, multiplication is evaluated before addition and subtraction. In 5 × 4 + 3, 5 × 4 is evaluated first. In 6 + 3 × 2, 3 × 2 is evaluated first.

  Also, sometimes numerical expressions are written using parentheses, like 4(2 + 1). Emphasize that the operation inside the parentheses is always done first and that when a number is directly in front of a set of parentheses, it means to multiply. So in the case of 4(2 + 1), the operation 2 + 1 within the parentheses needs to be evaluated first, and then the answer is multiplied by 4.

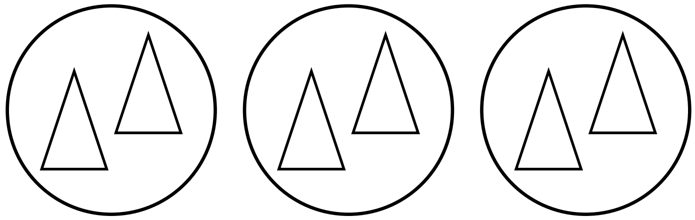
  Continue to model identifying the operation that needs to be evaluated first in examples such as the ones below by circling, underlining, highlighting, etc. Be sure to include the number on each side of the operator when identifying the expression that needs to be evaluated first.

- Ask students to circle, underline, highlight, etc. the expression that needs to be evaluated first in examples such as the ones below.

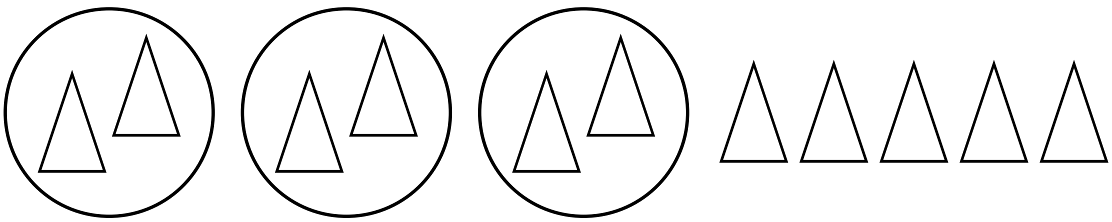
5(3 + 2)	6 – 2 + 5	3 + 4 × 6	5 × 2 – 6
5 × 3 + 4	9 – 2 × 4	6(8 – 3)	4(2 + 4)

- ❑ **Evaluate two-step numerical expressions involving multiplication with addition or subtraction.**

  - Explain that 5 + 3 × 2 is an example of a numerical expression with two steps. There are special rules to follow when multiplication is in the numerical expression. Multiplication comes before addition and subtraction. Model solving 3 × 2 with manipulatives or a drawing. Explain that 3 × 2 can be thought of as 3 groups of 2.

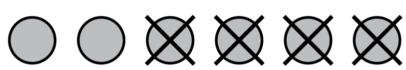
    Explain that the next step in the expression is to add the groups to 5. This gives a total of 11. Therefore, 5 + 3 × 2 = 11.

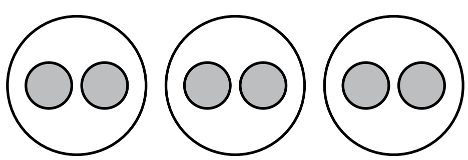
    Continue to model evaluating a variety of two-step numerical expressions involving multiplication with addition or subtraction.

  - Ask students to evaluate numerical expressions involving multiplication and addition or subtraction using manipulatives or drawings. For example, ask students to evaluate expressions such as 3 × 4 + 5 and 7 + 5 × 4.

  - Explain that when parentheses are included in a numerical expression, the operation inside the parentheses is always done first, and if a number is directly in front of a set of parentheses, as in 3(6 – 4), it means to multiply. Model evaluating 3(6 – 4) for students using manipulatives. Explain that 6 – 4 is inside parentheses, so it is evaluated first, which results in 2.

    Since 3 is directly in front of the parentheses, the next step is to multiply 3 × 2. Three times two can be reworded as three groups of two. This gives a total of 6. Therefore, 3(6 – 4) = 6.

    Continue to model evaluating a variety of two-step numerical expressions involving multiplication with addition or subtraction.

6.N.2 Operations

- Ask students to evaluate numerical expressions such as the following with parentheses using manipulatives or drawings.

6(2 + 3)	4(8 – 4)	9(9 – 7)	2(6 – 3)

Prerequisite Extended Indicators

MAE 5.A.1.d—Evaluate two-step numerical expressions involving addition or subtraction and multiplication using order of operations, limited to the digits 1–5 (e.g., 4 × (5 - 2), 4 + 2 × 3).

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

addition, multiplication, numerical expression, parentheses, subtraction

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-2-topic-b-lesson-3

http://nlvm.usu.edu/en/nav/frames_asid_189_g_2_t_2.html?open=activities&from=category_g_2_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

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Mathematics—Grade 6
Ratios and Proportions

6.R.1 Ratios and Rates

6.R.1.a

Determine ratios from concrete models, drawings, and/or words.

Extended: Determine ratios from concrete models and drawings.

Scaffolding Activities for the Extended Indicator

❑ **Identify a ratio between two quantities from a pattern.**

- Use a pattern of foot stomping and hand clapping to demonstrate a ratio. For example, model the pattern stomp, stomp, clap, stomp, stomp, clap, stomp, stomp, clap, and so on. Then explain the ratio of stomps to claps using the figure shown.

  $$\underbrace{\text{stomp, stomp,}}_{2}\ \underbrace{\text{clap}}_{1}$$

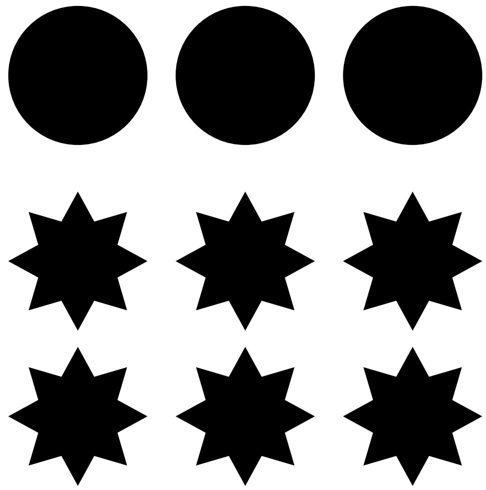
  Make note that there are 2 stomps to every 1 clap, so the ratio of stomps to claps is 2 to 1, or 2:1. It is **not** correct to list the ratio as 1:2. Be sure to emphasize that the order of the numbers is important in ratios. Since the stomp was listed first in “stomps to claps,” the number of stomps must be listed first. A different ratio can be made for claps to stomps, which would be 1:2.

- Ask students to identify a ratio written with a colon when given the same ratio in word form.

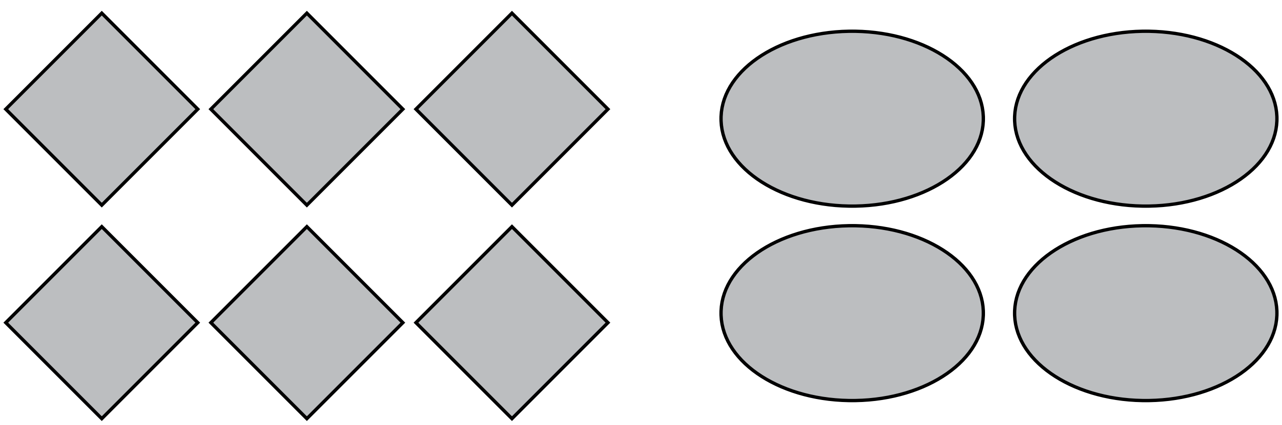
- Ask students to identify a ratio from a given pattern.

- **Identify a ratio between two quantities using models and drawings.**

  - Use manipulatives to demonstrate finding a ratio of quantities. For example, the figure shown has 3 circles and 6 stars. To find a ratio, the order of the ratio is needed. If the ratio is circles to stars, the quantity of circles is listed first (3:6).



Continue to demonstrate how to find the ratio between two quantities using a variety of models and drawings. Progress from demonstrating with manipulatives in which the order of manipulatives is reversed to model different ratios to demonstrating with drawings in which two ratios are identified from the same figure. For example, using the figure shown, demonstrate the ratio of diamonds to ovals (6:4) and the ratio of ovals to diamonds (4:6).



  - Ask students to identify a ratio between two quantities using manipulatives.
  - Ask students to identify a ratio between two quantities given a drawing.

Prerequisite Extended Indicators

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 4.N.1.a—Identify representations of whole numbers up to 100.

Key Terms

order, pattern, quantity, ratio

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/6/RP/A/1/tasks/2158

https://www.engageny.org/resource/grade-6-mathematics-module-1-topic-a-lesson-2

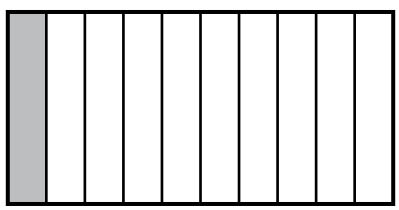
6.R.1.c

Find a percent of a quantity as a rate per 100 and solve problems involving finding the whole, given a part and the percent.

Extended: Recognize $\frac{1}{10}$ and $\frac{1}{100}$ as ratios and convert to equivalent percents.

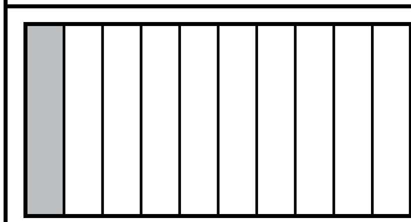
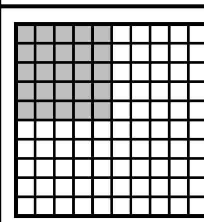
Scaffolding Activities for the Extended Indicator

- ❑ **Identify a model that represents a ratio of 1/10.**
  - Use objects of visual models to demonstrate ratios.
  - Explain that ratios are ways to compare two numbers and tell how much of one thing you have compared to another. Ratios can be written in different forms. For example, the ratio 1 to 10 can be written as 1:10 and as the fraction $\frac{1}{10}$. However, when a ratio is written as a fraction, the denominator represents the equal parts of a whole.
  - Show students an example of a ratio involving a ratio of $\frac{1}{10}$. The model shows the ratio $\frac{1}{10}$ because one bar is shaded out of the total 10.



Continue to demonstrate models representing the ratio $\frac{1}{10}$. Guide students in determining if the model represents the correct ratio.

  - Show a variety of ratios representing $\frac{1}{10}$.
  - Ask students to sort models into categories of ratios representing $\frac{1}{10}$ and not representing $\frac{1}{10}$. Explain that the denominator 10 represents the whole broken into 10 equal parts.

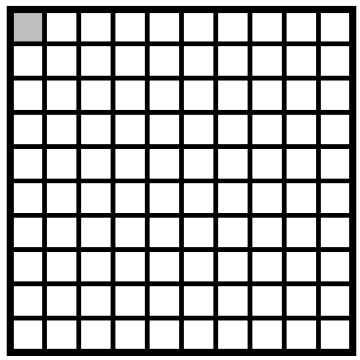
$\frac{1}{10}$	Not $\frac{1}{10}$
	

6.R.1 Ratios and Rates

- ❑ **Identify a model that represents a ratio of $\frac{1}{100}$.**

  - Use objects and visual models to demonstrate ratios with the denominator 100.

    Show students an example of a ratio representing $\frac{1}{100}$. The model shows the ratio $\frac{1}{100}$ because one part is shaded out of a total of 100.

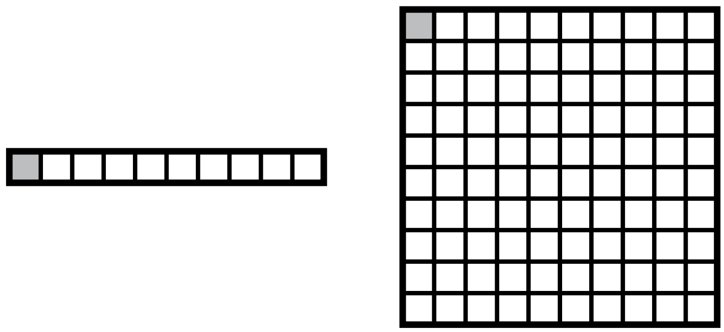
    Continue to demonstrate models representing the ratio $\frac{1}{100}$. Guide students in determining if the model represents the correct ratio.

  - Ask students to shade $\frac{1}{100}$ on a hundreds grid.

- **Recognize $\frac{1}{10}$ and $\frac{1}{100}$ on a hundreds grid.**

  - Use a hundreds grid to demonstrate $\frac{1}{10}$ and $\frac{1}{100}$.

    Explain that the hundreds grid represents the shaded area compared to the total area. Introduce the hundreds grid. Explain to students that the first grid is one bar from the hundreds grid. One part is shaded out of 10 to model $\frac{1}{10}$. The second grid has 1 part shaded out of 100 to model $\frac{1}{100}$.

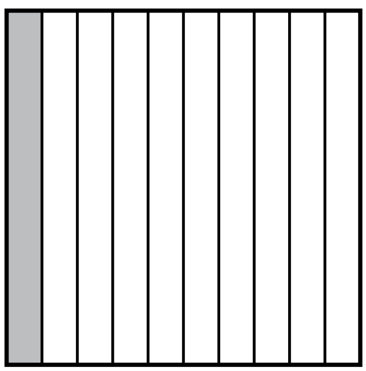
    Continue to demonstrate models representing the ratio $\frac{1}{10}$ and $\frac{1}{100}$. Guide students in determining if the model represents the ratio $\frac{1}{10}$ or $\frac{1}{100}$.

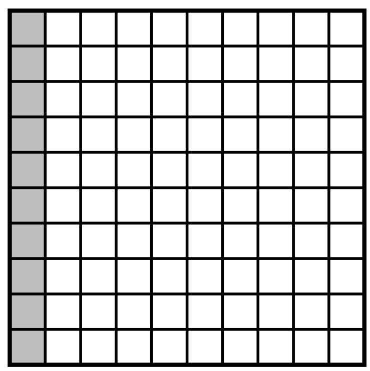
  - Ask students to identify $\frac{1}{10}$ and $\frac{1}{100}$ on a hundreds grid.

- **Convert $\frac{1}{10}$ and $\frac{1}{100}$ to an equivalent percent.**

  - Show students that $\frac{1}{10}$ is the same as 10 percent.

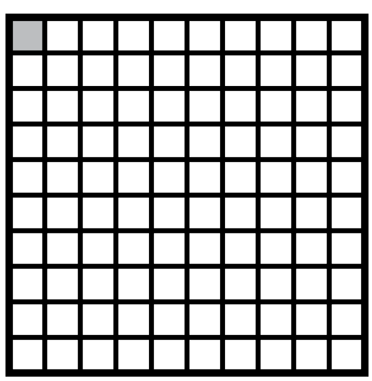
    Demonstrate how a visual model that represents $\frac{1}{10}$ is equivalent to a visual model that represents $\frac{10}{100}$. The first visual model shows $\frac{1}{10}$ because one whole bar is shaded out of 10. The second visual model shows $\frac{10}{100}$ because there are 10 parts shaded out of 100. Each model shows the same amount shaded out of a whole. Explain that $\frac{10}{100}$ is equivalent to 10 percent. The denominator 100 shows how many pieces the grid is divided into to make up a whole number. “Percent” means “per hundred”, which means that we want to find out how many pieces of the whole we would have if there were 100 pieces possible. If ten pieces of the whole is shaded, then we have 10 percent.





    Continue this demonstration with $\frac{1}{100}$, explaining that this is equivalent to 1 percent.

    Show students a visual model of 1 percent using a hundreds grid. Write 1% to show $\frac{1}{100}$ and its equivalent percentage.



  - Ask students to demonstrate shading 1 percent and 10 percent using a hundreds grid.
  - Ask students to match models shaded and labeled with $\frac{1}{10}$ and $\frac{1}{100}$ to their equivalent percentages.

Prerequisite Extended Indicators

MAE 5.N.2.a—Use models to represent equivalent fractions with denominators up to 10 (e.g., $\frac{2}{4} = \frac{1}{2}$, $\frac{3}{3}$ = 1 whole).

MAE 4.N.1.d—Use decimal notation for fractions from 0 to 1 with a denominator of 10 (e.g., $\frac{2}{10}$ = .2), and identify those decimals on a number line from 0 to 1.

Key Terms

denominator, fraction, grid, percent, ratio

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/4/NF/C/5/tasks/154

https://www.engageny.org/resource/grade-6-mathematics-module-1-topic-a

6.R.1.d

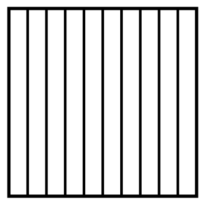
Convert among fractions, decimals, and percents using multiple representations.

Extended: Using a model, convert halves, fourths, and tenths to decimals and identify the corresponding percentages for the fractions 1/4, 1/2, and 3/4.

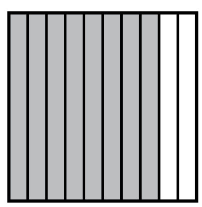
Scaffolding Activities for the Extended Indicator

❑ **Convert tenths into decimal numbers using a model.**

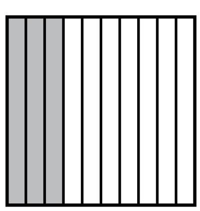
- Use a model to show how to convert fractions with tenths into decimal numbers. Present a figure as shown. Explain that the square represents the whole, and each part is $\frac{1}{10}$ of the whole.



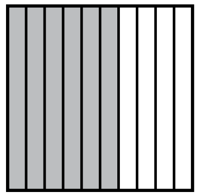
Explain that $\frac{1}{10}$ is equal to 0.1 because the 1 in 0.1 is in the tenths place. Shading in 8 of the $\frac{1}{10}$ parts represents $\frac{8}{10}$. Eight-tenths is equal to the decimal number 0.8 since there are 8 of the 0.1 parts shaded. Continue to demonstrate by shading in other fractional parts of the whole and equating the number of tenths shaded as a fraction and as a decimal.



- Ask students to convert tenths into decimal numbers when given a model and the equivalent fraction as shown.

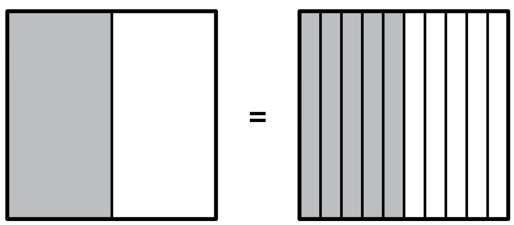
Model	Fraction	Decimal Number
	$\frac{3}{10}$	

- Ask students to convert tenths to decimals when given a model as shown.

Model	Decimal Number
	

- [ ] **Convert halves into decimal numbers using a model.**

- Use a model to demonstrate converting halves into decimal numbers. Explain that base-ten models can be used to convert halves to tenths. This helps to create decimals. Present a figure as shown. Explain that the square on the left represents $\frac{1}{2}$ and the square on the right has the same amount shaded, representing $\frac{5}{10}$.



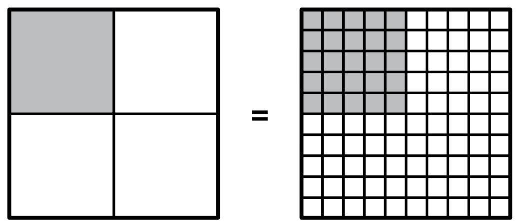
One-half, or $\frac{1}{2}$, of the square is the same as five-tenths, or $\frac{5}{10}$. Since $\frac{1}{10} = 0.1$, that means that $\frac{5}{10} = 0.5$. So one-half is equal to 0.5 as a decimal number. Continue to demonstrate using a variety of models representing one-half and converting $\frac{1}{2}$ to 0.5.

- Ask students to identify a model representing $\frac{5}{10}$.

- Ask students to convert halves to decimals using a model.

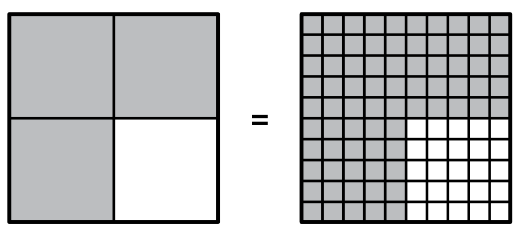
- **Convert fourths into decimal numbers using a model.**

  - Use a model to demonstrate converting one-fourth into a decimal number. Explain that a 100 grid can be used to convert fourths to hundredths. This helps us convert to decimals. Present a figure as shown. Explain that the square represents one whole. The square on the left has 1 part shaded out of 4 equal parts, representing $\frac{1}{4}$. The square on the right has 25 out of 100 equal parts shaded, representing $\frac{25}{100}$.

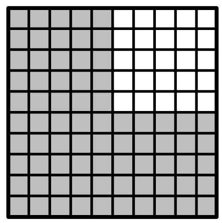
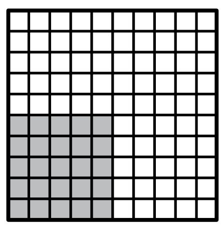
    

    The two large squares each have the same amount shaded, which means that $\frac{1}{4} = \frac{25}{100}$. Each of the small squares on the model for $\frac{25}{100}$ is $\frac{1}{100}$ of the whole. The fraction $\frac{1}{100}$ can be written as the decimal number 0.01. So, 25 hundredths is written as 0.25. Therefore, $\frac{1}{4} = 0.25$.

  - Repeat the process to demonstrate that $\frac{3}{4} = \frac{75}{100} = 0.75$, using the figure shown.

- Ask students to convert fourths to decimals when given a model and the equivalent fractions as shown.

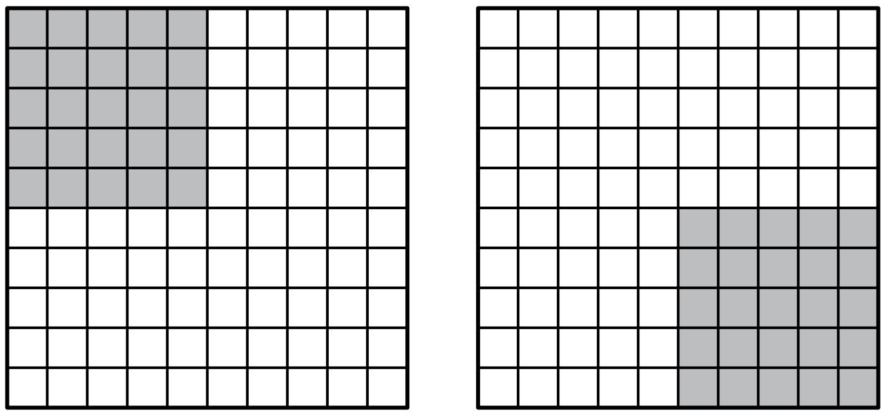
Model	Fraction	Decimal Number
	$\frac{75}{100}$	
	$\frac{25}{100}$	

- Ask students to convert halves, fourths, and tenths to decimals using a model.

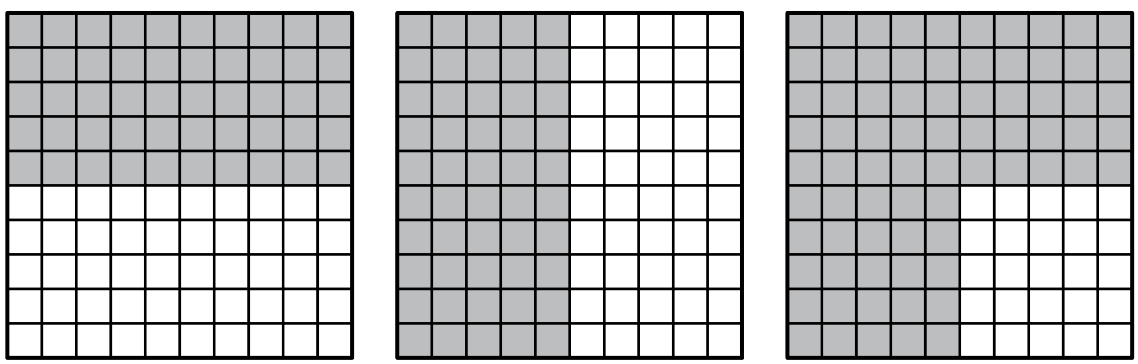
❑ **Recognize percentages as representing a part of a whole divided into 100 parts.**

- Introduce the concept of percentage as a part of something using real-life objects as models. Discuss relevant examples of percentages in a familiar context.
- Use base-ten blocks and a hundreds grid to demonstrate the concept of percentage as a part of a whole 100. Place 9 base-ten rods and 10 unit cubes on a hundreds grid and indicate that 100 percent of the grid is covered. (Note: use the term percent and the percent symbol, %, interchangeably.) Remove 1 base-ten rod and 5 unit cubes. Remind students that 100 is the whole and that now 85 parts of the whole (85 out of 100) are covered, which represents 85 percent. Continue demonstrating percentage as parts of 100 by using different combinations of base-ten blocks and unit cubes.
- Ask students to show 8 percent by covering 8 squares on the hundreds grid with 8 unit cubes.

- ❑ **Identify 25 percent, 50 percent, and 75 percent on a hundreds grid.**
  - Use a hundreds grid to model 25 percent. Highlight or cover 25 unit squares on a hundreds grid. Identify the covered section of the grid as 25 percent. Count the unit squares to reinforce the idea that 25 out of 100 squares are covered, which represents 25 percent. Demonstrate different ways a cutout square can be placed on the hundreds grid and cover 25 percent.



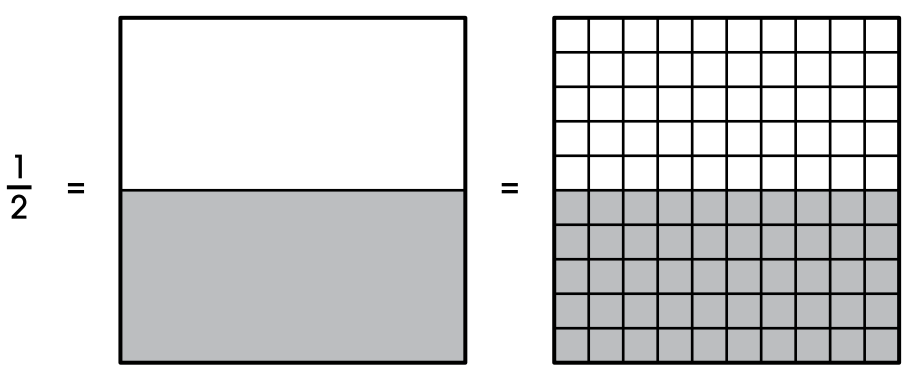
Repeat the same process to represent 50 percent and 75 percent on a hundreds grid.



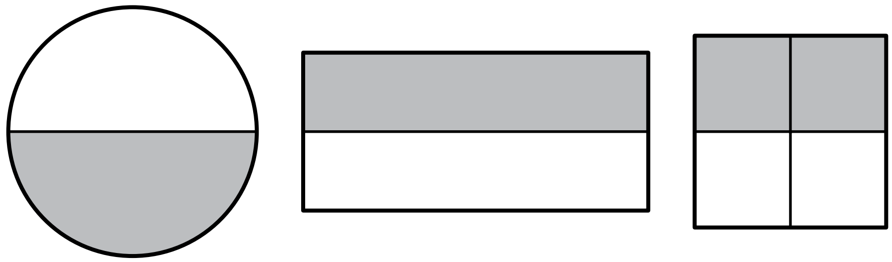
  - Ask students to identify 25 percent, 50 percent, and 75 percent on a hundreds grid.

- ❑ **Identify other visual models that represent 25 percent, 50 percent, and 75 percent.**
  - Demonstrate that $\frac{1}{2}$ of a square is the same as $\frac{1}{2}$ of a 100-square grid, which is 50 squares. Continue to reinforce the idea that 50 out of 100 equals $\frac{1}{2}$, which equals 50 percent.

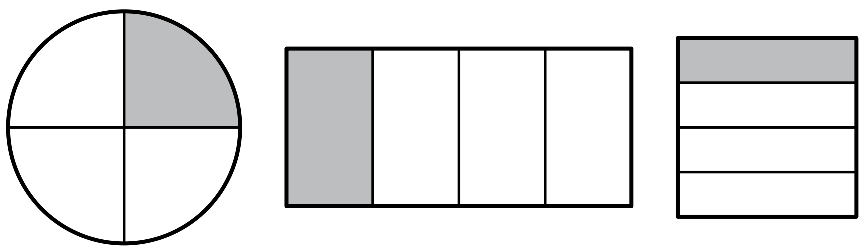
50%



Show other circle, rectangle, and square fractional models without grids that equal 50 percent.



- Repeat the same process to demonstrate that $\frac{1}{4}$ of a square is the same as $\frac{1}{4}$ of a 100-square grid, which is 25 squares and represents 25 percent. Show other circle, rectangle, and square fractional models without grids that equal 25 percent.

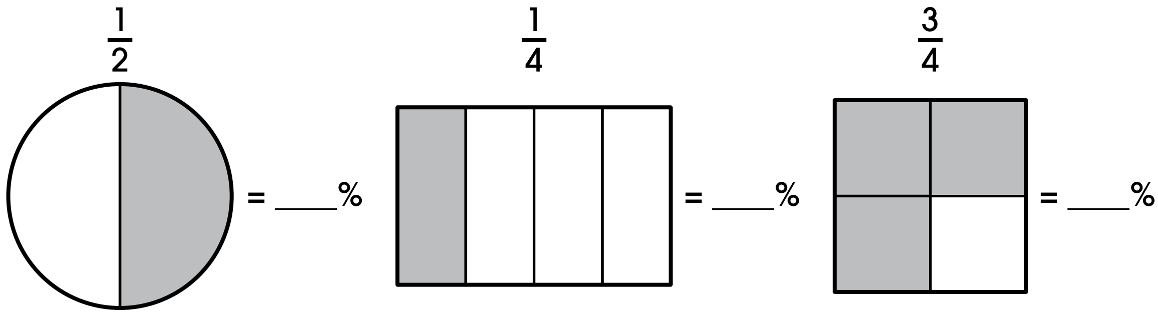


- Repeat the process to demonstrate that $\frac{3}{4}$ of a square is the same as $\frac{3}{4}$ of a 100-square grid, which is 75 squares and represents 75 percent. Show other circle, rectangle, and square fractional models without grids that equal 75 percent.

- Ask students to identify fractional models without grids that represent 25 percent, 50 percent, and 75 percent.

❑ **Given the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, identify the corresponding percentage.**

- Demonstrate identifying the correct percentage when given the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.



- Ask students to identify the correct percentage when given the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

Prerequisite Extended Indicators

MAE 4.N.1.d—Use decimal notation for fractions from 0 to 1 with a denominator of 10 (e.g., $\frac{2}{10}$ = .2), and identify those decimals on a number line from 0 to 1.

MAE 3.N.2.f—Use a model to compare unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

MAE 6.R.1.c—Recognize $\frac{1}{10}$ and $\frac{1}{100}$ as ratios and convert to equivalent percents.

Key Terms

decimal number, fourth, fraction, half, percentage, tenth

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/4/NF/C/5/tasks/154

http://tasks.illustrativemathematics.org/content-standards/4/NF/C/5/tasks/103

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Models/

http://nlvm.usu.edu/en/nav/frames_asid_264_g_2_t_1.html?from=category_g_2_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/grade-6-mathematics-module-1-topic-d-lesson-24

6.R.1.e

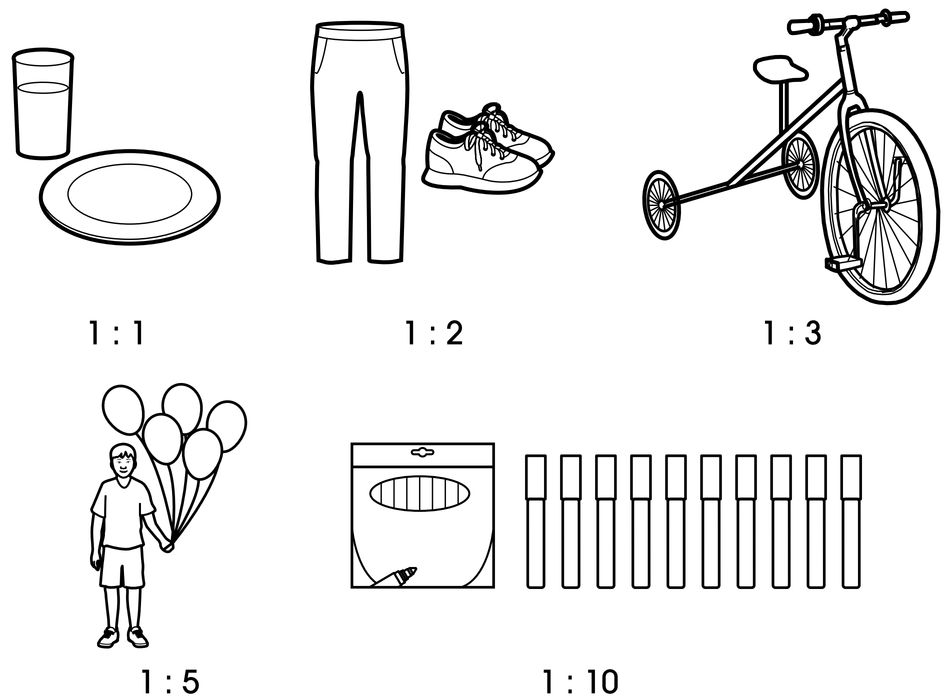
Solve authentic problems using ratios, unit rates, and percents.

Extended: Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

Scaffolding Activities for the Extended Indicator

❑ **Identify graphic representations that represent the ratios 1:1, 1:2, 1:3, 1:5, and 1:10 in an authentic context.**

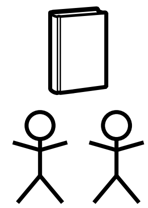
- Use authentic objects or pictures to show models of 1:1, 1:2, 1:3, 1:5, and 1:10 ratios, and explain that a ratio indicates the quantity of one object compared to another.



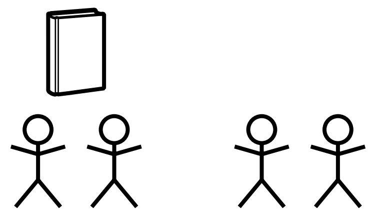
- Ask students to identify the ratios 1:1, 1:2, 1:3, 1:5, and 1:10 using authentic objects or pictures.

- **Use pictures to solve authentic problems with ratios 1:1, 1:2, 1:3, 1:5, and 1:10.**

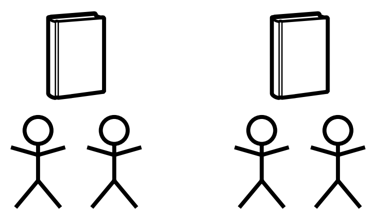
  - Use an authentic word problem and a picture to model solving a ratio problem. For example, “During partner reading there is 1 book for every 2 students. How many books will 4 students need?” Draw a picture of 1 book and 2 students to represent the given ratio.

    Then reread the question and draw two more students.

    Explain that the ratio 1 book to 2 students needs to be the same for both pairs of students and draw in the book for the second pair of students. Reread the question one more time to identify that the problem is asking for the total number of books, which is 2.

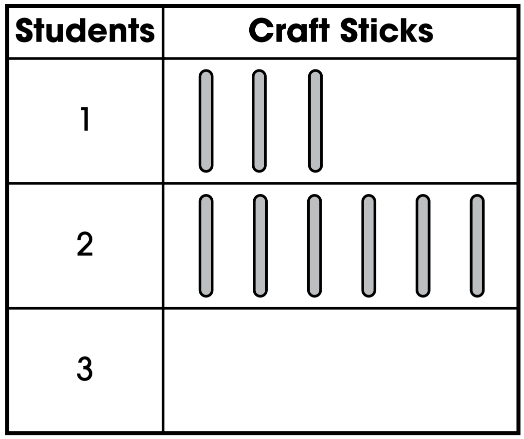
    

    Repeat the process using various word problems and various ratios including 1:1, 1:2, 1:3, 1:5, and 1:10.

  - Ask students to solve a ratio problem given a word problem and a picture that represents the scenario.

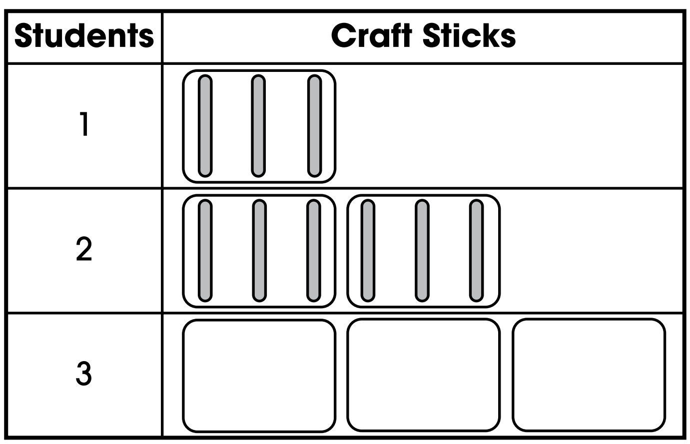
❑ **Solve authentic problems with the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.**

- Represent a ratio problem in a table. For example, “Each student will need three craft sticks for the art project. How many craft sticks will be needed for three students?”



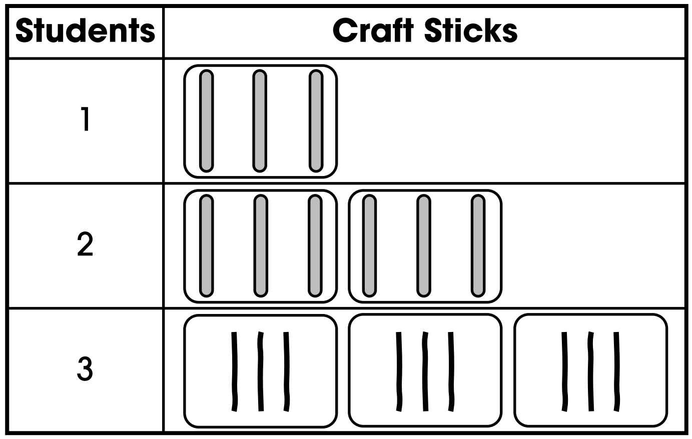
Ratio – 1:3

Model solving the problem by first circling groups of three craft sticks to represent the ratio 1:3. Each circle, or group of 3, is for one student. Then, draw in 3 circles to represent the number of groups of craft sticks needed for three students.



Ratio – 1:3

Next, draw 3 craft sticks or tally marks in each circle to represent the ratio 1 student to 3 craft sticks. Reread the question to identify that the problem is asking for the number of craft sticks needed for 3 students, and count to find the answer is 9.



Ratio – 1:3

Repeat the process using different authentic problems and ratios including 1:1, 1:2, 1:3, 1:5, and 1:10.

- Ask students to solve a ratio problem given an authentic problem and a ratio table that represents the scenario.

Prerequisite Extended Indicator

MAE 6.R.1.a—Determine ratios from concrete models and drawings.

Key Terms

ratio, ratio table

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-1-topic-overview

http://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/496

http://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/61

6.R.2 Represent

6.R.2.f

Plot the pair of values from a ratio table on the coordinate plane.

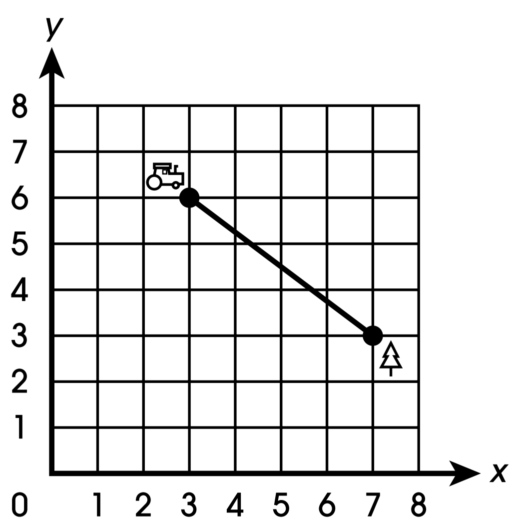
Extended: Identify the line on a coordinate grid that represents the values given in a ratio table.

Scaffolding Activities for the Extended Indicator

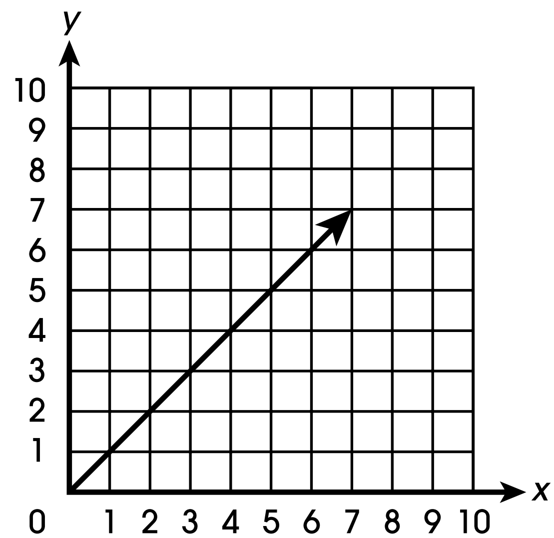
❑ **Identify the *x*-values represented by a line on a coordinate grid.**

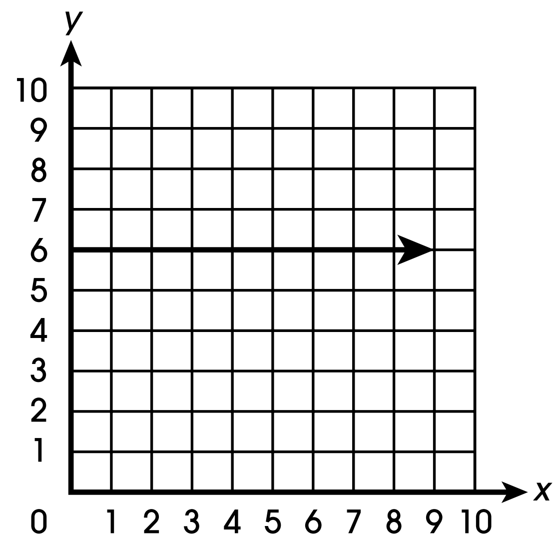
- Use a coordinate grid to demonstrate how to find the *x*-value represented by the line.
- Explain that the numbers on a coordinate grid are used to locate points. Each point along a line can be identified by an ordered pair of numbers: a number on the *x*-axis called an *x*-coordinate and a number on the *y*-axis called a *y*-coordinate. Ordered pairs are written in parentheses as (*x*-coordinate, *y*-coordinate).

  Present a coordinate grid as shown. Demonstrate finding the *x*-coordinate of the location of the tractor first, which is where the line that is shown starts. Explain that you can find the point located on the line next to the tractor and follow the grid line down toward the *x*-axis. Since the point on the line aligns with the 3 on the *x*-axis, the *x*-value of the tractor is 3. Repeat this process to find the *x*-value of the point of the tree and other *x*-values represented by the line.



- Explain to students that there are many positions of lines on coordinate grids, but the *x*-values are always found the same way. Present various coordinate grids with different positioning of lines to students and determine the *x*-values of the lines.

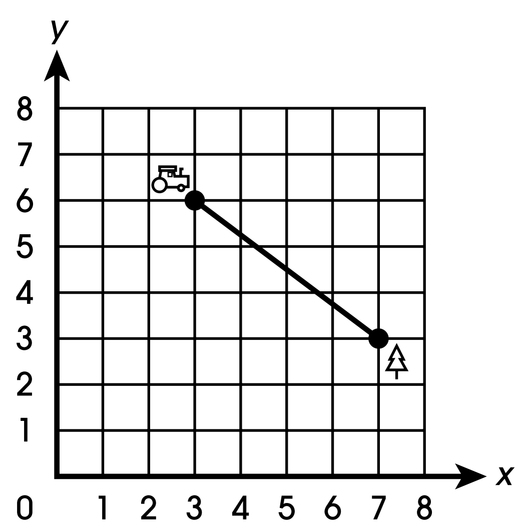




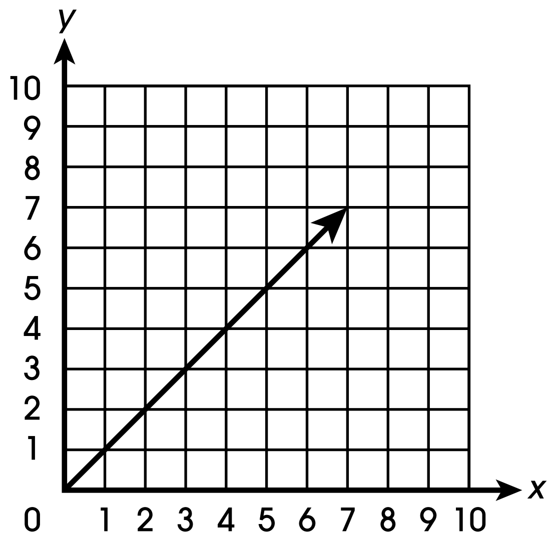
- Ask students to locate different *x*-values represented by lines on different coordinate grids.

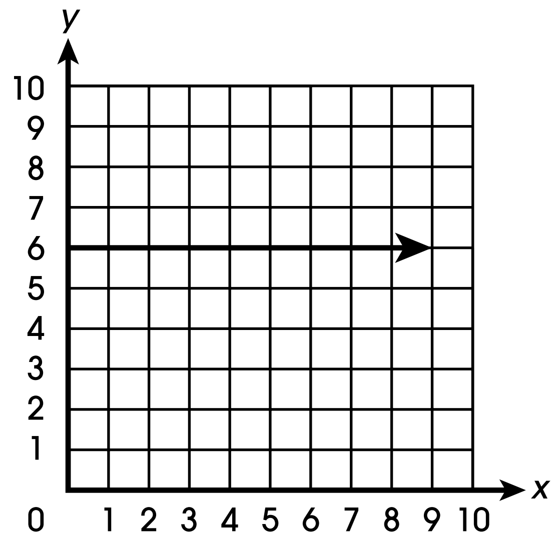
❑ **Identify the *y*-value represented by a line on a coordinate grid.**

- Use a coordinate grid to demonstrate how to find the *y*-value represented by a line. Present a coordinate grid as shown. Demonstrate finding the *y*-value of the location of the tractor first, which is where the line that is shown starts. Explain that you can find the point located on the line next to the tractor and follow the grid line left toward the *y*-axis. Since the point on the line aligns with the 6 on the *y*-axis, the *y*-value of the tractor is 6. Repeat this process to find the *y*-value of the point of the tree and other *y*-values represented by the line.



- Explain to students that there are many positions of lines on coordinate grids, but the *y*-values are always found the same way. Present various coordinate grids with different positioning of lines to students and determine the *y*-values of the lines.





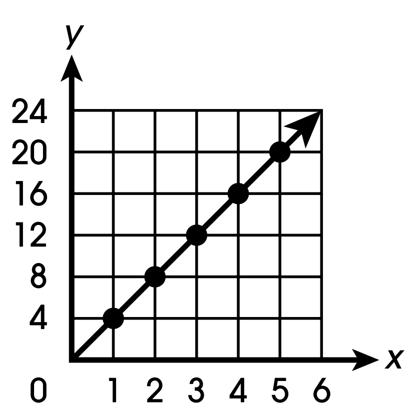
- Ask students to locate different *y*-values represented by lines on different coordinate grids.

❑ **Identify the line on a coordinate grid that represents the values given in a ratio table.**

- Using a coordinate grid and a ratio table, show students how to plot the values shown in the ratio table onto a coordinate grid.

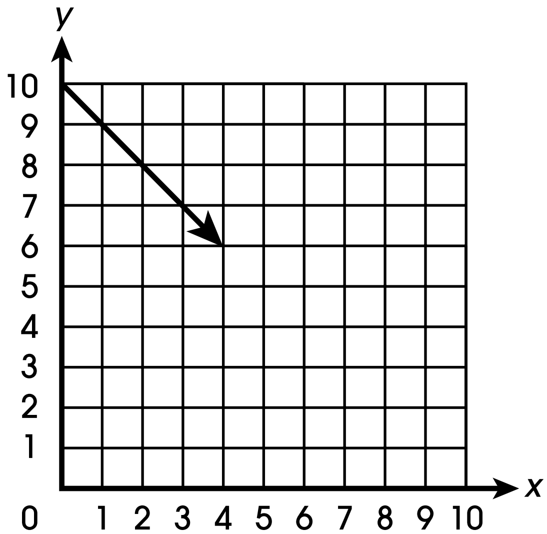
  Indicate the ratio table. Explain that a ratio table is a table that is used to represent the relationship between two separate quantities. Explain that the top row has *x*-values, and the bottom row has *y*-values. Each column in the ratio table makes a coordinate pair. For example, the first column in the table has an *x*-value of 1 and a *y*-value of 4. This can be plotted on a coordinate grid by starting at 0 and then moving to the right one time for the *x*-value and moving up from 1 until you reach 4 on the *y*-axis, which is the *y*-value. Continue modeling how to plot and identify points using a ratio table and a coordinate grid. Indicate that the points shown on the line are the same coordinates shown in the ratio table.

x	1	2	3	4	5
y	4	8	12	16	20



6.R.2 Represent

- Present various ratio tables and various coordinate grids with lines in different positions to students. Work together to match the ratio tables with the corresponding lines.
- Ask students to match the line presented on a coordinate grid with the correct ratio table.



A.

x	0	1	2	3	4
y	10	9	8	7	6

B.

x	4	3	1	3	4
y	1	2	8	7	6

C.

x	10	1	2	3	4
y	0	2	4	6	8

Prerequisite Extended Indicator

MAE 6.R.1.a—Determine ratios from concrete models and drawings.

MAE 5.G.2.a—Identify the origin, *x*-axis, and *y*-axis of a coordinate plane.

MAE 5.G.2.b—Identify the *x*- or *y*-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

coordinate, coordinate grid, coordinate pair, ratio, ratio table, *x*-axis, *x*-value, *y*-axis, *y*-value

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/G/A/1/tasks/489

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-2/lesson.html

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-3/lesson.html

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Mathematics—Grade 6
Algebra

6.A.1 Algebraic Processes

6.A.1.a

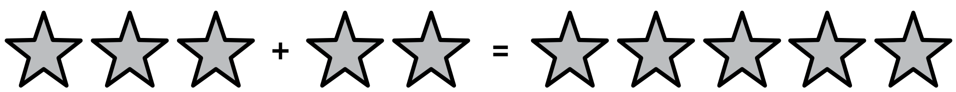
Recognize and generate equivalent algebraic expressions involving the distributive property and combining like terms.

Extended: Identify equivalent expressions with one variable by combining like terms, limited to digits 1–9 (e.g., 2n + 3n = 5n).

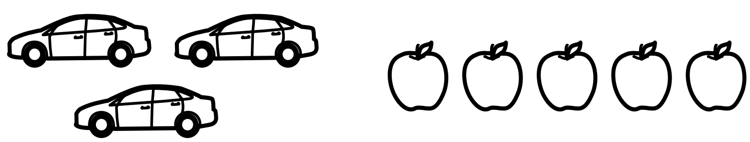
Scaffolding Activities for the Extended Indicator

❑ **Represent like terms with manipulatives.**

- Use similar objects to demonstrate that groups of the same object can be combined. A group of three stars can be combined with (or added to) a group of two stars to make a group of five stars.



- Show students objects that are not the same. Since cars and apples are different, they cannot be combined or added together to get one group of similar objects.



- Use manipulatives around the classroom (e.g., crayons, markers, erasers, pens, pencils, paper clips) to continue modeling combining like objects.

4 crayons + 2 crayons = ?
2 pencils + 5 markers = ? (cannot combine)
3 erasers + 2 erasers = ?
1 paper clip + 3 pens = ? (cannot combine)

- Ask students to combine groups of real-life objects, math manipulatives, and pictures of objects to represent like terms.

❑ **Combine like terms.**

- Explain that, like objects and pictures, variables in math expressions can be combined when they are the same. A variable replaces an unknown number in a math expression. The variable, n, is used in the math expression $2n + 3n$. Since the variable n is the same in both parts (or terms) of the math expression, the $2n$ can be added to the $3n$. Since $2 + 3 = 5$, $2n + 3n = 5n$.
- Explain that for the same reason cars and apples cannot be combined, different variables cannot be combined. For example, $4p + 3w$ cannot be combined because the variables are different.
- Model identifying expressions with like terms that can be combined.

$4p + 3p$	$8y + 2b$	$2m + 4k$
$3w + 7t$	$2n + 5n$	$3w + 4w$

- Ask students to identify like terms.

$2m + 4b$	$5w + 4w$	$3a + 5a$
$7n + 2h$	$4k + 2w$	$8m + 2m$
$2y + 7w$	$3y + 4y$	$9b + 4b$

- Model combining like terms by using such familiar methods as a counting strategy, an addition strategy, or a calculator. Continue to emphasize why the terms can be combined.

$3b + 4b =$	$2m + 4m =$
$5w + 5w =$	$6y + 3y =$

- Ask students to combine like terms.

$2n + 7n =$	$3a + 5a =$
$4y + 4y =$	$2d + 4d =$
$8w + 2w =$	$3m + 2m =$

Prerequisite Extended Indicator

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 4.A.1.e—Identify an addition or subtraction equation in an authentic mathematical situation using a variable for an unknown, limited to an unknown change or unknown result (e.g., $3 + n = 10$, $12 - 6 = n$).

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

Key Terms

addition, combine, expression, like terms, number sentence, term, variable

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_161_g_1_t_1.html?from=category_g_1_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/grade-7-mathematics-module-3-topic-lesson-6

6.A.1.b

Given the value of the variable, evaluate algebraic expressions with non-negative rational numbers with respect to order of operations, which may include absolute value.

Extended: Given the positive integer value of the single variable, evaluate an addition or subtraction expression.

Scaffolding Activities for the Extended Indicator

❑ **Evaluate addition expressions for a given variable value.**

- Present the expression 16 + *x* on cards. Explain that *x* is called a variable and represents a number in math expressions. Consider demonstrating with manipulatives and/or relating the expression to a real-world scenario such as, “There are sixteen cars in a group and more cars are going to be added to the group. The number to be added is not yet known, and the unknown number to be added is represented with a variable, such as *x*.”

16	+	*x*

Demonstrate that the *x* can be replaced with any number by writing numbers on cards and covering the variable card with a numerical value card. Show *x* = 1 by covering the *x* with the card and emphasizing that the value of *x* is now 1. Repeat the process with other numerical values such as 2 and 3.

16	+	1
16	+	2
16	+	3

Repeat the process with different numbers to demonstrate that *x* can have any value. Emphasize that the variable can be replaced only by a numerical value, not by other math symbols (e.g., <, >, +, ÷).

- Continue to model replacing the variable with a numerical value and evaluating one-step addition expressions. Be sure to include examples with the variable as the first addend and the variable as the second addend. For example, model evaluating 5 + *x* and *x* + 5. Model evaluating the expressions using appropriate computation strategies including, but not limited to, manipulatives, pictures, number lines, and calculators.

- Present a set of cards labeled with both numerical values and math symbols and a one-step addition algebraic expression. Ask students to identify the cards that could be used to replace the variable.
- Ask students to evaluate one-step addition expressions for a given value of the variable.

❑ **Evaluate subtraction expressions for a given variable value.**

- Use numerical value and symbol cards to demonstrate how to evaluate a subtraction expression. Present the expression 9 – *x* on cards. Consider demonstrating with manipulatives and/or relating the expression to a real-world scenario such as, “Avery has nine pencils and gives some of them away. The number of pencils he will give away is unknown. The unknown number is represented by *x*.”

  $$\boxed{9} \quad \boxed{-} \quad \boxed{x}.$$

  Explain that *x* is a variable that can represent any number value. Demonstrate taking the *x* card away and replacing it with a numerical value card of 7. Evaluate the expression 9 – 7 to get 2.

  $$\boxed{9} \quad \boxed{-} \quad \boxed{7} \quad = 2$$

  Remove the number 7 card and replace it with the *x*. Repeat the process using different numerical value cards to demonstrate that the variable *x* can be any number.

- Continue to model replacing the variable with a numerical value and evaluating one-step subtraction expressions. Be sure to include examples with the variable as the minuend and the variable as the subtrahend. For example, model evaluating 15 – *x* and *x* – 2. Model evaluating the expressions using appropriate computation strategies including, but not limited to, manipulatives, pictures, number lines, and calculators.
- Present three cards that show a one-step subtraction expression with a variable. Then present a selection of cards with numerical values and symbols (e.g., <, >, +, ÷) written on them and ask students to identify the cards that could be used to replace the variable.
- Ask students to evaluate one-step subtraction expressions for a given value of the variable.

Prerequisite Extended Indicators

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

MAE 3.A.1.c—Solve one-step addition and subtraction equations using the digits 0–9, limited to equations with an unknown change or unknown result.

Key Terms

addition, evaluate, expression, subtraction, value, variable

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-c-overview

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-f-lesson-18

6.A.1.c

Use substitution to determine if a given value for a variable makes an equation or inequality true.

Extended: Use substitution to determine if a given value for a variable makes an equation true.

Scaffolding Activities for the Extended Indicator

❑ **Recognize that variables represent numeric values in expressions and equations.**

- Explain that one thing variables (letters) can be used for is as a symbol to represent any one of a set of numbers and these numbers are called values. Variables can be used in expressions and equations to represent specific unknown values. Also, explain that, since a variable is representing an unknown value, the choice of the variable or letter doesn't matter, as shown in these two equations.

$$6 + w = 30$$

$$6 + y = 30$$

- Explain that the variables in both equations represent the same value because the solutions to both equations are the same. The only difference is that a different variable are used to represent the unknown value in each equation.
- Present a problem where a variable could be used to represent a given mathematical situation. For example, 6 times the sum of some number and 8. In this case, the phrase can be changed to a variable expression by replacing "some number" with a variable. The phrase becomes $6(x + 8)$ when it is written as an expression that uses a variable for the unknown.
- Ask students to write an expression using a variable for a given statement. For example, change the statement "The product of 3 and a number" or "The sum of a number and 9 is equal to 12" or "The product of a number and 8 is equal to 16" or "5 less than some number."
- Demonstrate recognizing expressions and equations that use variables. For example, present a list of expressions and equations where some have variables and some do not have variables. Which expressions and equations have variables? Model the process of identifying the expressions and equations with variables.

$8 – 3$	$20 – 4(3)$	$5 + a = 11$	$18 \div 2 = 9$
$8 + s$	$12 – t = 5$	$9 – 6 = 3$	$5 \times 4 = 10b$

- Ask students to write examples of expressions or equations with variables.

❑ **Substitute values for variables in expressions.**

- Explain to students that when an expression has a variable or unknown value, a number can be given as a value to substitute for the variable in the expression. Then, the expression can be evaluated. The value of the expression will change when the value of the variable changes.

- Model substituting for the variable *w* in each expression when $w = 6$.

5 + *w*	**3*w***	**13 – *w***	**18 ÷ *w***	**(2 + *w*) (3)**
5 + (6) 11	3(6) 18	13 – (6) 7	18 ÷ (6) 3	(2 + 6) (3) (8) (3) 24

- Substitute 10 for *s* to determine the value of the expression.

$$22 - s + 1$$

- Ask students to use the different values for *b* to substitute in the expression $b + 8$ and match with the number that results when evaluating the expression with each value for *b* that is given.

$b = 3$	18
$b = 5$	15
$b = 10$	13
$b = 7$	11

❑ **Determine whether a given value for a variable makes an equation true.**

- Use different values for a variable in an equation to show students how to determine whether the resulting equation is true. Model substituting the different values for *x* shown in the given equation.

  $x = 3$ $\qquad$ $x = 7$ $\qquad$ $x = 5$

$$2x + 3 = 13$$

When using $x = 3$, the result looks like this.

$$2(3) + 3 = 13$$
$$6 + 3 = 13$$
$$9 = 13$$

Substituting 3 for *x* results in an equation that is not true, so *x* cannot equal 3 in this equation.

When using $x = 7$, the result looks like this.

$$2(7) + 3 = 13$$
$$14 + 3 = 13$$
$$17 = 13$$

Substituting 7 for *x* results in an equation that is not true, so *x* cannot equal 7 in this equation.

When using $x = 5$, the result looks like this.

$$2(5) + 3 = 13$$
$$10 + 3 = 13$$
$$13 = 13$$

Substituting 5 for x results in an equation that is true. So, x does equal 5 in this equation.

- Demonstrate determining whether a given value for a variable makes an equation true. This table can be used to have students substitute values in different equations to show which values make equations true. Place the word "True" in the correct row and column for the value of x that makes the given equation true.

	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$x + 3 = 6$				
$5x = 5$				
$17 – x = 13$				
$12 + x = 14$				

- Ask students to match each equation with the value for y that makes each equation true.

$4y = 12$	9
$y + 11 = 20$	8
$20 – y = 12$	7
$35 \div y = 5$	3

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 4.A.1.e—Identify an addition or subtraction equation in an authentic mathematical situation using a variable for an unknown, limited to an unknown change or unknown result (e.g., $3 + n = 10$, $12 - 6 = n$).

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

Key Terms

equation, expression, substitute, substitution, unknown, value, variable

Additional Resources or Links

https://im.kendallhunt.com/MS/students/1/6/6/index.html

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Expressions/

6.A.1.d

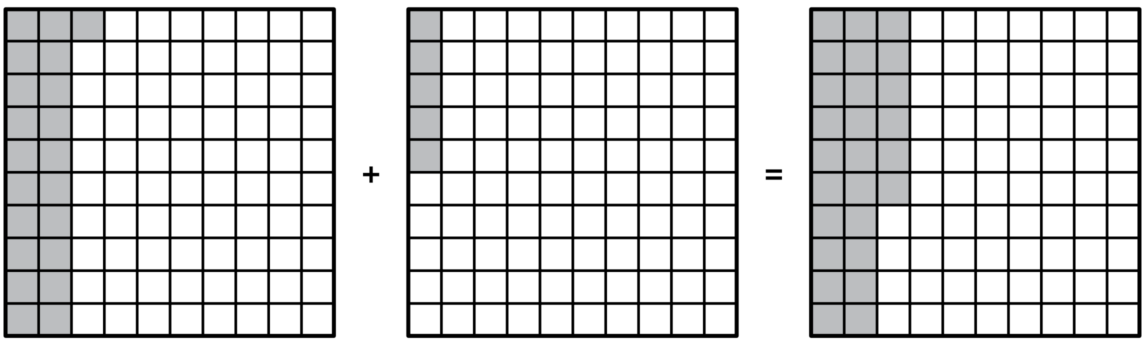
Solve one-step equations with non-negative rational numbers using addition, subtraction, multiplication, and division.

Extended: Add and subtract two decimal numbers without regrouping, limited to hundredths.

Scaffolding Activities for the Extended Indicator

❑ **Add numbers 0–1 with decimal numbers to the hundredth without regrouping.**

- Use a hundreds grid to demonstrate adding decimal numbers less than 1. Explain that a hundreds grid represents one whole, and each small square is $\frac{1}{100}$ of the whole, or 0.01. Shading in 21 of the squares represents 0.21, and shading in 5 of the squares represents 0.05. The model shown represents 0.21 + 0.05 = 0.26.



0.21 + 0.05 = 0.26

Continue to demonstrate solving a variety of addition problems using hundreds grids and sums less than 1.

- Ask students to identify the sum of two numbers 0–1 with decimal numbers to the hundredth represented on a hundreds grid.
- Ask students to add numbers 0–1 with decimal numbers to the hundredth without regrouping using hundreds grids.
- Use the standard algorithm to demonstrate adding decimal numbers less than 1. Present the addition problem 0.82 + 0.13 as shown.

$$\begin{array}{r} 0.82 \\ \underline{+\ 0.13} \end{array}$$

Emphasize that the decimals in each number must align vertically so that each place value is added to the same place value in the other number. So, for this example, the digits 2 and 3 in the hundredths place are added together to make 5 hundredths, and the digits 8 and 1 in the tenths place are added together to make 9 tenths. There are zeros in the ones place, so the final sum is 0.95. Emphasize that when writing the sum, the decimal point should align with the other decimal points.

$$\begin{array}{r} 0.82 \\ +\ 0.13 \\ \hline 0.95 \end{array}$$

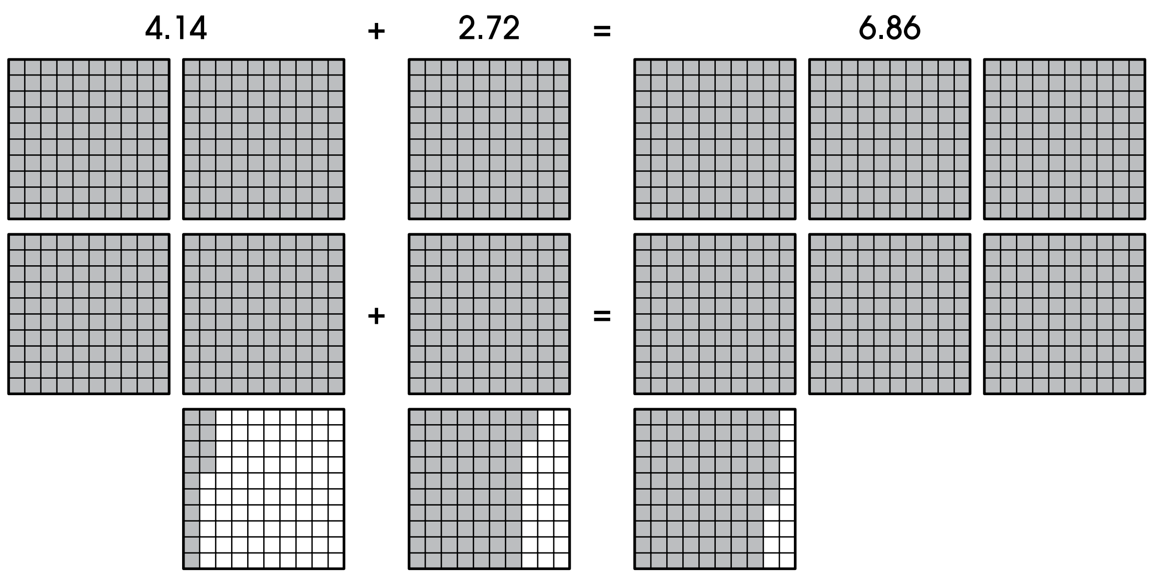
It might be helpful to use visual supports including but not limited to writing the addition problem on grid paper, writing the addition problem on a place-value mat, or using base-ten blocks to represent the problem on a place-value mat.

Continue to demonstrate solving a variety of addition problems involving decimal numbers less than 1, with hundredths and no regrouping.

- Ask students to add numbers 0–1 with decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

❑ **Add numbers greater than 1 with decimal numbers to the hundredth without regrouping.**

- Use strategies such as the hundreds grid and the standard algorithm to demonstrate adding numbers greater than 1 with decimal numbers to the hundredth. Present the problem 4.14 + 2.72, and demonstrate finding the sum using hundreds grids as shown.



As the numbers become greater, it gets more difficult to add them using hundreds grids. It might be helpful to first add together the whole numbers (i.e., the numbers to the left of the decimal point) and then use the hundreds grids for the tenths and hundredths places only. For this example, 4 + 2 = 6, and the bottom row of the figure can be used to find 0.86. Explain that combining the sum of the whole numbers and the sum of the decimals results in 6.86.

Then demonstrate how to use the standard algorithm to add numbers greater than 1 with decimal numbers to the hundredth. Present the problem 14.02 + 3.94 as shown and demonstrate finding the sum of each place value. Be sure to emphasize lining up the decimal points.

$$\begin{array}{r} 14.02 \\ +\ 3.94 \\ \hline 17.96 \end{array}$$

Continue to demonstrate solving a variety of addition problems with numbers greater than 1 with decimal numbers to the hundredth using models and the standard algorithm with visual supports as needed (e.g., grid paper, place-value mats, base-ten blocks).

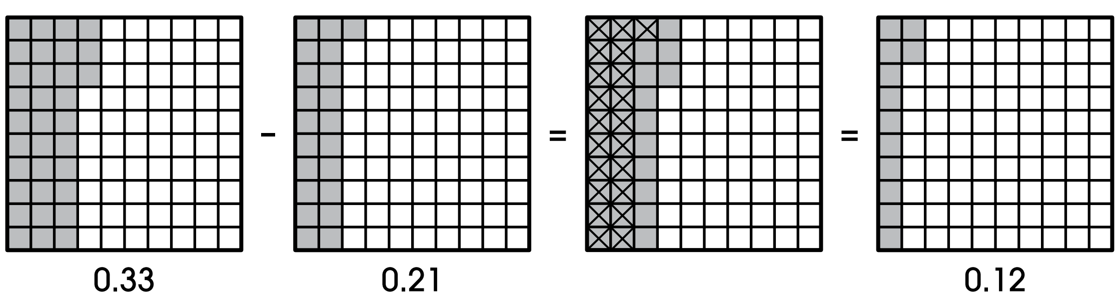
- Ask students to add numbers greater than 1 with decimal numbers to the hundredth without regrouping using a hundreds grid model.
- Ask students to add numbers greater than 1 with decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

❑ **Subtract decimal numbers to the hundredth without regrouping.**

- Use strategies such as the hundreds grid and the standard algorithm to demonstrate subtracting decimal numbers to the hundredth. Present the problem 23.89 – 3.77, and demonstrate using the standard algorithm to subtract 23.89 – 3.77. Emphasize the alignment of the decimal points, as well as the alignment of the digits of the same place value. Demonstrate starting with the place furthest to the right to calculate: 9 – 7 = 2 in the hundredths column, then 8 – 7 = 1 in the tenths column, 3 – 3 = 0 in the ones column, and 2 – 0 = 2 in the tens column. The difference can be written as shown.

$$\begin{array}{r} 23.89 \\ -\ 3.77 \\ \hline 20.12 \end{array}$$

Explain to students that hundred grids can be used for subtraction of decimal numbers only when the problem does not involve regrouping. For the problem 17.33 – 14.21, first demonstrate subtracting the whole numbers 17 – 14 = 3. The final answer will start with 3. Then use the hundreds grids to show 0.33 – 0.21.



Explain that putting together the difference of the whole numbers and the difference of the decimals results in 17.33 – 14.21 = 3.12.

Continue to demonstrate solving a variety of subtraction problems with decimal numbers to the hundredth without regrouping.

- Ask students to subtract decimal numbers to the hundredth without regrouping using a hundreds grid model.
- Ask students to subtract decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

Prerequisite Extended Indicators

MAE 5.N.3.g—Add and subtract two decimal numbers without regrouping, limited to 0–10 with at most one decimal place (e.g., 5.2 + 3.7).

MAE 5.A.1.c—Estimate the sum of two decimal numbers, limited to 0–10 with at most one decimal place (e.g., 5.2 + 3.7 is about 9).

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

Key Terms

add, decimal number, difference, hundreds grid, hundredth, place value, subtract, sum, tenth

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/NBT/B/7/tasks/1293

https://nysed-prod.engageny.org/resource/grade-5-mathematics-module-1-topic-d-overview

6.A.1.e

Solve one-step inequalities with whole numbers using addition, subtraction, multiplication, and division and represent solutions on a number line (e.g., graph $3x > 3$).

Extended: Identify a solution to an inequality on a number line from 0 to 10, limited to whole numbers (e.g., $x < 9$, $x \geq 3$).

Scaffolding Activities for the Extended Indicator

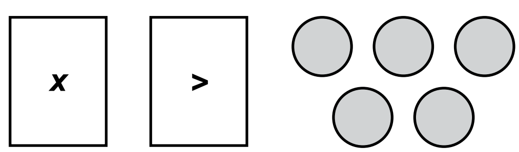
❑ **Identify an inequality in the form of *x* > (the integers 0 to 10).**

- Introduce the concept of an inequality as an expression showing that two quantities are not equal. The quantities are compared using inequality symbols. Use a table to display the inequality symbols, examples of inequalities, and word forms of the inequalities. Make connections between the symbols and the words that describe the symbols. The symbol "opens" to the greater value. When a variable is on the left side of an inequality and a number is on the right side, the variable is "less than" the number when the symbol opens to the right. The variable is "greater than" the number when the symbol opens to the left.

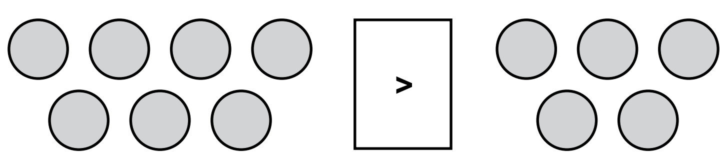
$<$	$>$	$\leq$	$\geq$
$x < 5$	$x > 5$	$x \leq 5$	$x \geq 5$
x is less than 5.	*x* is greater than 5.	*x* is less than or equal to 5.	*x* is greater than or equal to 5.

Explain that reading an inequality uses the specific vocabulary shown. Demonstrate how to read inequalities such as $x > 4$, $x < 9$, $x \geq 2$, and $x \leq 1$.

- Use manipulatives, a card with a variable, and a set of four cards with the inequality symbols to demonstrate that an inequality can have more than one solution that makes the inequality true.



Replace the variable card with manipulatives, such as 7 tokens, that make the number sentence true. Replace the 7 tokens with 9 tokens and indicate that the number sentence is still true. Be sure to reinforce the idea that there is more than one possible solution to an inequality.



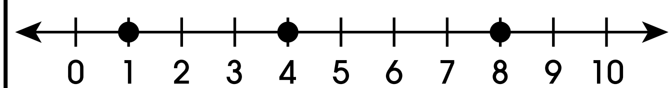
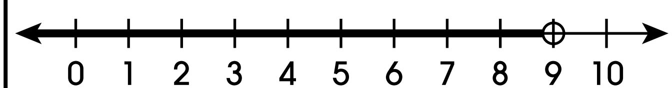
Continue the process with all the inequality cards.

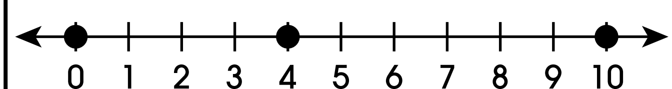
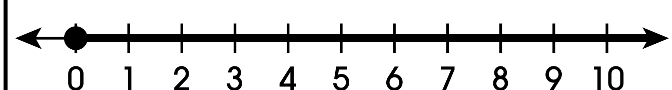
- Ask students to interpret an inequality and to identify manipulatives that are a solution to an inequality.

$x > 6$	$x \geq 10$	$x < 2$	$x \leq 7$

❑ **Identify a solution to an inequality on a number line (0 to 10).**

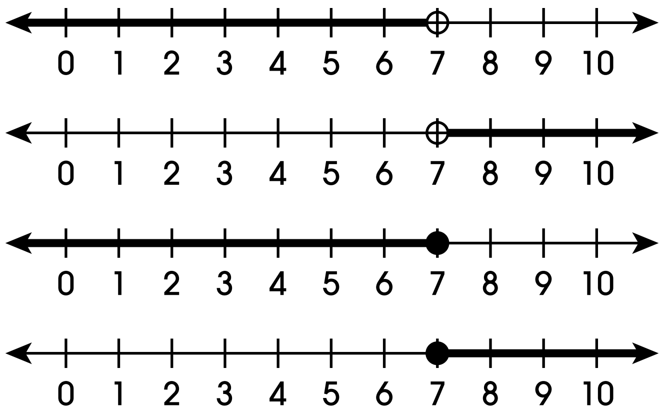
- Use tables to display example inequalities, a description of the solution, graphs of **some** solutions, and graphs of **all** solutions. Explain that the inequalities have too many solutions to list but that solutions can be graphed on a number line to show all the solutions. Some of the solutions may be plotted on the number line. All the solutions are shown with an arrow and either a closed circle or an open circle as the endpoint.

Inequality	$x < 9$
Description of solution	all numbers less than 9
Graph of <u>some</u> values of *x* that are solutions	
Graph of <u>all</u> values of *x* that are solutions	

Inequality	$x \geq 0$
Description of solution	all numbers greater than or equal to 0
Graph of <u>some</u> values of *x* that are solutions	
Graph of <u>all</u> values of *x* that are solutions	

In each graph, there is a circle drawn at the number given in the inequality. When an endpoint is <u>not</u> a solution, an open circle is used. When an endpoint is a solution, a closed circle is used. If the inequality uses < or >, the circle is open. If the inequality uses ≤ or ≥, the circle is closed. If the solutions are less than the number, the arrow points to the left. If the solutions are greater than the number, the arrow points to the right.

Continue to demonstrate number lines with various quantities and inequality symbols. For example, present four possible solutions to $x < 7$. Ask a series of questions to determine the correct solution. Are the solutions less than or greater than 7? Which direction should the arrow point to show solutions less than 7? The answers to the first two questions eliminate the second and fourth number lines. Does the solution include the number 7? The answer to the last question is no, so the circle should be open. The correct solution to $x < 7$ is the first number line. It may be helpful to create a checklist of questions to ask when determining the solution to an inequality on a number line.



- Ask students to identify the graph of the solution to an inequality.

Prerequisite Extended Indicators

MAE 6.N.1.c—Identify models of integers from –10 to 10 using drawings, words, manipulatives, number lines, and symbols.

MAE 6.N.1.e—Compare and order halves with halves, quarters with quarters, and tenths with tenths from 0 to 1 on a number line and compare and order integers from -10 to 10 on a number line.

MAE 4.N.1.b—Use symbols <, >, and = to compare whole numbers up to 50.

Key Terms

endpoints, equal, inequality, number line, solution, variable

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-lesson-1

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-overview

https://curriculum.illustrativemathematics.org/MS/students/1/7/8/index.html

6.A.2 Applications

6.A.2.a

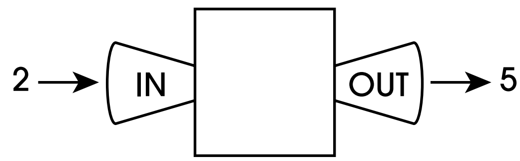
Create algebraic expressions (e.g., one operation, one variable as well as multiple operations, one variable) from word phrases.

Extended: Match a simple word phrase with an input-output box.

Scaffolding Activities for the Extended Indicator

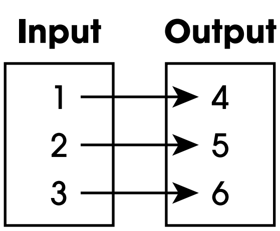
❑ **Match an addition word phrase with an input/output box.**

- Use a number machine (also called a function machine) to demonstrate an addition input/output pattern or rule. If a 2 goes in the machine, a 5 comes out.



  Since the number 2 gets larger when it comes out of the machine, the machine must add something to the 2. To get from 2 to 5, the machine must add 3. So, for this machine, if you put a 4 in, a 7 would come out because 4 + 3 = 7. The word phrase for this machine is "add 3," which can also be called the rule. Show a variety of number machines that have rules to add a value to the input number, such as "add 5" and "add 1."

- Ask students to match an addition word phrase with an input/output box. For example, present the following figure.



  This input/output box shows that when a 1 goes in, a 4 comes out; when a 2 goes in, a 5 comes out; and when a 3 goes in, a 6 comes out. Ask students to choose the word phrase that matches the input/output box.

  "add 2"
  "add 3"
  "add 4"

  Students should choose "add 3" as the correct word phrase because each input has 3 added to it to find the corresponding output. The rule in this input/output pattern is "add 3."

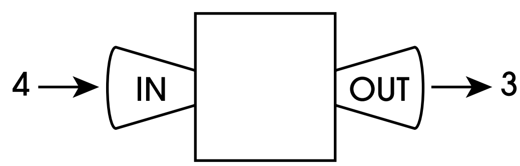
- ❑ **Match a subtraction word phrase with an input/output box.**

  - Use a number machine, an input/output box, or an input/output table to demonstrate a subtraction input/output pattern or rule. For example, present the following table.

Input	Output
7	5
8	6
9	7

Since each number in the input column corresponds to a smaller number in the output column, this table involves subtraction. Each number in the output column is 2 less than the corresponding number in the input column, so the word phrase (or rule) for this pattern is "subtract 2." Show a variety of subtraction examples.

  - Ask students to match a subtraction word phrase with a number machine, an input/output box, or an input/output table. For example, give students the following number machine.



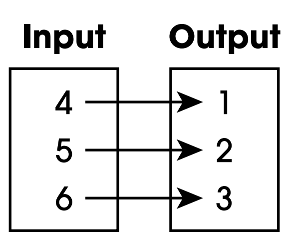
Have students choose one of the following word phrases to match the rule of the number machine. It may help to ask the question "What does the machine do to the number 4 to make it 3?"

"subtract 1"
"subtract 2"
"subtract 3"

Students should choose "subtract 1" as the rule for this input/output number machine.

- ❑ **Match a simple word phrase with an input/output box.**

  - Use a number machine, an input/output box, or an input/output table to model how to decide whether a number pattern shows an addition rule or a subtraction rule. For example, present the following input/output box.



First, show how to decide whether the phrase starts with “add” or “subtract” and then demonstrate finding the number value to add to the rule. Since each output number here is less than the corresponding input number, the word phrase will start with the word “subtract.” To find the number value needed, ask the question, “What number is subtracted from each number to get the new number?” In this case, that number is 2, so the rule for this example is “subtract 2.” Continue to demonstrate the process of identifying the pattern or rule with a variety of input/output boxes using both addition and subtraction.

- Ask students to match a simple word phrase with a number machine, an input/output box, or an input/output table. For example, present the following table.

Input	Output
1	6
2	7
3	8

Give students three word phrases to choose from: “add 3,” “add 4,” and “add 5.” Students should choose the word phrase “add 5” as the rule for this example.

Prerequisite Extended Indicator

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 3.A.1.c—Solve one-step addition and subtraction equations using the digits 0–9, limited to equations with an unknown change or unknown result.

Key Terms

add, input, output, pattern, rule, subtract, table

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/growing%20staircases.pdf

http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

http://tasks.illustrativemathematics.org/content-standards/5/OA/B/3/tasks/1895

6.A.2.b

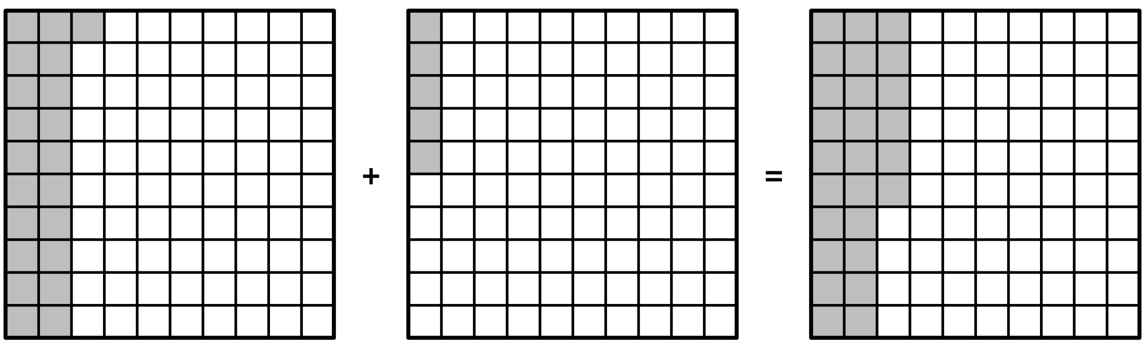
Write equations (e.g., one operation, one variable) to represent authentic situations involving non-negative rational numbers.

Extended: Solve authentic problems with addition and subtraction of decimal numbers to the hundredth, without regrouping.

Scaffolding Activities for the Extended Indicator

❑ **Using visual models, add and subtract decimal numbers to the hundredth, without regrouping.**

- Use a hundreds grid to demonstrate adding decimal numbers less than 1. Explain that a hundreds grid represents one whole, and each small square is 1/100 of the whole, or 0.01. Shading in 21 of the squares represents 0.21 and shading in 5 of the squares represents 0.05. The model shown represents 0.21 + 0.05 = 0.26.



0.21 + 0.05 = 0.26

- Continue to demonstrate solving a variety of addition problems using hundreds grids and sums less than 1. Use a variety of numbers so that some problems have numbers that go to the hundredths place, without regrouping.
- Ask students to identify the sum of two numbers 0–1 with decimal numbers to the hundredth place represented on a hundreds grid, without regrouping.
- Use the standard algorithm to demonstrate adding decimal numbers less than 1. Present the addition problem 0.82 + 0.15 as shown.

$$\begin{array}{r} 0.82 \\ \underline{+\ 0.15} \end{array}$$

Emphasize that the decimals in each number must align vertically so that each place value is added to the same place value in the other number. So, for this example, the digits 2 and 5 in the hundredths place are added to make 7 hundredths. The digits 8 and 1 in the tenths place are added to make 9 tenths. The final sum is 0.97. Emphasize that when writing the sum, the decimal point should align with the other decimal points.

$$\begin{array}{r} 0.82 \\ \underline{+\ 0.15} \\ 0.97 \end{array}$$

It might be helpful to use visual supports including but not limited to writing the addition problem on grid paper, writing the addition problem on a place-value mat, or using base-ten blocks to represent the problem on a place-value mat. Continue to demonstrate solving a variety of addition problems involving decimal numbers less than 1, with tenths or hundredths and no regrouping.

- Ask students to add and subtract with decimal numbers to the hundredths place without regrouping using the standard algorithm and visual supports as needed.

❑ **Solve authentic problems with addition and subtraction of decimal numbers to the hundredth, without regrouping.**

- Use story problems to demonstrate how to solve authentic problems with addition and subtraction of decimal numbers to the hundredth without regrouping.

Marcy has a new plant. It is 1.10 centimeters tall.

One month later, the plant is 6.35 centimeters tall.

How much did the plant grow in one month?

Explain that this problem involves subtraction because the difference between the two heights is needed. Model solving the subtraction problem using a hundreds grid or the standard algorithm to identify that the plant grew 5.25 centimeters in one month.

$$\begin{array}{r} 6.35 \\ \underline{-\ 1.10} \\ 5.25 \end{array}$$

- Continue to demonstrate solving a variety of real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping.

6.A.2 Applications

- Ask students to solve real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping. For example, present the problem shown with three answer options. Students should identify \$4.79 as the correct answer.

Brian has \$3.24 in his pocket.

He found \$1.55 in his room.

What is the total amount of money Brian has now?

$$\begin{array}{r} \$3.24 \\ +\ \$1.55 \\ \hline \end{array}$$

\$2.31

\$4.00

\$4.79

Prerequisite Extended Indicators

MAE 5.N.3.g—Add and subtract two decimal numbers without regrouping, limited to 0–10 with at most one decimal place (e.g., 5.2 + 3.7).

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

MAE 3.A.1.d—Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

Key Terms

add, decimal number, difference, hundredth, place value, subtract, sum, tenth

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_264_g_1_t_1.html?from=topic_t_1.html

https://tasks.illustrativemathematics.org/content-standards/5/NBT/B/7

6.A.2.c

Write inequalities (e.g., one operation, one variable) to represent authentic situations involving whole numbers.

Extended: Identify an inequality that represents a solution to a problem involving an authentic situation (e.g., x < 9, x ≥ 3).

Scaffolding Activities for the Extended Indicator

❑ **Identify a solution to an authentic situation involving the inequalities > or <.**

- Explain that an inequality is a statement that represents two quantities that are not equal and that the solution to an inequality is any value that makes the inequality true. Emphasize that the inequality symbol "opens" to the greater value. Present a table as shown to display two inequalities and word forms that may be used to describe them.

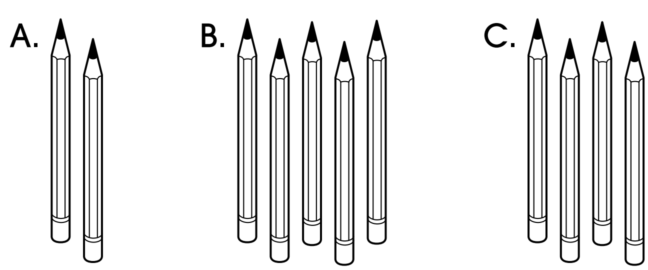
$x > 4$	$x < 4$
x is greater than 4. *x* is more than 4.	*x* is less than 4. *x* is fewer than 4.

Demonstrate identifying solutions to the inequality $x > 4$. The inequality symbol opens to the *x*, so any solution to the inequality must be a value for *x* that is greater than or more than 4. List possible solutions to the inequality $x > 4$ (e.g., 5, 15, 100).

Demonstrate identifying solutions to the inequality $x < 4$. The inequality symbol opens to the 4, so any solution to the inequality must be a value for *x* that is less than or fewer than 4. List possible solutions to the inequality $x < 4$ (e.g., 3, 2, 1).

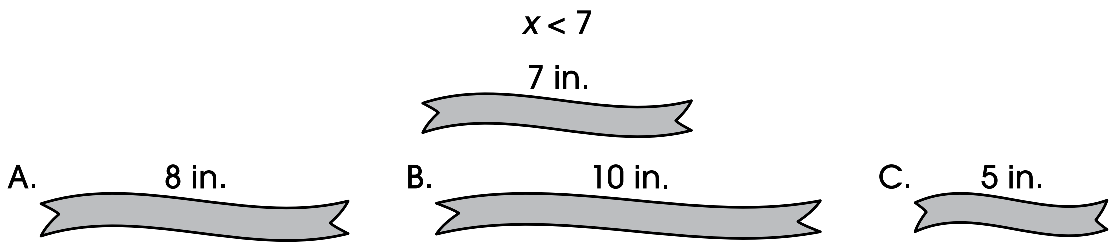
Explain that inequalities can be used to represent solutions to real-world problems. Refer to the inequality $x > 4$ and explain that this inequality can be used to represent the following problem: "Nina has more than 4 pencils. How many pencils could Nina have?" Present three options as shown and describe each option, explaining why the quantity may or may not correctly answer the problem. Be sure to emphasize why C is not the correct option.

$x > 4$



6.A.2 Applications

- Ask students to identify the model that represents the solution to the inequality $x < 7$ in the following word problem: “Jake has a piece of ribbon that is less than 7 inches long. How many inches long could his piece of ribbon be?”



- Ask students to identify the inequality that represents the solution to a word problem. For example, present the problem “The number of coins Kiara has is greater than 3. How many coins could Kiara have?” and the student should identify $x > 3$ as the solution.

  A. $x > 3$
  B. $x = 3$
  C. $x < 3$

❑ **Identify a solution to an authentic situation involving the inequalities ≥ or ≤.**

- Present a table as shown to display two inequalities and word forms that may be used to describe them.

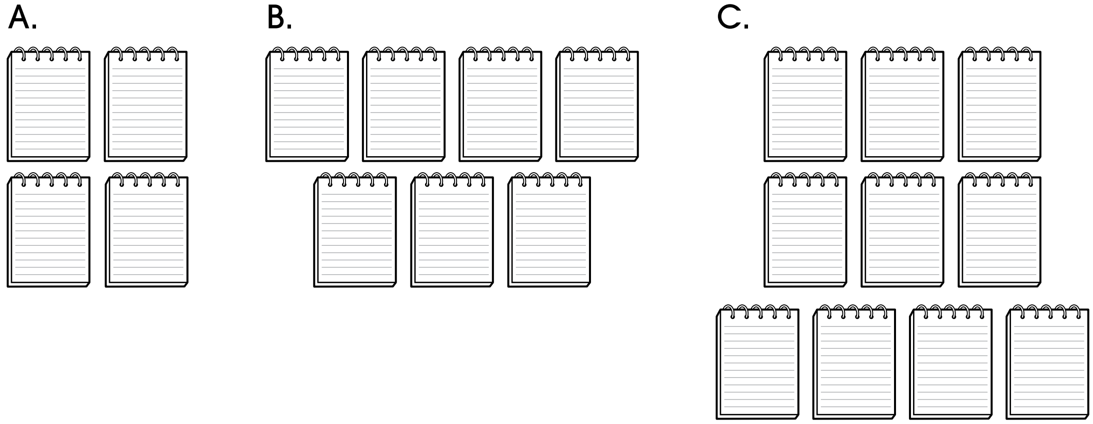
$x \geq 6$	$x \leq 6$
x is greater than or equal to 6. x is at least 6.	x is less than or equal to 6. x is at most 6.

  Demonstrate identifying solutions to the inequality $x \geq 6$. The inequality symbol opens to the x, so any solution to the inequality can be greater than 6. Explain that the bar under the symbol means that the solution could also be equal to 6. List possible solutions to the inequality $x \geq 6$ (e.g., 6, 10, 20).

  Demonstrate identifying solutions to the inequality $x \leq 6$. The inequality symbol opens to the 6 and has a bar underneath, so any solution to the inequality must be less than 6 or equal to 6. List possible solutions to the inequality $x \leq 6$ (e.g., 6, 4, 0). Emphasize that 6 is a possible solution to both the inequalities $x \geq 6$ and $x \leq 6$.

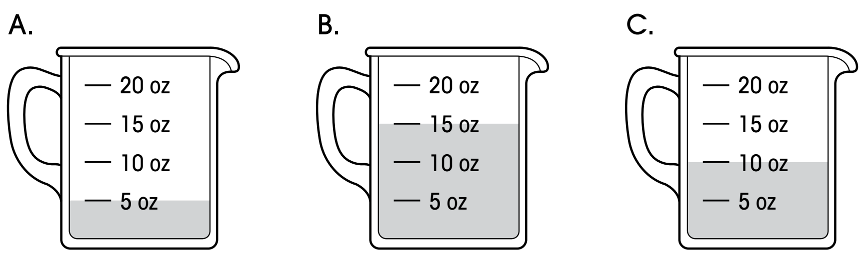
  Refer to the inequality $x \leq 6$ and explain that this inequality can be used to represent the following problem: “Eduardo keeps at most 6 notebooks in his backpack. How many notebooks could Eduardo have in his backpack?” Present three options as shown and describe each option, explaining why the quantity may or may not correctly answer the problem.

$x \leq 6$



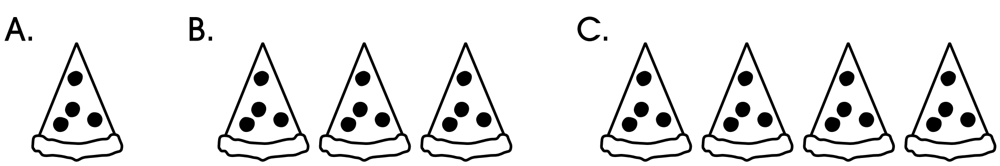
- Ask students to identify the model that represents the solution to the inequality $x \geq 12$ in the following word problem: "Mae pours at least 12 ounces of water into a measuring cup. How many ounces of water could Mae pour into the measuring cup?"

$x \geq 12$



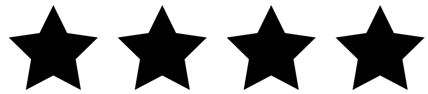
- Ask students to identify more than one model that represents a solution to the inequality $x \leq 3$ in the following word problem: "Craig orders a pizza. He wants to eat 3 or fewer slices of pizza. How many slices of pizza could Craig eat?"

$x \leq 3$



6.A.2 Applications

- Ask students to identify the inequality statement that is true in the following scenario, “Jason draws stars, as shown.”



Which statement is true?

A. Jason drew less than 2 stars.
B. Jason drew less than 4 stars.
C. Jason drew less than 5 stars.

Prerequisite Extended Indicators

MAE 6.A.1.e—Identify a solution to an inequality on a number line from 0 to 10, limited to whole numbers (e.g., $x < 9$, $x \geq 3$).

MAE 4.N.1.b—Use symbols <, >, and = to compare whole numbers up to 50.

Key Terms

equal, greater than, inequality, less than, solution

Additional Resources or Links

https://curriculum.illustrativemathematics.org/MS/students/1/7/8/index.html

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-h-lesson-33

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-10

Mathematics—Grade 6
Geometry

6.G.1 Attributes

6.G.1.a

Identify and create nets to represent two-dimensional drawings of prisms and pyramids.

Extended: Use two-dimensional representations (e.g., drawings, nets) and/or three-dimensional models to identify cubes, cylinders, cones, rectangular prisms, pyramids, and spheres.

Scaffolding Activities for the Extended Indicator

❑ **Identify characteristics of three-dimensional objects.**

- Use pattern blocks or cutout shapes of squares, circles, and triangles to demonstrate the concept of something being two-dimensional. Explain that the shapes are called "two-dimensional" because they have length and width but no depth (i.e., they are flat). Another way to think of two-dimensional is something that can be easily drawn on a piece of paper. Draw the shapes on paper and indicate the length and width.

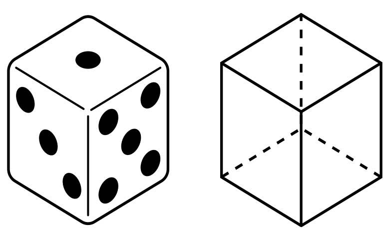
  Explain to students that "three-dimensional" means a shape has length, width, and depth. Present three-dimensional real-world objects or geometric solid figures and indicate the three dimensions.

- Compare a variety of objects that represent two-dimensional shapes with three-dimensional shapes and demonstrate identifying a shape as two-dimensional or three-dimensional. For example, compare a cutout square with a die or a cutout circle with a can.

- Present students with figures that represent two-dimensional shapes and three-dimensional objects and ask students to select the three-dimensional objects.

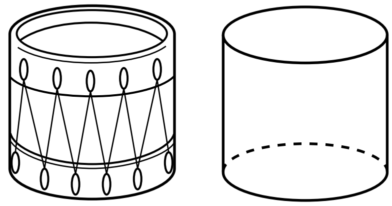
❑ **Identify cubes, cylinders, cones, rectangular prisms, pyramids, and spheres with two-dimensional representations and three-dimensional models.**

- Use a drawing of a cube and a real-world object to show what a cube looks like.



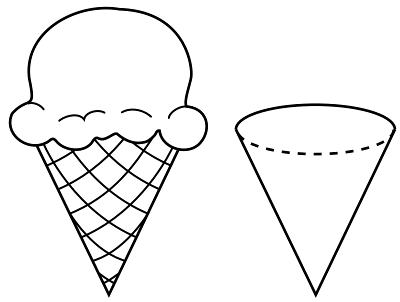
  A cube is a three-dimensional shape that has square faces. The faces are the six flat sides of the cube. Since it is difficult to draw something three-dimensional on paper, drawings of three-dimensional shapes usually have dashed lines or faded lines to help show the edges of the shape that cannot be seen without turning the shape over. Some examples of cubes are building blocks, ice cubes, and cardboard boxes with square faces.

- Use a drawing of a cylinder and a real-world object to show what a cylinder looks like.



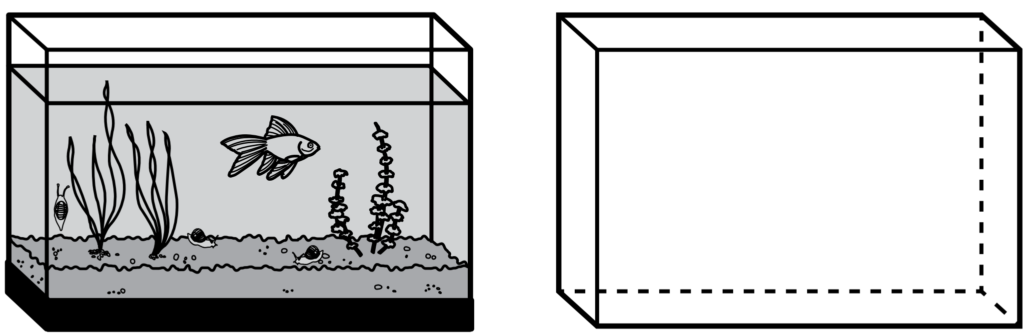
A cylinder is a three-dimensional shape that has two faces that are circles. In the drum shown above, the circles are the top and bottom. Some other examples of cylinders are buckets, cans of soup, and batteries.

- Use a drawing of a cone and a real-world object to show what a cone looks like.



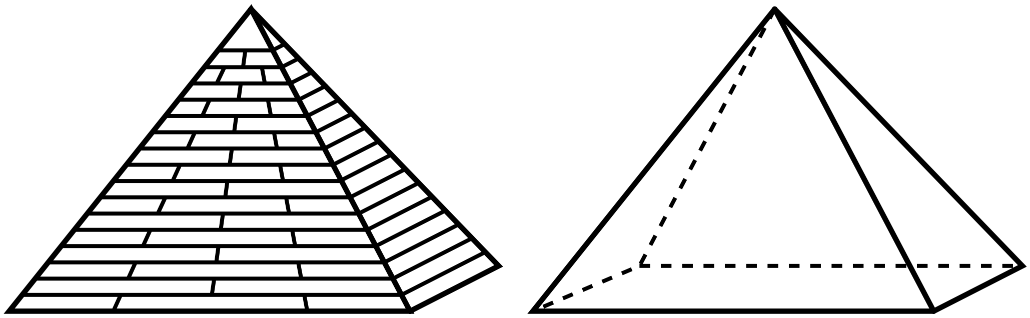
A cone is a three-dimensional shape that has one circle as a face and comes to a point (vertex) opposite the circle. Some examples of cones are traffic cones, party hats, and megaphones.

- Use a drawing of a rectangular prism and a real-world object to show what a rectangular prism looks like.



A rectangular prism is a three-dimensional shape that has 6 rectangular faces in which all the pairs of opposite faces are congruent. It has 8 vertices, 6 faces, and 12 edges. Some examples of rectangular prisms are fish tanks and shoeboxes.

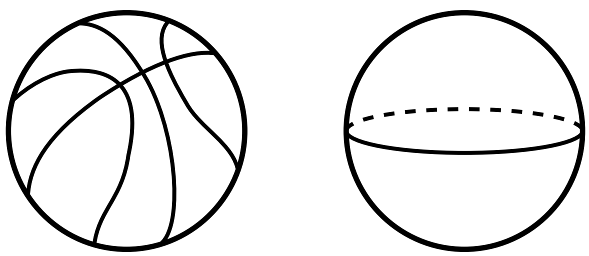
- Use a drawing of a pyramid and a real-world object to show what a pyramid looks like.



  A pyramid is a three-dimensional shape that has a polygonal base and flat triangular faces, which join at a point called the apex. An example of a pyramid is the Great Pyramid of Giza in Egypt.

- Use a drawing of a sphere and a real-world object to show what a sphere looks like.

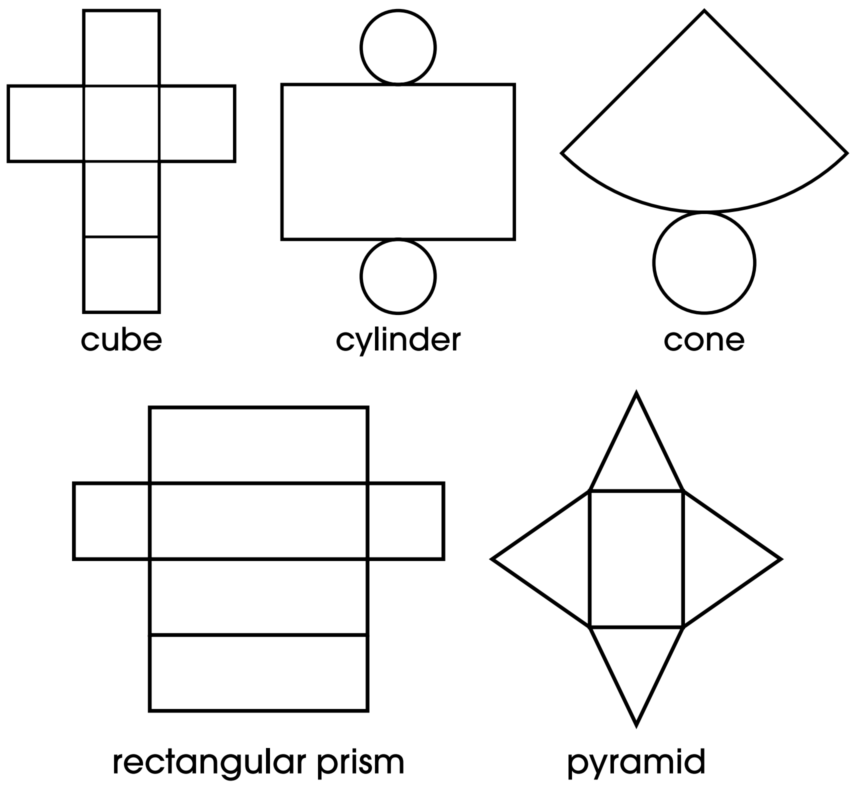
  A sphere is a three-dimensional shape with a curved surface. Unlike other three-dimensional shapes, a sphere has no faces, no edges, and no vertex. Some examples of spheres are marbles, a ball, an orange, and Earth.



- Ask students to identify a cylinder, a cone, a rectangular prism, a pyramid, and a sphere from a collection of three-dimensional models.
- Ask students to identify a cylinder, a cone, a rectangular prism, a pyramid, and a sphere from a collection of two-dimensional drawings.

- ❑ **Identify a cube, a cylinder, a cone, a rectangular prism, and a pyramid from a net.**

  - Use a net to make a three-dimensional model of a cube, a cylinder, a cone, a rectangular prism, a pyramid, and a sphere. Then show how each shape can be unfolded into the net shown. The net is a two-dimensional representation of the three-dimensional model when it is unfolded and laid flat. Emphasize the shape of each face. For the cube, each square of the net is a face of the cube. For the cylinder, the two circles are the top and bottom faces. For the cone, the circle shows the face of the cone that is flat. For a rectangular prism, each rectangle or square of the net is a face of the rectangular prism. For the pyramid, the rectangle is the base, and the four triangles are the faces. Explain to students that spheres are unable to be unfolded and laid flat since they do not have any faces or edges to do so with.



    When appropriate, show examples and offer opportunities to form three-dimensional models of cubes, cylinders, cones, rectangular prisms, and pyramids using a variety of sizes and two-dimensional net patterns.

  - Ask students to identify a cube, a cylinder, a cone, a rectangular prism, and a pyramid from a net.

6.G.1 Attributes

Prerequisite Extended Indicators

MAE 5.G.1.a—Identify the faces, edges, and vertices of cubes and other rectangular prisms.

MAE 5.G.1.b—Identify the difference between two-dimensional (flat) and three-dimensional (solid) figures.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

base, cone, cube, cylinder, face, net, pyramid, rectangular prism, sphere, three-dimensional, two-dimensional

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB1SUP-C1_Geometry3D-201304.pdf

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB1SUP-C7_MarDesc_3-D_Shapes-201304.pdf

https://www.engageny.org/resource/kindergarten-mathematics-module-2-topic

6.G.3 Measurement

6.G.3.a

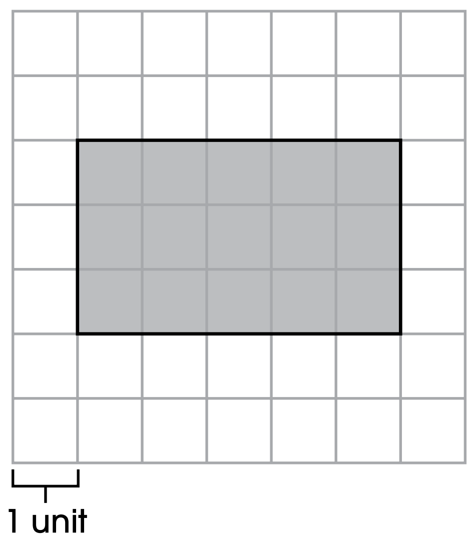
Determine the area of quadrilaterals and triangles by composition and decomposition of these shapes, as well as applications of properties and formulas. Quadrilaterals include parallelograms and trapezoids.

Extended: Find the area of a rectangle using its whole-number side lengths.

Scaffolding Activities for the Extended Indicator

❑ **Find the area of a rectangle by counting whole-number unit squares.**

- Use a rectangle drawn on a grid to count the number of unit squares that cover a rectangle. Draw and shade in a rectangle that is 3 units tall and 5 units wide on a grid. Explain that area is the number of unit squares that cover a shape.



Demonstrate creating a 3 × 5 array that represents the square units shaded to form the rectangle.

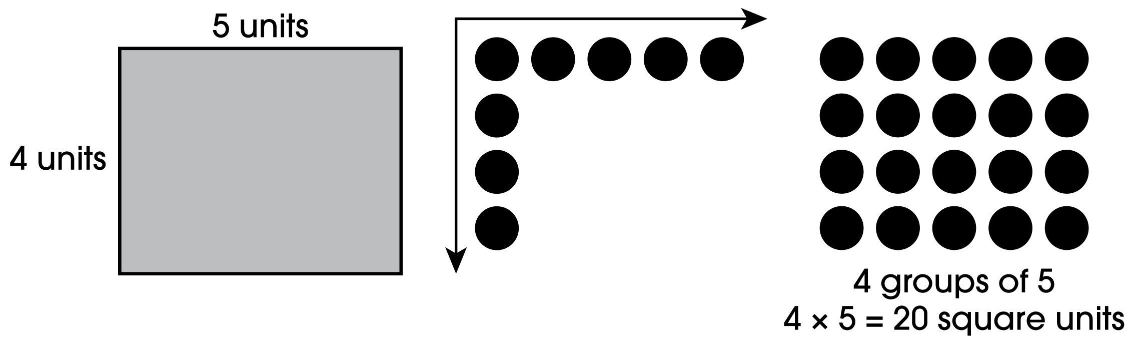
x	x	x	x	x	5
x	x	x	x	x	5
x	x	x	x	x	+ 5
					15 square units

Determine the area of the rectangle or the size of the array by counting, skip counting by 5, or using repeated addition. Identify 15 square units as the area of the rectangle.

- Ask students to construct arrays that represent the areas of rectangles presented on a grid.
- Ask students to calculate the areas of rectangles presented on a grid.

❑ **Find the area of a rectangle by using its whole-number side lengths.**

- Present a 4-by-5 rectangle labeled with side lengths. Explain that the side lengths indicate the number of rows and columns of square units that cover the rectangle. Create an array of dots to represent the square units.



- Demonstrate calculating the area of the rectangle by using the side lengths with an appropriate computation strategy, including but not limited to skip counting by 5 or by using a calculator, repeated addition, automaticity of multiplication facts, or a multiplication chart.
- Continue to demonstrate finding the area of a rectangle when given the side lengths. Continue to use arrays when necessary to support computation strategies and progress to finding the area without visual representations when appropriate.
- Ask students to create arrays when given the side lengths of a rectangle.
- Ask students to calculate the area of a rectangle when given whole-number side lengths.

Prerequisite Extended Indicators

MAE 5.A.1.a—Multiply the numbers 1–9 by single-digit numbers and 10, and multiply two-digit numbers 11–20 by single-digit numbers 1–5.

MAE 3.G.2.b—Find the area of a square or rectangle by counting whole-number unit squares.

MAE 3.G.2.c—Find the area of a square or rectangle with whole-number side lengths by counting unit squares and showing that repeated addition is the same as multiplying the side lengths.

Key Terms

array, length, multiply, rectangle, sides, square unit, width

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-4-topic-lesson-3

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-c-overview

6.G.3.b

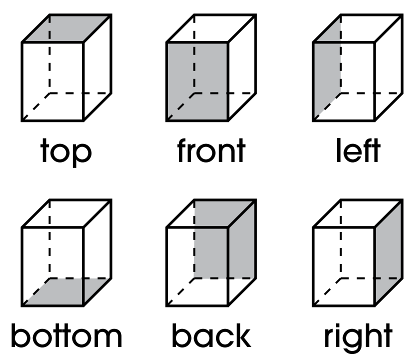
Determine the surface area of rectangular prisms and triangular prisms using nets as well as application of formulas.

Extended: Find the surface area of a rectangular prism by counting unit squares in a net of the figure.

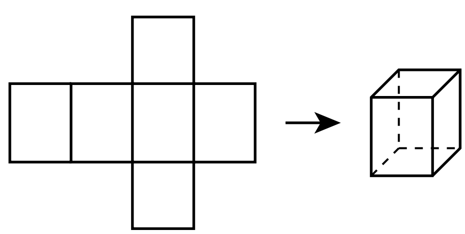
Scaffolding Activities for the Extended Indicator

- ❑ **Identify the six faces (sides) of a rectangular prism represented in a net figure.**

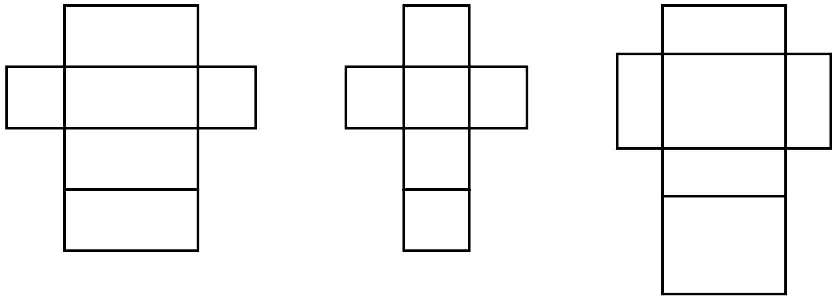
  - Explain that three-dimensional shapes are solid shapes. A rectangular prism is one type of three-dimensional shape. Show real-life examples and images of rectangular prisms (e.g., tissue box, cereal box, fish tank).
  - Use a rectangular prism manipulative or one made from a net to show the six faces. Explain that faces are the flat sides of the rectangular prism and that each face in a rectangular prism is a rectangle. Label each of the faces with tape or sticky paper.



  - Ask students to identify the six faces on a model of a rectangular prism.
  - Demonstrate folding the net of a rectangular prism into a three-dimensional model. Repeat with several examples of nets. Be sure to emphasize the rectangular shape of each section of the flat figure (net) and the rectangular shape of each face of the three-dimensional figure as the nets are folded and unfolded.

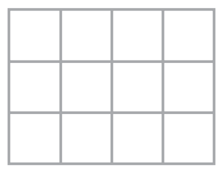


- Ask students to identify the six rectangles in a net of a rectangular prism.



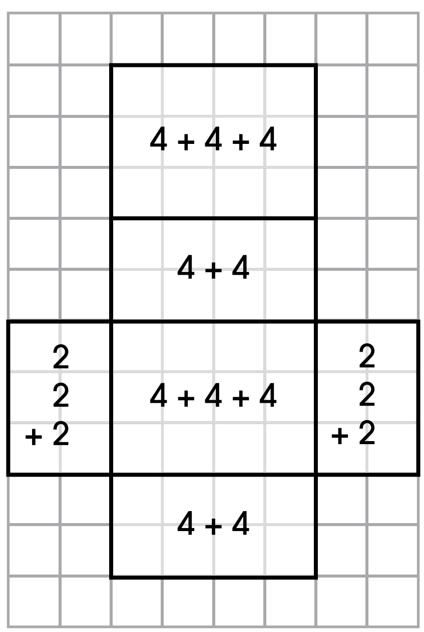
❑ **Find the surface area of a rectangular prism by counting unit squares.**

- Explain that the area of a rectangle is the count of how many square units cover the surface of the rectangle. Demonstrate counting the squares of the rectangle shown one row at a time and adding to find the total area equals 12 square units.



$4 + 4 + 4 = 12$

- Ask students to find the area of a rectangle by counting unit squares.
- Explain that the area of all the faces of a rectangular prism combined is called the surface area. Use the example below to demonstrate finding the surface area of a rectangular prism when given a net on a grid.



$$\begin{array}{c} 2+2+2=6 \\ 2+2+2=6 \\ 4+4+4=12 \\ 4+4=8 \\ 4+4+4=12 \\ 4+4=8 \\ \hline 52 \end{array}$$

Count the left side and the right side and then count the four remaining faces by starting at the top and moving down. Record the results of each side and then add to find the total surface area of 52 square units. Another strategy could be placing tick marks or dots in each square as it is counted and recording the results for each face to be added with a calculator. Continue modeling finding the surface areas of rectangular prisms by using appropriate counting and computation strategies, including counting, skip counting, repeated addition, and arrays.

- Ask students to find the surface area of a rectangular prism by counting unit squares in a net for each of the faces.

Prerequisite Extended Indicators

MAE 3.G.2.b—Find the area of a square or rectangle by counting whole-number unit squares.

MAE 3.G.2.c—Find the area of a square or rectangle with whole-number side lengths by counting unit squares and showing that repeated addition is the same as multiplying the side lengths.

MAE 6.G.1.a—Use two-dimensional representations (e.g., drawings, nets) and/or three-dimensional models to identify cubes, cylinders, cones, rectangular prisms, pyramids, and spheres.

Key Terms

area, face, rectangle, rectangular prism, side, square unit, surface area, three-dimensional shape

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-d-lesson-15

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-d-lesson-17

6.G.3.c

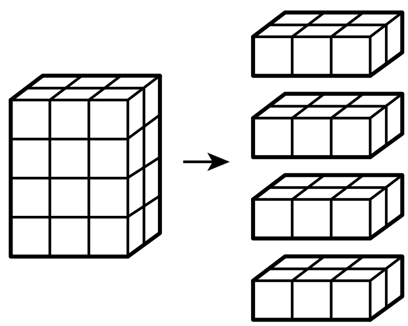
Apply volume formulas for triangular prisms.

Extended: Use the volume formula to determine the volume of a rectangular prism, limited to whole-number side lengths.

Scaffolding Activities for the Extended Indicator

❑ **Find the volume of a rectangular prism by counting unit cubes.**

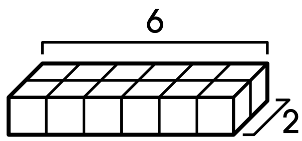
- Use a square or rectangular box to demonstrate that volume can be found by filling the box with unit cubes. Explain that volume is the amount of space inside an object. Stack unit cubes on top of one another and next to each other to fill the box. The number of unit cubes that fit into the box is the volume of the box.
- Use models of rectangular prisms built from unit cubes to show how to count the cubes to find the volume. Model finding the volume of a rectangular prism by counting the unit cubes in one layer and then skip counting to find the total. Explain that each layer has the same number of unit cubes.



- Ask students to determine the number of unit cubes in one layer of a rectangular prism.
- Ask students to determine the number of layers, or height, of a rectangular prism.
- Ask students to determine the total number of unit cubes, or volume, of a rectangular prism by counting the unit cubes. Encourage skip counting when appropriate.

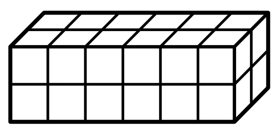
- [ ] **Find the volume of a rectangular prism using the volume formula.**

- Use unit cube models of rectangular prisms to point out that each layer is a rectangle. Demonstrate finding the area of a rectangle by multiplying the length and the width.



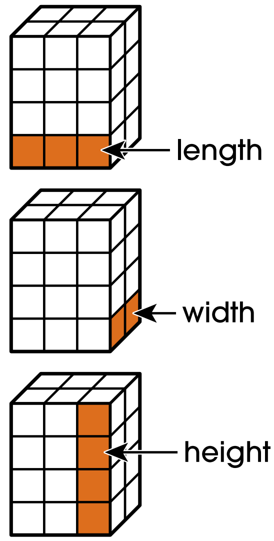
$$\begin{array}{r} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ +\ 2 \\ \hline 12 \end{array} \qquad 2 \times 6 = 12$$

  Next, explain that all the layers of rectangular prisms have stacked identical rectangular prisms. Model how the volume of a rectangular prism can be found by adding the area of the different rectangle layers that make up the prism. For example, a 2 × 2 × 6 rectangular prism has two layers, each with an area of 2 × 6, or 12 square units. To find the volume, add 12 + 12. The volume is found by repeated addition of the layers. Therefore, the volume is 12 + 12, or 24 cubic units.



- Ask students to determine the area of a rectangular face (or one layer) of a rectangular prism.
- Ask students to determine the number of layers, or height, of the rectangular prism.
- Ask students to determine the volume of the rectangular prism by using repeated addition of the area of one of the layers.

- Use unit cube models of rectangular prisms to demonstrate finding the volume of a rectangular prism by using the formula length × width × height. Model counting unit cubes to find the length, the width, and the height. Colored tape can be used to identify each measurement on a rectangular prism.



  Model appropriate computation strategies including the use of a calculator. It may be helpful to check calculated answers by counting individual unit cubes after a rectangular prism has been deconstructed.

- Ask students to determine the length, width, and height of a rectangular prism by counting unit cubes.
- Ask students to determine the volume of a rectangular prism by using the volume formula.

Prerequisite Extended Indicators

MAE 6.G.1.a—Find the area of a rectangle using its whole-number side lengths.

MAE 5.G.4.c—Use concrete and/or visual models to measure the volume of rectangular prisms by counting unit cubes.

MAE 5.G.4.d—Find the volume of a cube or another rectangular prism with whole-number side lengths by counting unit cubes and showing that repeated addition is the same as multiplying the side lengths (e.g., 9 + 9 + 9 = 27 unit cubes in a 3 × 3 × 3 cube).

Key Terms

cubic unit, face, height, layer, length, multiply, rectangle, rectangular prism, volume, width

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-5/file/117906

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB5SUP-D2_MeasVolume-201309.pdf

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Mathematics—Grade 6
Data

6.D.2 Analyze Data and Interpret Results

6.D.2.a

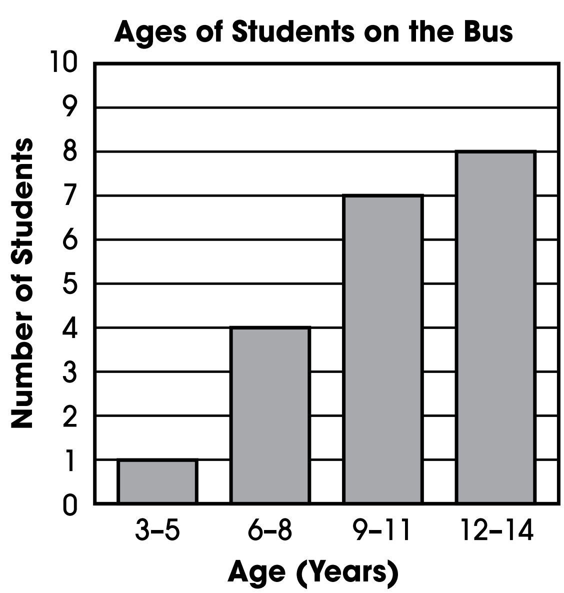
Represent data using dot plots, box-and-whisker plots, and histograms.

Extended: Identify characteristics (e.g., title, labels, intervals, quantities) of a histogram and identify a histogram that matches a data set.

Scaffolding Activities for the Extended Indicator

❑ **Identify characteristics of a histogram.**

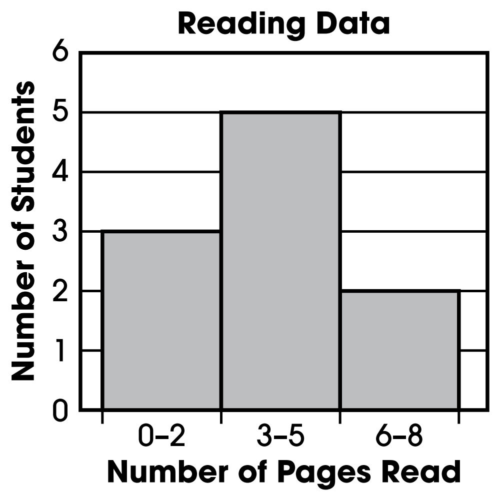
- Explain that a histogram is a chart that shows data grouped by intervals. Intervals include all the values between two numbers. Demonstrate an interval on a number line. For example, 0–2 is an interval that includes the numbers 0, 1, and 2. The interval 3–5 includes the numbers 3, 4, and 5.
- Explain that histograms need a title, labels, intervals, and quantities to display data and provide all necessary information. Present a histogram to students, as shown. Indicate its title and explain that it gives an overview of what is being shown in the histogram. Indicate its labels and explain how these provide further detail about what is being displayed with data. Indicate its intervals and explain that the vertical intervals show how much of something is being shown with the data, and the horizontal axis shows the range of the intervals. Indicate the quantities provided for each interval and explain that each bar is displaying the data.



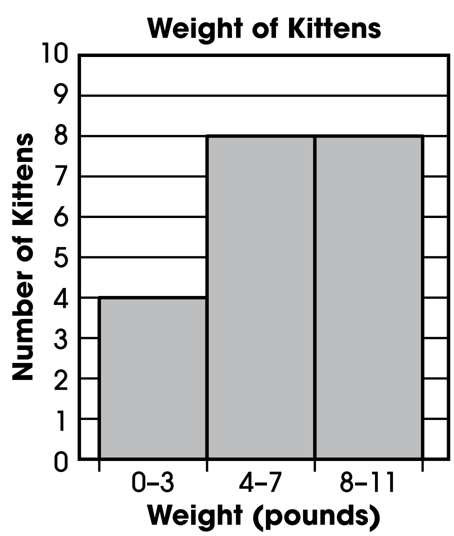
- Present various histograms to students that have different titles, labels, intervals, and quantities shown. Model identifying and labeling the different characteristics.
- Ask students to identify characteristics of a histogram.

- ❑ **Interpret information from a histogram.**

  - Explain that a histogram is a chart that shows data grouped by intervals. Intervals include all the values between two numbers. Demonstrate an interval on a number line. For example, 0–2 is an interval that includes the numbers 0, 1, and 2. The interval 3–5 includes the numbers 3, 4, and 5.
  - Present a histogram and show that the intervals are found on the *x*-axis. Explain that reading the labels on both axes will show what the intervals represent. Identify the intervals and describe them in the context of the labeling. For example, “This histogram has intervals of 0–2, 3–5, and 6–8. The label on the *x*-axis is ‘Number of Pages Read’ and the label on the *y*-axis is ‘Number of Students.’ The interval 0–2 indicates how many students read 0, 1, or 2 pages. The interval 3–5 indicates how many students read 3, 4, or 5 pages.”

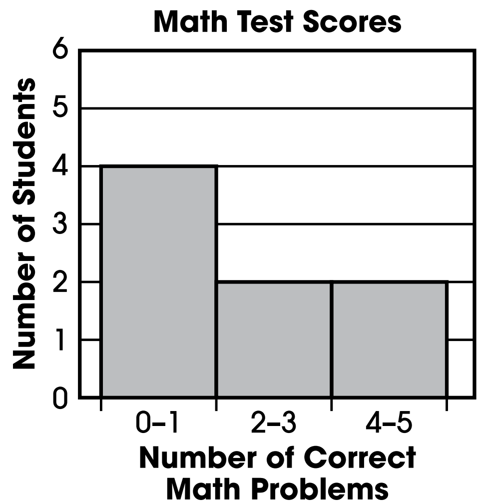


  - Ask students to identify the intervals and axis labels on a histogram.
  - Ask students to answer questions about the intervals of a histogram within the context of the labeling. For example, present the histogram shown and ask the following questions: “To which interval should data be added if a kitten weighs 3 pounds?” and “Will this histogram tell us the weight of a dog?”



- ❑ **Identify a histogram that matches a data set.**

  - Demonstrate how to determine whether a histogram matches a data set. Present a histogram as shown.



Present a table that matches the data in the histogram. Demonstrate matching the values in the table to the values on the histogram. Progress to presenting two tables (one table that correctly matches the histogram and one table that does not match the histogram) and asking a series of questions to determine which table matches the histogram. Repeat the process with a choice of three tables.

Correct

Number of Correct Math Problems	Number of Students
0–1	4
2–3	2
4–5	2

Not Correct

Number of Correct Math Problems	Number of Students
0–1	1
2–3	2
4–5	3

Not Correct

Number of Correct Math Problems	Number of Students
0–1	4
2–3	4
4–5	4

- Given a histogram and a set of three tables with the same numbers in all cells but different column headings (one table with column headings that match the histogram labels), ask students to identify which table matches the given histogram.
- Given a histogram and a set of three tables with column headings that match the histogram labels but with numbers that are different from table to table (only one table has numbers that match the histogram), ask students to identify which table matches the given histogram.

Prerequisite Extended Indicators

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.D.2.a—Solve problems with addition or subtraction of whole numbers using information from pictographs, bar graphs, and line plots.

MAE 3.D.1.a—Identify characteristics (e.g., title, labels, key, scale, quantities, categories) on a bar graph, pictograph, and circle graph.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Key Terms

data set, interval, label, range, title, *x*-axis, *y*-axis

Additional Resources or Links

https://www.engageny.org/file/45621/download/math-g6-m6-topic-a-lesson-4-teacher.pdf?token=j114C_-P

https://curriculum.illustrativemathematics.org/MS/students/1/8/6/index.html

6.D.2.b

Solve problems using information presented in dot plots, box-and-whisker plots, histograms, and circle graphs.

Extended: Solve problems using information presented in histograms and circle graphs, limited to halves, thirds, and fourths of a circle.

Scaffolding Activities for the Extended Indicator

- ❑ **Identify the components of a histogram.**

  - Present a histogram and a bar graph and explain the difference between the two graphic representations. Explain that a bar graph is a graph where the information on the *x*-axis is grouped by categories (e.g., names, colors, places, objects) and shows single values, while a histogram is a graph that shows data values grouped by intervals. An interval includes all the values between two numbers. Intervals can be modeled on a number line. For example, 0–2 is an interval that includes the numbers 0, 1, and 2. The interval 3–5 is an interval that includes the numbers 3, 4, and 5. Point to the intervals on a histogram to show that the intervals are found on the *x*-axis. Explain that for both a bar graph and a histogram, the information on the graph is interpreted by reading the title and the labels on the *x*- and *y*-axes.

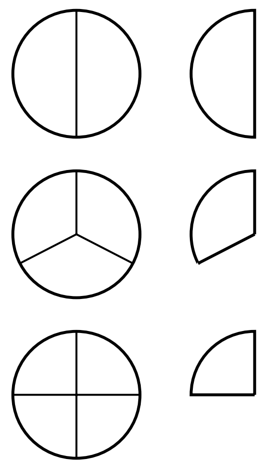
    Present the histogram shown. Identify the intervals on the histogram and describe them in the context of the labeling. For example, “This histogram has intervals of 0 to 1, 2 to 3, and 4 to 5. The label on the *x*-axis is ‘Number of Goals Scored,’ and the label on the *y*-axis is ‘Number of Soccer Games.’ So when the interval is 0 to 1, that shows how many games a soccer team scored 0 or 1 goals. When the interval is 2 to 3, that shows how many games the soccer team scored 2 or 3 goals.”

  - Ask students to identify on a given bar graph or histogram whether categories or intervals are represented by the data on the *x*-axis.

  - Ask students to answer questions about the labels and intervals on a histogram. For example, “What numerical values are included in the first interval?”

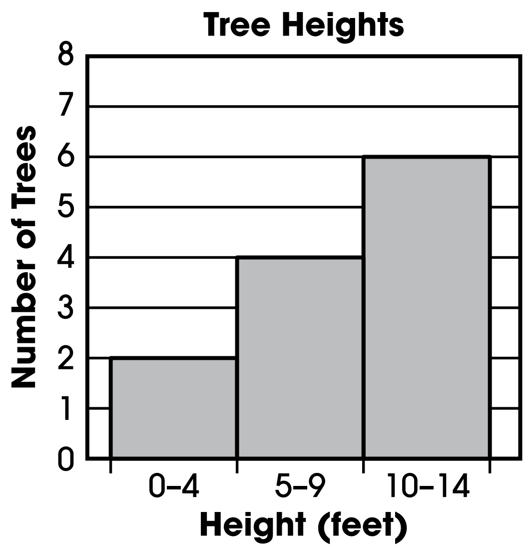
- ❑ **Recognize halves, thirds, and fourths of circles.**

  - Use circle manipulatives or cutouts to model thirds and fourths. Explain that when a circle is divided into three equal pieces, the pieces are called “thirds,” and when a circle is divided into four equal pieces, the pieces are called “fourths.” Emphasize that $\frac{1}{3}$ of a circle is larger than $\frac{1}{4}$ of a circle. It may also be helpful to reference the right angle at the vertex or $\frac{1}{4}$ of a circle and the obtuse angle at the vertex of $\frac{1}{3}$ of a circle to differentiate between the shapes and sizes of the fractional pieces.



  - Ask students to identify a circle divided into halves, thirds, and fourths when shown a collection of circles.
  - Ask students to identify which fractional piece is larger or smaller when comparing halves, thirds, and fourths.

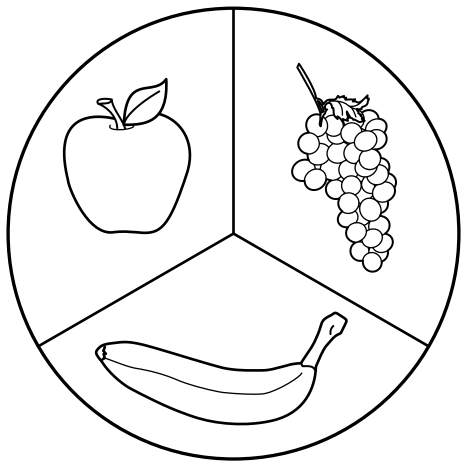
- **Solve problems by using information presented in histograms and circle graphs.**
  - Demonstrate solving problems based on data found in a histogram. Present the histogram shown. Model answering the following questions: “How many trees are between 5 and 9 feet tall?” and “Which interval has the least number of trees?”



  - Ask students to answer questions based on intervals, such as “How many ______ are in the interval ______ ?” within context.
  - Ask students to answer questions regarding which intervals or values are represented more/ most or less/least.

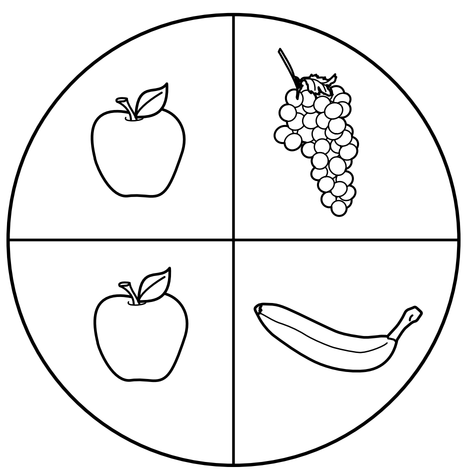
- Explain that the fractional pieces in a circle graph represent a quantity or an amount Explain that the fractional pieces in a circle graph represent a quantity or an amount compared to the whole. Present a circle graph depicting students' favorite fruits where $\frac{1}{3}$ like apples, $\frac{1}{3}$ like grapes, and $\frac{1}{3}$ like bananas. In this circle graph, each fruit was selected equally by the students because all the fractional pieces are equal. Guide students in answering the question "What fraction of the students picked grapes for a favorite fruit?" Repeat the question for apples and bananas.

Students' Favorite Fruit



Present a similar graph in which $\frac{2}{4}$ or half of the students like apples, $\frac{1}{4}$ like grapes, and $\frac{1}{4}$ like bananas. Explain that the fractional pieces are all equal, but since two fractional pieces show an apple, more students like apples than bananas or grapes. Guide students in answering these questions: "What fruit do most students like the best?" "What fraction of the students picked apples as the favorite fruit?" "What fraction of the students picked bananas as the favorite fruit?"

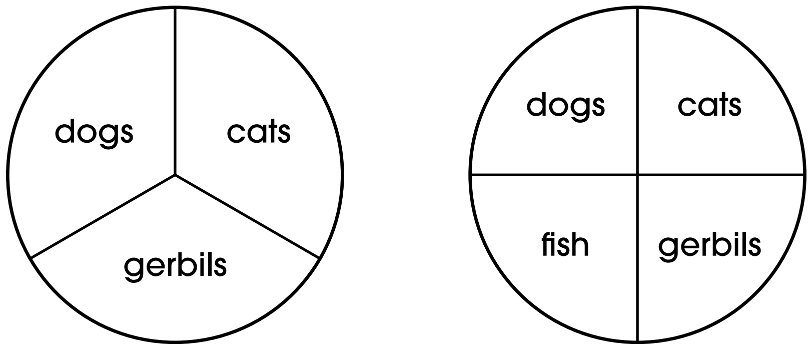
Students' Favorite Fruit



6.D.2 Analyze Data and Interpret Results

- Present two circle graphs as shown to demonstrate comparisons that can be made between the information in two circle graphs. Point out that one circle graph is divided into fourths and one circle graph is divided into thirds. Since thirds are larger than fourths, comparisons can be made between the information in the two circle graphs. Guide students in answering questions like this that compare information on the two circle graphs: “In which class does a larger fraction of students like dogs?”

5th Grade Favorite Pets **6th Grade Favorite Pets**



- Continue to present a variety of circle graphs divided into thirds or fourths. Be sure to use questions that address identifying the fractional part, comparing the size of fractional parts, and adding fractional parts with like denominators.
- Ask students to solve problems by using information presented in circle graphs divided into halves, thirds, and fourths.

Prerequisite Extended Indicators

MAE 6.D.2.a—Identify characteristics (e.g., title, labels, intervals, quantities) of a histogram and identify a histogram that matches a data set.

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.D.2.a—Solve problems with addition or subtraction of whole numbers using information from pictographs, bar graphs, and line plots.

MAE 3.D.1.a—Identify characteristics (e.g., title, labels, key, scale, quantities, categories) on a bar graph, pictograph, and circle graph.

Key Terms

bar graph, circle graph, category, fourth, *fraction, half,* histogram, interval, label, less, more, third, title, *x*-axis, *y*-axis

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-6-topic-lesson-4

https://curriculum.illustrativemathematics.org/MS/students/1/8/6/index.html

https://www.engageny.org/resource/grade-2-mathematics-module-8/file/15916

6.D.2.c

Find and interpret the mean, median, mode, and range for a set of data.

Extended: Find the mode and/or range of a set of ordered whole-number data.

Scaffolding Activities for the Extended Indicator

❑ **Identify the object that occurs the most in a set.**

- Present a set of four objects in which three objects are the same and one object is different. For example, present three pencils and one stapler. Identify the pencil as the object that appears the most in this set of school supplies. Present two pencils, one marker, and one stapler, and again identify the pencil as the object that appears the most.
- Repeat the process with a drawing of one circle and four squares, and identify the square as the shape that appears the most. Present a drawing of one circle, one triangle, and two squares, and again identify the square as the shape that occurs the most. Continue to demonstrate finding the object that appears the most in a set with larger sets and a variety of objects and drawings.
- Ask students to identify the object that appears the most in a set of four objects.
- Ask students to identify the object that appears the most in a set of five or more objects.

❑ **Find the mode of a set of ordered whole-number data.**

- Describe the mode of a data set as the number that is listed the most often. Make connections between “mode” and “most.” Demonstrate finding the mode in a whole-number ordered set of data when the frequency of the mode is much greater than the frequency of the other numbers in the data set. For example, present the set of numbers shown and identify the number 8 as the mode.

  {8, 8, 8, 8, 14}

  Continue to demonstrate finding the mode when the frequency of the mode is closer to the frequency of the other numbers in the data set. For example, present the data set {12, 14, 17, 17, 17} and then present the data sets {15, 19, 20, 20, 23} and {3, 3, 7, 7, 7, 12, 15}.
- Ask students to identify the mode of a set of four numbers when three of the numbers are the same.
- Ask students to identify the mode of a set of ordered whole-number data.

6.D.2 Analyze Data and Interpret Results

- ☐ **Identify real-world scenarios in which a range of numbers are relevant or useful.**
  - Present a set of four objects in which all four objects are different numbers/sizes. For example, present four shoes with four different shoe sizes. Identify the highest shoe size and the lowest shoe size. Subtract the lowest shoe size from the highest shoe size to identify the range of shoe sizes.
  - Repeat the process with varying numbers of crackers in snack bags. Present the number of crackers that are in each bag. Identify the bag with the highest number of crackers and the bag with the lowest number of crackers. Subtract the lowest number of crackers from the highest number of crackers to find the range of crackers in the snack bags.
  - Present a drawing of points scored in a basketball game. Find the highest and lowest scores in the set of points. Find the range by subtracting the lowest score from the highest score. Continue to demonstrate finding the range within a set when given a variety of objects or numbers.
  - Ask students to identify the highest number and lowest number in a data set.
  - Ask students to identify the range of a set of ordered whole-number data.
- ☐ **Find the range of a set of ordered whole-number data.**
  - Describe the range of a data set as the difference between the highest and lowest numbers. Demonstrate finding the range in a whole-number ordered set of data. For example, present the set of numbers shown and identify 8 as the highest number and 3 as the lowest number. Find the difference between 8 and 3 by subtracting 3 from 8 to find the range of the data set is 5.

    $$\{3, 4, 4, 6, 8\}$$

  - Continue to demonstrate finding the range with other numbers in a data set. For example, present the data set {2, 3, 5, 5, 9} and then present the data sets {4, 6, 3, 8, 10} and {9, 3, 4, 8, 1}.
  - Ask students to identify the highest number and lowest number in a data set.
  - Ask students to identify the range of a set of ordered whole-number data.

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

data, highest, lowest, mode, most, range, set

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/pick%20a%20pocket.pdf

https://www.insidemathematics.org/sites/default/files/materials/through%20the%20grapevine.pdf

https://www.map.mathshell.org/download.php?fileid=1619

6.D.2.d

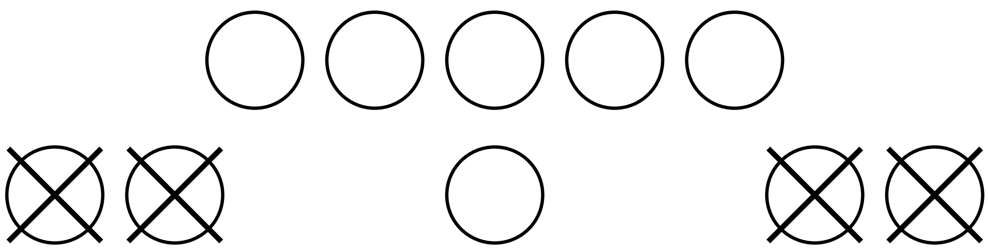
Compare the mean, median, mode, and range from two sets of data.

Extended: Find the median of a set of ordered whole-number data.

Scaffolding Activities for the Extended Indicator

❑ **Locate the middle object in a set of objects aligned horizontally.**

- Identify the middle of a group of objects. Present three objects and indicate the object in the middle. Continue to demonstrate identifying the middle object in a set of five objects and a set of seven objects. Explain that when there are more objects, a strategy can be used to locate the middle.
- Demonstrate finding the middle circle in a drawing of five circles aligned horizontally. For example, draw five circles as shown and cross out the circles one at a time from each end to find the middle circle of the set. Repeat the process with drawings of a set of seven circles and a set of nine circles.



- Ask students to locate the middle object in groups of three, five, seven, and nine objects aligned horizontally.
- Ask students to locate the middle shape in drawings of three, five, seven, and nine shapes aligned horizontally.

❑ **Find the median of a set of ordered whole-number data.**

- Describe the median of a data set as the number in the middle of the set when the numbers are ordered from least to greatest. Present the numbers 6, 7, and 8 and identify 7 as the median or the number in the middle. Present the numbers 5, 6, 7, 8, and 9 and explain that 7 is also the median in this set of numbers.

  Continue to demonstrate finding the median by presenting larger sets of ordered data. For example, in the set below, draw a line through the four numbers on each side of 18 to show 18 as the median of the set of data.

  6, 9, 13, 16, 18, 20, 21, 25, 28
  ~~6, 9, 13, 16~~, 18, ~~20, 21, 25, 28~~

Another strategy is to use two pieces of paper to cover one number at a time from the left and one number at a time from the right of the data set until only the number 18 remains in the middle.

6 9 13 16 18 20 21 25 28

Present the data set 14, 15, 19, 24, and 25. Demonstrate crossing one number off at a time from the left and from the right until only 19 remains. Explain that the strategy of crossing off numbers can be used to find the median of a set of numbers that are ordered from least to greatest. Continue to model with ordered data sets of three, five, seven, and nine numbers.

- Ask students to identify the median in an ordered data set of three numbers.
- Ask students to identify the median in an ordered data set of five, seven, or more numbers.

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 6.D.2.c—Find the mode and/or range of a set of ordered whole-number data.

Key Terms

data, median, middle, set

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/pick%20a%20pocket.pdf

https://www.insidemathematics.org/sites/default/files/materials/through%20the%20grapevine.pdf

6.D.3 Probability

6.D.3.a

Identify a list of possible outcomes for a simple event.

Extended: Identify a list of possible outcomes for a simple event, limited to four possible outcomes.

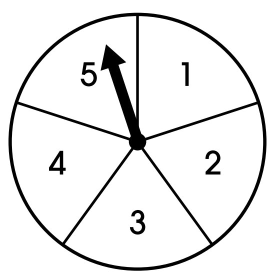
Scaffolding Activities for the Extended Indicator

❑ **Demonstrate that an event can have different outcomes by doing an experiment, e.g., spinner, coin toss, number cube, etc.**

- Demonstrate scenarios with manipulatives and drawings to identify the basic probability of outcomes. Use red and black tokens to define and demonstrate possible outcomes. For example, place one black token and one red token on a table. Pick up one of the tokens. Explain that picking up one token is an event and that the color of that token is an outcome of the event. Repeat with the other color token. Emphasize that there are two possible outcomes for this event because there are two different colors for the tokens.

- Continue to demonstrate determining the possible outcomes for an event by presenting a collection of manipulatives of three different colors or three different shapes. Explain that in this scenario there are three possible outcomes determined by the three different colors or shapes. Indicate each of the three possible outcomes if one manipulative is chosen at a time.

- Demonstrate determining the number of possible outcomes using a spinner. For example, present a spinner or a drawing of a spinner and indicate the number of possible outcomes. In this example, there are five possible outcomes.

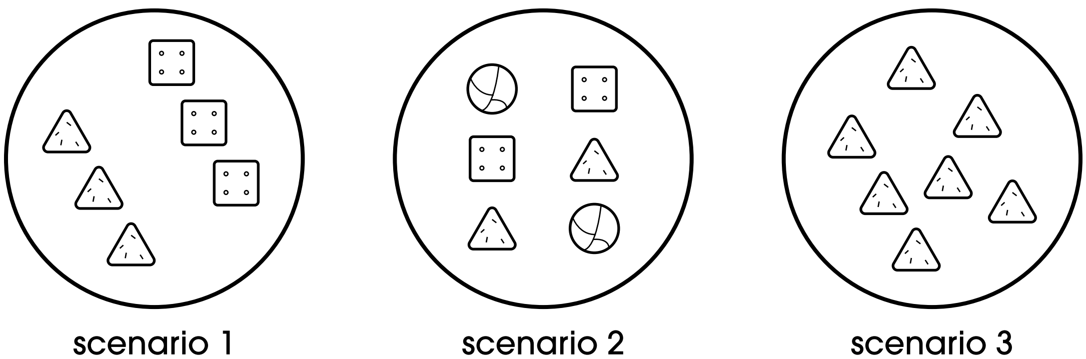
  Continue to demonstrate determining the possible outcomes and the number of possible outcomes using real-life scenarios (e.g., choice of different colors of the shirts or shoes) and other manipulatives.

- Ask students to determine the possible outcomes for an event (e.g., drawing objects from a bag, spinning a spinner, rolling a numbered cube).

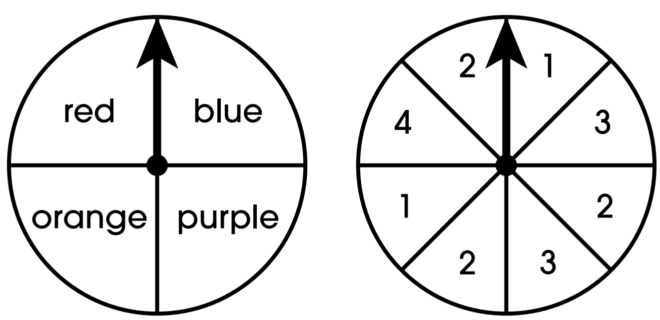
- Ask students to determine the number of possible outcomes for an event (e.g., drawing objects from a bag, spinning a spinner, rolling a numbered cube).

- [ ] **Identify possible outcomes for simple events.**

  - Use manipulatives to demonstrate a situation where students can determine all the possible outcomes for an event. For example, an opaque bag and 2 each of yellow, blue, red, and green marbles to describe a simple event when someone pulls a marble out of the bag without looking. Describe to students that since there are four different colors of marbles, there are four different possible results when a marble is pulled out of the bag. Those four possible outcomes are yellow, blue, red, and green. Any other color, such as orange, is not a possible outcome of this event.
  - Demonstrate determining the possible outcomes by using manipulatives and/or drawings of three different scenarios. For example, present the groups of snacks shown and discuss the list of possible outcomes of selecting different snacks for each group. Explain that the list of possible outcomes is dependent on the types of snacks that are available in each group. For example, in Scenario 3, the only possible outcome will be to get a chip as a snack but in Scenario 1, you could get a chip or a cracker.



  - Discuss other real-life situations where there can be different numbers of possible outcomes. For example, a soccer game is played, and the outcome can be that Team A beats Team B, Team B beats Team A, or Team A and Team B score the same number of goals and the game is a tie. So, there are three possible outcomes before the game is played.
  - Ask students to identify the number of possible outcomes when shown different spinners. For example, two different spinners are shown. The first spinner does not have any repeated values, so there are 4 possible outcomes. Even though the second spinner is divided into 8 sections, there are numbers that are repeated more than once, such as there being three 2s, and two 1s, which will impact how many possible outcomes there are. The second spinner has 4 possible outcomes.



6.D.3 Probability

Prerequisite Skills

identify objects as the same or different.

Key Terms

event, outcome, possible outcomes, probability, simple event

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_186_g_3_t_5.html?open=activities&from=topic_t_5.html

http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html?from=topic_t_5.html

https://illuminations.nctm.org/Search.aspx?view=search&st=d&gr=6-8&page=3

6.D.3.c

Express the degree of likelihood (possible, impossible, certain, more likely, equally likely, or less likely) of simple events.

Extended: Identify the probability of an event as always, sometimes, or never.

Scaffolding Activities for the Extended Indicator

❑ **Identify the probability of an event as always, sometimes, or never.**

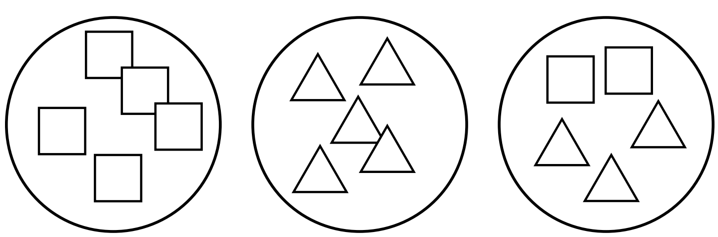
- Use manipulatives to demonstrate event probability. For example, use a chart, an opaque bag, and five each of yellow, blue, and green marbles to create scenarios to demonstrate the probability of always, sometimes, and never. Record the results in a table as shown for the following scenarios.

Turn	Yellow	Blue	Green
1			
2			
3			
4			
5			

  To demonstrate the probability of always and never, put only yellow marbles in the bag. Pick one marble at a time from the bag and record the results in the table. Return the marble to the bag each time and repeat the process at least three more times. Conclude that if the marbles are all yellow, the results will always be yellow. Extend the thinking to conclude that if the marbles are all yellow, the result will never be blue. The result will also never be green. If needed, repeat the entire demonstration using the blue marbles.

  To demonstrate the probability of sometimes, put the same numbers of blue, yellow, and green marbles in the bag. Pick one marble from the bag and record the result in the table. Return the marble to the bag and repeat the process until all the colors have been drawn. Conclude that if there are three colors of marbles in the bag, sometimes the result of the draw will be blue, sometimes it will be yellow, and sometimes it will be green.

- Demonstrate determining the probability of an event as always, sometimes, or never by using drawings that represent three different scenarios. For example, present the groups of shapes shown and discuss the probability of drawing a square from each group. Explain that the probability of drawing a square is “always” if the group only includes squares. The probability is “never” if the group does not have squares. The probability is “sometimes” if the group includes squares and triangles.



- Discuss real-life situations that occur always, sometimes, or never. For example, Avery rides his bike to school one day a week. The name of the day he rides always ends in the letter *y*, sometimes begins with the letter *T*, and never begins with the letter *B*.
- Ask students to identify the probability of an event as always, sometimes, or never when shown a collection of colored manipulatives in a bag. For example, place only blue marbles in the bag and ask students to determine whether the marble selected will always, sometimes, or never be blue.
- Ask students to select a scenario that represents always, sometimes, or never when given drawings of three different scenarios. For example, present three drawings, with the first drawing showing a group of circles, the second showing a group of rectangles, and the third showing a group of circles and rectangles. Ask students to identify which drawing represents the probability of sometimes selecting a circle.

Prerequisite Skill

MAE 6.D.3.a—Identify a list of possible outcomes for a simple event, limited to four possible outcomes.

Key Terms

always, event, never, outcome, probability, sometimes

Additional Resources or Links

https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-lesson-1/file/61366

https://www.engageny.org/resource/grade-7-mathematics-module-5-topic-lesson-3/file/61411

Alternate Mathematics
Instructional Supports
for
NSCAS Mathematics Extended Indicators
Grade 6