NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 3

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

NEBRASKA DEPARTMENT OF EDUCATION

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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 3
Number

3.N.1 Numeric Relationships

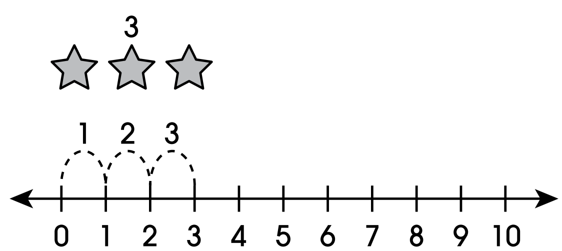
3.N.1.a

Read, write, and demonstrate multiple equivalent representations for the numbers up to 10,000 using objects or visual representations including standard form and expanded form.

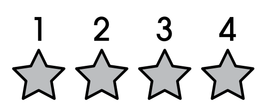
Extended: Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Scaffolding Activities for the Extended Indicator

- ❑ **Identify equivalent representations of whole numbers 1–10.**
  - Use a number line and objects to demonstrate counting to 3 on the number line and counting the number of stars. Count out loud to 3 and write "3."



  - Ask students to identify the word form of a whole number represented by a given set of objects. For example, ask students, "How many stars are there?"



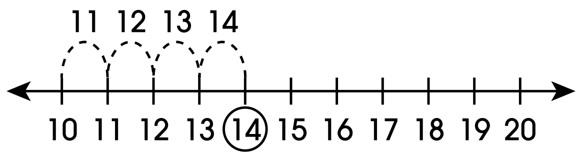
A	B	C
Two	Three	Four

  - Ask students to identify the word form and to write or identify the whole number represented by a given set of objects.

❑ Identify equivalent representations of whole numbers 11–20.

- Use a group of objects and a number line to show equivalent representations. For example, present 14 stars. Circle the group of 10. Then count on to 14 and write "14."

Demonstrate using a number line. Since the group of 10 was circled, and therefore established, count on from 10 to 14 on the number line.



- Ask students to identify visual models that represent values between 10 and 20.
- Ask students to identify the word form and to write or identify the whole number represented by a given set of objects.

Prerequisite Skills

- Perform the counting sequence by counting forward from any given number to 10.
- Demonstrate cardinality (i.e., the last number name said indicates the number of objects counted) in which up to 10 objects are counted.
- Use one-to-one correspondence when counting up to 10 objects.

Key Terms

group, set, standard form, whole number, word form

Additional Resources or Links

https://www.engageny.org/resource/kindergarten-mathematics-module-5-topic-lesson-1

https://www.engageny.org/resource/kindergarten-mathematics-module-5-topic-lesson-2

3.N.1.b

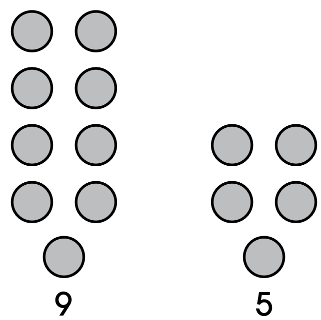
Represent and justify comparisons of whole numbers up to 10,000 using number lines and reasoning strategies.

Extended: Compare and order whole numbers 1–20 using number lines or quantities of objects.

Scaffolding Activities for the Extended Indicator

❑ **Compare whole numbers 1–20 using number lines or quantities of objects.**

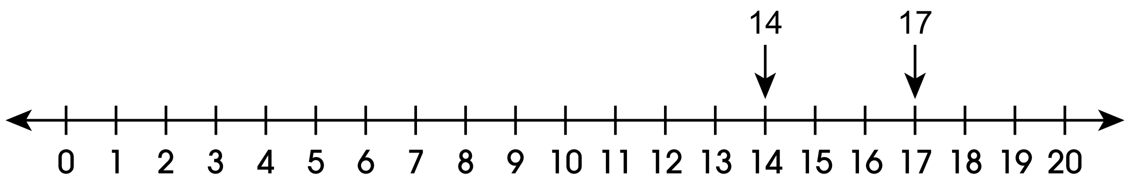
- Use manipulatives to compare whole numbers. For example, count the 9 objects on the left and the 5 objects on the right in the figure below. Demonstrate by counting and comparing visually that the group of 9 has more objects than the group of 5 and that 9 is greater than 5. Repeat by comparing other groups of objects and indicating which group has the lesser number.



- Ask students to compare two groups of objects. Which group has more objects? Which number is greater? Which group has fewer objects? Which number is less?
- Use a number line to compare the quantity of a set of objects. Demonstrate with manipulatives and a number line that more objects will equal a number that is farther away from 0 on the number line and that fewer objects will equal a number that is closer to 0 on the number line. For example, present 2 objects and locate the number 2 on a created or purchased number line slider board. Add more objects, and then locate that number on the number line. Continue adding objects and emphasizing that more objects means the number is farther from 0 on the number line. Repeat the process by starting with 20 objects and gradually taking away objects. Emphasize that fewer objects means the number is closer to 0 on the number line.
- Present a set of 6 objects. Indicate the number 6 on a number line, and then add more objects. Ask students to indicate the direction on the number line (to the right or farther away from 0) where the number for the additional objects will be located. Repeat by asking students to indicate where to find a number on a number line that equals fewer objects (to the left or closer to 0).
- Use number lines to compare two numbers. Present a set of 12 objects and a number line with the number 12 indicated. Also present 6 objects and a number line with the number 6 indicated. Compare the two sets of objects and the two number lines, demonstrating how to determine which number is greater and which number is less.

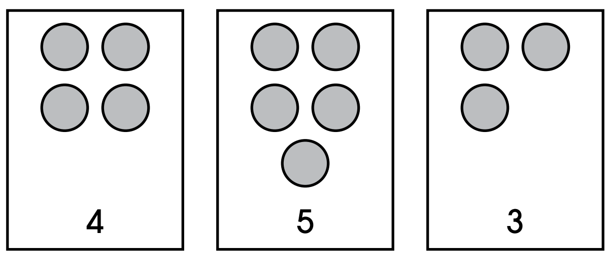
- Ask students to compare two numbers when presented with two sets of objects and two number lines with the number of objects indicated on the number line.

- Progress to comparing two numbers on one number line. For example, present a line with arrows pointing to 14 and 17 and ask students to identify which value is greater or which value is less.

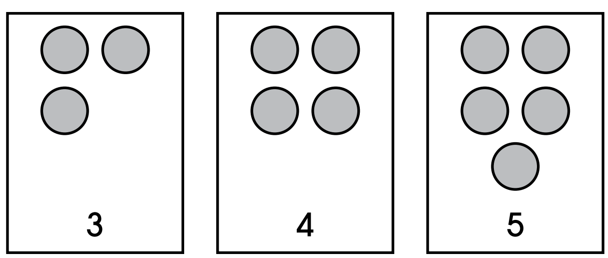


❑ **Order whole numbers 1—20 using number lines or quantities of objects.**

- Use manipulatives to order whole numbers. For example, present three cards with different quantities of objects.



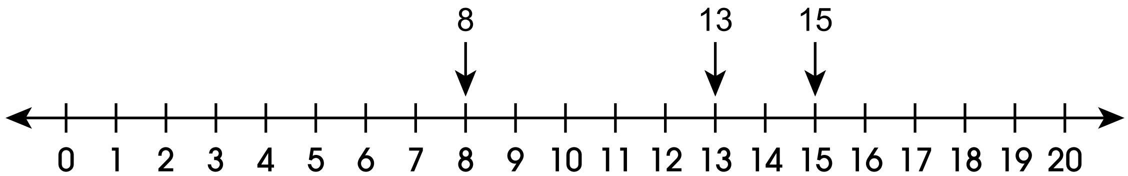
Demonstrate placing the cards in order from least to greatest.



- Ask students to order three number cards with objects from least to greatest.

3.N.1 Numeric Relationships

- Use a number line to order whole numbers. For example, present a number line with three numbers indicated. Write the numbers in order from least to greatest, explaining that the number with the least value is located farthest to the left (or closest to 0) and the number with the greatest value is located farthest to the right (or farthest from 0). Demonstrate selecting from three options the set of numbers correctly ordered from least to greatest.



Numbers in order: 8, 13, 15

15, 8, 13
8, 13, 15
15, 13, 8

- Ask students to order a set of three numbers from least to greatest when given a number line. When appropriate, progress to ordering a set of three numbers from least to greatest without a number line or visual representation of objects.

Prerequisite Extended Indicator

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Key Terms

greater, greatest, least, less, more, order

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/1/NBT/B/3/tasks/1102

http://nlvm.usu.edu/en/nav/frames_asid_334_g_1_t_1.html?from=topic_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

3.N.2 Fractions

3.N.2.a

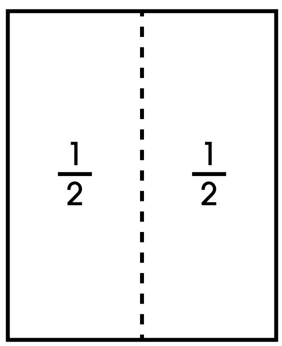
Partition two-dimensional figures into equal areas and express the area of each part as a unit fraction of the whole.

Extended: Partition two-dimensional figures (circles, triangles, rectangles, and squares) into two or four equal shares and express the area of each part as a unit fraction of the whole using $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$.

Scaffolding Activities for the Extended Indicator

❑ **Identify equal-sized parts on a partitioned circle, triangle, rectangle, and square.**

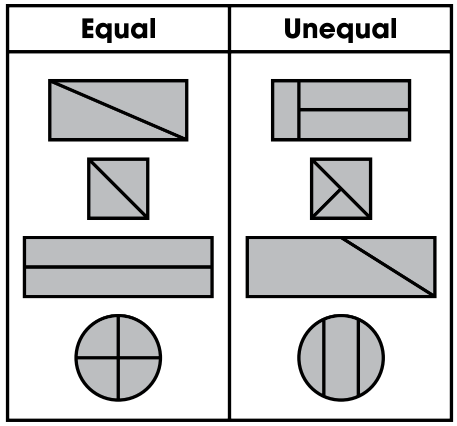
- Use cutout shapes to demonstrate folding a shape into equal-sized parts.
- Show students a piece of paper that is a square. Fold the square in half to demonstrate equal parts. Label each part of the folded square $\frac{1}{2}$.



Continue to demonstrate by folding a different square into thirds and then fourths. Demonstrate with other shapes. (e.g., rectangles, circles, triangles).

Next, fold shapes so unequal parts are created by the fold (e.g., fold a triangle parallel to the base, fold one corner of a square). Discuss how these parts cannot be labelled with equal fractions because the parts are not equal size.

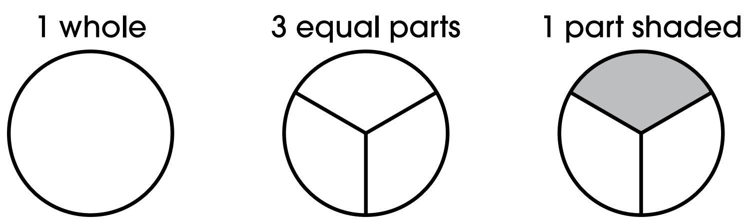
Demonstrate sorting shapes into the two categories equal-sized parts or unequal-sized parts. Continue to reinforce that only equal-sized parts are fractions and identify the fraction of each part.



- Ask students to identify whether a shape is divided into equal-sized parts or unequal-sized parts.
- Ask students to sort shapes into the two categories: equal-sized parts or unequal-sized parts.

❑ **Identify the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ as part of a whole.**

- Use drawings to demonstrate fractions as part of a whole.
- Draw a circle and label it “1 whole.” Draw another circle the same size. Draw lines on the circle to show three equal parts. Label this circle “3 equal parts.” Demonstrate counting the three parts of the circle and write the number three as the denominator for a fraction. Draw another circle the same size and draw lines to show three equal parts. Count the parts and write the number 3 as the denominator of a fraction. Shade in one part. Ask the students to identify how many parts of the circle are shaded. Write the number 1 as the numerator of the fraction.



- Repeat the process by drawing a new circle of the same size and draw lines to show three equal parts.
- Ask students to choose which section to shade. Reinforce that it doesn’t matter which section of the circle is shaded; the fractional part is still one-third.
- Continue to demonstrate the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, using circles, triangles, rectangles, and squares.

- Ask students to identify the number of equal-sized fractional parts in a circle, triangle, rectangle, or square that is divided into halves, thirds, or fourths.
- Ask students to identify the unit fraction $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ that is represented with fraction manipulatives or drawings of circles, triangles, rectangles, or squares.

Prerequisite Extended Indicator

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

circle, denominator, equal, fraction, triangle, numerator, part, rectangle, shape, square, unequal, whole

Additional Resources or Links

https://www.insidemathematics.org/inside-problem-solving/part-and-whole

https://im.kendallhunt.com/k5/teachers/grade-3/unit-5/lesson-2/lesson.html

3.N.2.b

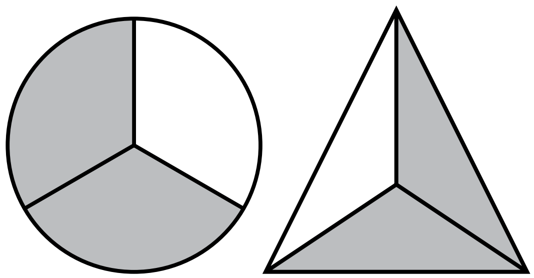
Find parts of a whole using visual fraction models.

Extended: Partition two-dimensional figures (circles, triangles, rectangles, and squares) into three, four, or five equal shares, and express the area of each part as a fraction of the whole using $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, or $\frac{4}{5}$.

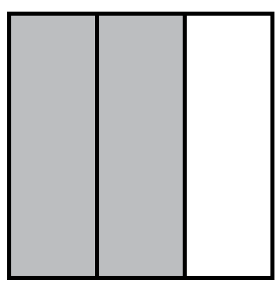
Scaffolding Activities for the Extended Indicator

❑ **Identify the fraction $\frac{2}{3}$ as part of a whole.**

- Use objects or visual models to demonstrate the fraction $\frac{2}{3}$ as part of a whole.
- Use a variety of shapes to demonstrate $\frac{2}{3}$ as part of the whole number one.
- Show students a variety of shapes as one whole. Separate those shapes into three equal parts. Shade two sections to represent $\frac{2}{3}$ of the whole.



- Continue to demonstrate a variety of examples involving identifying $\frac{2}{3}$ as part of a whole with multiple shapes.
- Use paper to evenly fold the shapes into three equal parts. Shade two parts.

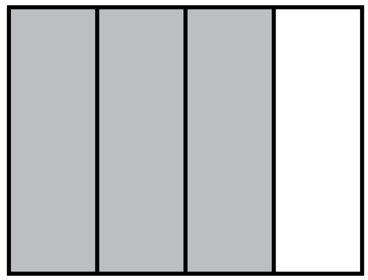


- Use objects to create visual models such as circles, triangles, rectangles, and squares to show $\frac{2}{3}$ of a whole.
- Demonstrate shading the fractional parts.
- Ask students to identify the number of equal-sized fractional parts in a whole circle, triangle, rectangle, or square that is divided into thirds.
- Ask students to identify various visual models that have $\frac{2}{3}$ of a whole shaded.

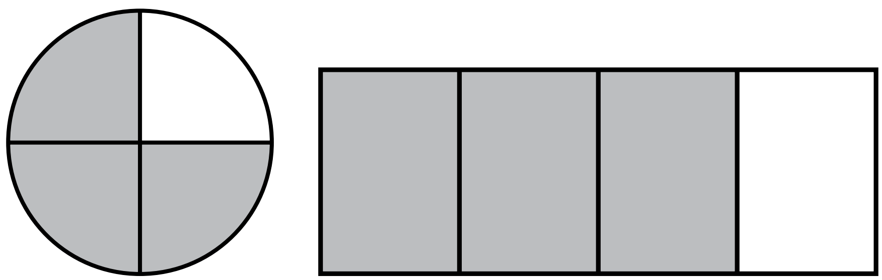
3.N.2 Fractions

- [ ] **Identify the fraction $\frac{3}{4}$ as part of a whole.**

  - Use objects or visual models to demonstrate the fraction $\frac{3}{4}$ as part of a whole.
  - Show students a shape represented as one whole. Separate that shape into four equal parts. Shade three sections to represent $\frac{3}{4}$ of the whole.



  - Continue to demonstrate what $\frac{3}{4}$ looks like as part of a whole, as shown, by shading the fractional parts of various visual models.

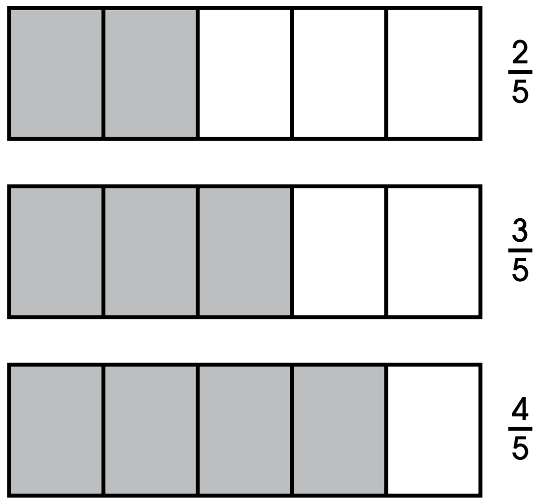


  - Repeat the process with different visuals of circles, triangles, rectangles, and squares.
  - Ask students to identify the number of equal-sized fractional parts in a whole circle, triangle, rectangle, or square that is divided into fourths.
  - Ask students to identify various visual models that have $\frac{3}{4}$ of a whole shaded.

3.N.2 Fractions

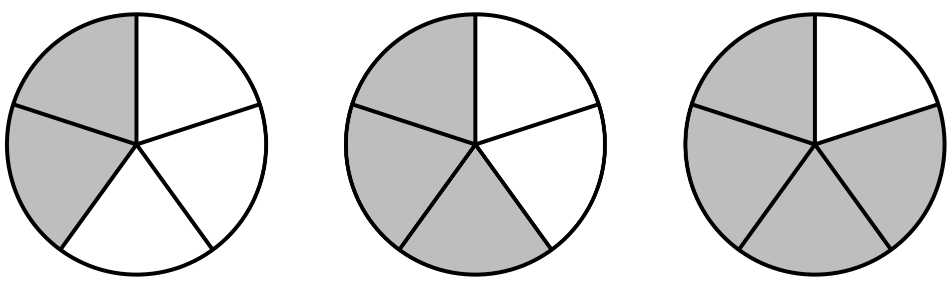
- **Identify the fractions $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ as part of a whole.**

- Use objects or visual models to demonstrate the fraction $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ as part of a whole.
- Use a variety of shapes to demonstraten $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ as part of the whole number one.
- Show students a shape as one whole. Separate that shape into five equal parts. Shade sections to represent the fractional part.



  Explain that the numerator, or top number of the fraction, will change as you shade another section but the denominator, or bottom number of the fraction, will remain the same.

- Continue to demonstrate a variety of examples involving identifying $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ as part of a whole.



- Use a variety of shapes to show $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ of a whole.

  Demonstrate shading the fractional parts.

- Ask students to identify a variety of shapes that have been divided into five equal parts, with either $\frac{2}{5}$, $\frac{3}{5}$, or $\frac{4}{5}$ of a whole shaded.
- Ask students to match already shaded visual models to the correct fraction of $\frac{2}{5}$, $\frac{3}{5}$, or $\frac{4}{5}$.

Prerequisite Extended Indicator

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.2.a—Partition two-dimensional figures (circles, triangles, rectangles, and squares) into two, three, or four equal shares and express the area of each part as a unit fraction of the whole using $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$.

Key Terms

denominator, fraction, fractional part, numerator, part, whole number

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB3SUP-A5_NumFractions-201304.pdf

https://www.engageny.org/resource/grade-3-mathematics-module-5-topic-b-lesson-6

3.N.2.c

Represent and understand a fraction as a number on a number line.

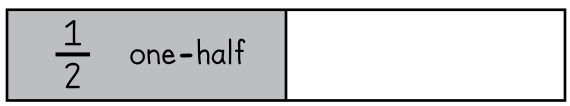
Extended: Represent halves and wholes on a number line.

Scaffolding Activities for the Extended Indicator

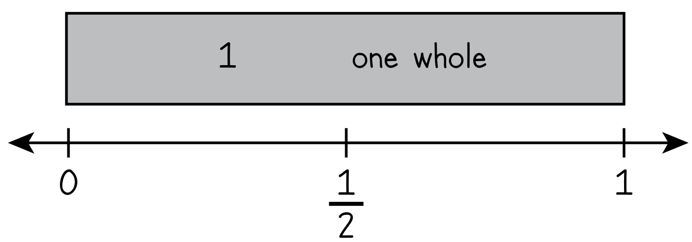
- ❑ **Create models of one whole and one-half using strips of paper and number lines.**

  - Show students one long rectangular strip of paper. Color in the whole piece. Label it with “1” and “one whole.”

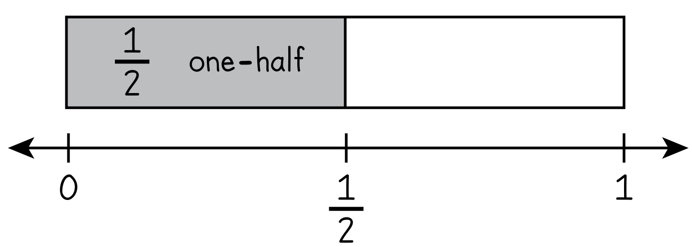
  - Show students how to fold a paper strip into two equal parts. Describe the parts as halves, stating each part is a fraction named one-half and together there are two parts or two halves that make a whole. Color half the strip and write the fraction “$\frac{1}{2}$” as well as the word “one-half.”



  - Ask students to create their own models of one whole and one-half using paper strips. Rectangles may already be partitioned.
  - Model how to create a number line from 0 to 1 using the paper strips.



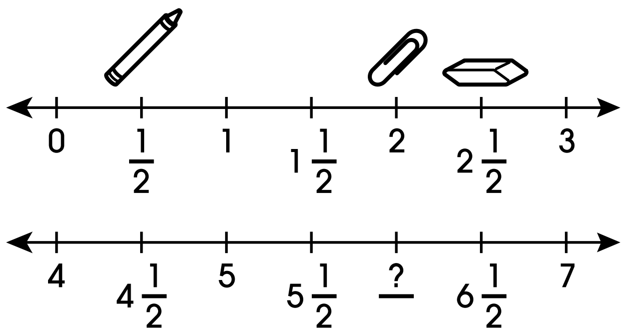
Repeat the procedure for one-half. Include a tick mark at 1/2 .



  - Ask students to create a number line from 0 to 1 using paper strips. Ask students to identify the correct label for each tick mark (0, $\frac{1}{2}$, 1).

❑ Identify halves and wholes on a number line.

- Demonstrate finding whole numbers and halves on a number line from 0 to 3 that is labeled in increments of one-half. Place or draw objects at different tick marks and then model identifying an object at a specific location (e.g., determine which object is located at two and one-half). Present a similar number line with one tick mark not labeled and demonstrate filling in the missing number.



- Ask students to identify an object located at a position on a number line that is labeled in increments of one-half.
- Ask students to fill in or select a missing number on a number line that is labeled in increments of one-half.

Prerequisite Skills

- Divide objects and images into two equal parts and use the language one-half, halves, and whole.

Key Terms

half, number line, whole

Additional Resources or Links

https://www.mathlearningcenter.org/apps/number-line

https://www.engageny.org/resource/grade-3-mathematics-module-5-topic-d-lesson-16/file/35386

3.N.2.e

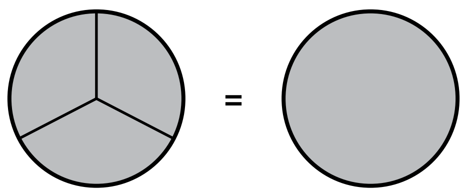
Justify whole numbers as fractions and identify fractions that are equivalent to whole numbers.

Extended: Given a model, represent a whole number (1, 2, or 3) as a fraction with a denominator of 2, 3, or 4.

Scaffolding Activities for the Extended Indicator

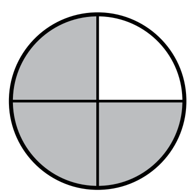
❑ **Use models to represent the whole number 1 as a fraction with a denominator of 2, 3, or 4.**

- Use circles to demonstrate the equivalence of fractions and the whole number 1. Use the graphic below to indicate that the 3 in the numerator represents the number of shaded pieces, the 3 in the denominator represents the number of equal-sized pieces the whole is divided into, and the whole number 1 represents the number of whole circles.



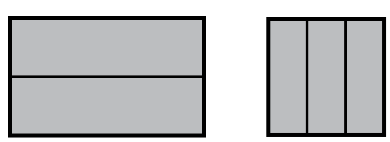
$\frac{3}{3} = 1$

Show counterexamples to students as well, such as $\frac{3}{4}$ does **not** equal 1 whole.



$\frac{3}{4}$ is not equal to 1

- Use rectangular and square models with denominators of 2, 3, and 4 to represent the whole number 1.



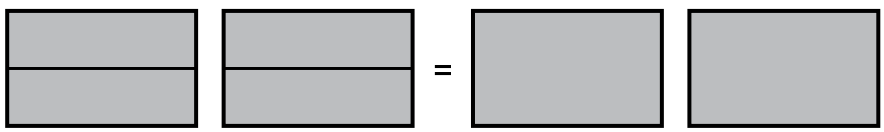
$\frac{2}{2} = 1$ $\frac{3}{3} = 1$

- Ask students to identify the fraction form of the whole number 1 represented with circles, squares, and rectangles with 2, 3, and 4 sections.

___ = 1　　___ = 1　　___ = 1

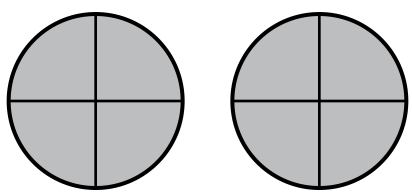
❑ **Use models to represent whole numbers 1, 2, or 3 as fractions with denominators of 2, 3, or 4.**

- Use rectangular models to demonstrate the equivalence of fractions and whole numbers. Use the graphic below to indicate that the 4 in the numerator represents the number of shaded pieces, the 2 in the denominator represents the number of pieces each whole is divided into, and the whole number 2 represents the number of whole rectangles.



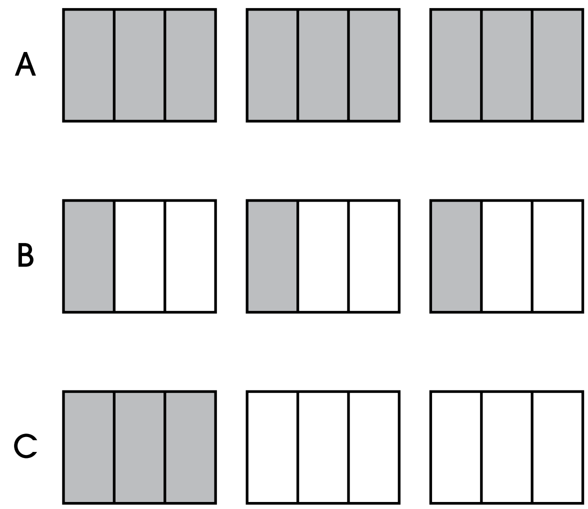
$$\frac{4}{2} = 2$$

- Ask students to identify a whole number up to 3 represented in fraction form with circles, squares, and rectangles divided into 2, 3, and 4 sections.



$$\frac{8}{4} = \underline{\qquad}$$

- Ask students to identify a whole number up to 3 in its fractional form in multiple-choice formats such as “Which model represents the whole number 3?”



Prerequisite Skills

- Draw lines to separate two-dimensional figures into equal areas and express the area of each part as a unit fraction of the whole.
- Decompose circles, squares, and rectangles into two, three, and four equal parts while using the terms halves, thirds, and fourths.
- Divide real-world objects into two, three, or four equal parts. Describe the parts using the language of halves, thirds, and fourths.

Key Terms

denominator, equal, fraction, numerator, whole number

Additional Resources or Links

https://www.mathlearningcenter.org/apps/fractions

http://tasks.illustrativemathematics.org/content-standards/3/NF/A/2/tasks/173

3.N.2.f

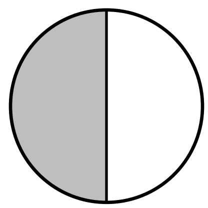
Compare and order fractions having the same numerators or denominators by reasoning about their size.

Extended: Use a model to compare unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Scaffolding Activities for the Extended Indicator

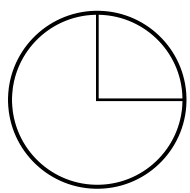
❑ **Identify the fractions one-half, one-third, and one-fourth as part of a whole.**

- Use models to show what one-half, one-third, and one-fourth of a whole look like. Present the figure as shown. Explain that the shaded part of the circle represents one-half, or $\frac{1}{2}$, of the circle. Emphasize that the two parts **must** be equal in size to represent $\frac{1}{2}$.

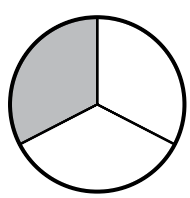


$\frac{1}{2}$

Present the figure as shown, and explain that the circle has two parts, but the parts are not equal in size, so they do not represent $\frac{1}{2}$.

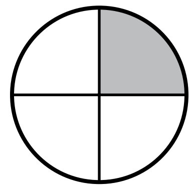


Show a circle that is divided into three parts of equal size. Then, to represent one-third, or $\frac{1}{3}$, shade one of the parts. The figure showing the fraction $\frac{1}{3}$ must have 3 parts of equal size, with one of the parts shaded.



$\frac{1}{3}$

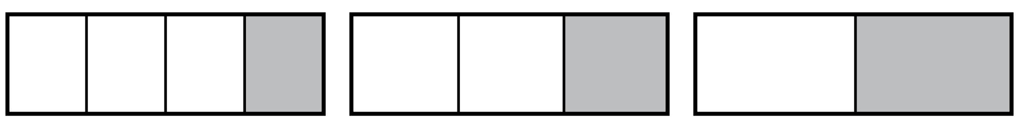
Next, show a circle that is divided into four parts of equal size, with one part shaded to represent $\frac{1}{4}$.



$\frac{1}{4}$

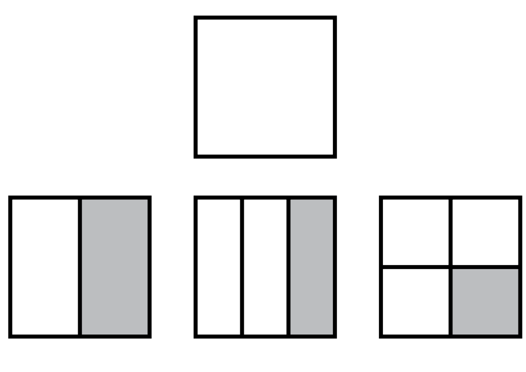
Continue to demonstrate models that represent $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a whole using a variety of shapes in different sizes.

- Ask students to identify the model that represents one-half as part of a whole when given two models to compare. Present one model divided into two equal parts with one part shaded and another model divided into two unequal parts with one part shaded. For example, ask students which rectangle shows $\frac{1}{2}$ shaded. Repeat with the fractions $\frac{1}{3}$ and $\frac{1}{4}$.
- Ask students to identify the fraction one-third as part of a whole when presented with the figures shown below. Repeat with the fractions $\frac{1}{2}$ and $\frac{1}{4}$.



❑ **Compare the unit fractions one-half, one-third, and one-fourth using a model.**

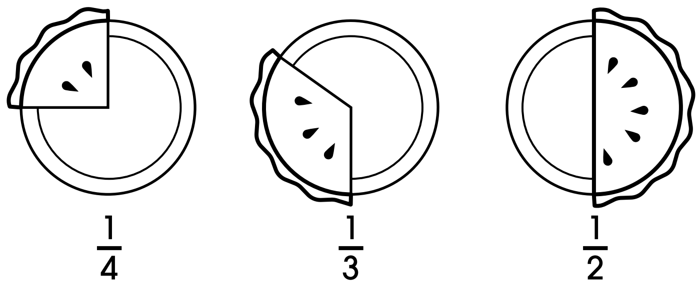
- Use a model to show the size differences of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Present a square as shown, and indicate that the square represents one whole. Demonstrate shading the whole square. Next, present the same square, and demonstrate shading $\frac{1}{2}$ of the whole. Repeat the process with $\frac{1}{3}$ and $\frac{1}{4}$.



Explain that one part was shaded in each square, but a different amount was shaded each time because the size of the parts was different. Indicate that the more parts the square is divided into, the smaller each part is.

Present models with shading representing $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of the whole, in that order. Explain that each model shows a smaller amount of shading than the model before it, so $\frac{1}{2}$ is greater than $\frac{1}{3}$, and $\frac{1}{3}$ is greater than $\frac{1}{4}$. Also, $\frac{1}{2}$ is greater than $\frac{1}{4}$.

It may be helpful to use a real-life situation to compare the sizes of unit fractions. Present the idea of cutting up a pie into pieces. Point out that $\frac{1}{4}$ of the pie is a smaller piece than $\frac{1}{3}$ of the pie or $\frac{1}{2}$ of the pie. Also, $\frac{1}{2}$ of the pie is a larger piece than $\frac{1}{3}$ of the pie.



Continue to demonstrate comparing the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ and using a variety of models and placing the models in order from least to greatest. Consider using a piece of bread or a piece of paper as models that can be easily manipulated.

- Ask students to compare two models that represent unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ and indicate which fraction is more or which fraction is less.
- Ask students to compare three models that represent unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ and place the models in order from least to greatest.

Prerequisite Extended Indicators

MAE 3.N.2.b—Partition two-dimensional figures (circles, triangles, rectangles, and squares) into three, four, or five equal shares, and express the area of each part as a fraction of the whole using $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, or $\frac{4}{5}$.

MAE 3.N.2.c—Represent halves and wholes on a number line.

Key Terms

divide, equal, greatest, least, less, more, part, unit fraction, whole

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-5-topic-c-overview

http://nlvm.usu.edu/en/nav/search.html?qt=ffraction

(Note: Java required for website. Most recent version recommended, but not needed.)

Mathematics—Grade 3
Algebra

3.A.1 Operations and Algebraic Thinking

3.A.1.a

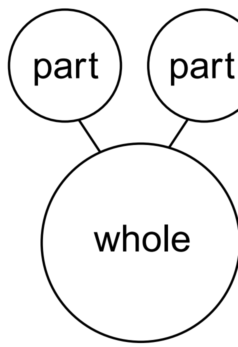
Add and subtract up to four-digit whole numbers with or without regrouping using strategies based on place value and algorithms.

Extended: Add and subtract without regrouping, limited to a maximum sum and minuend of 20.

Scaffolding Activities for the Extended Indicator

❑ **Add numbers with sums of 20 or less without regrouping.**

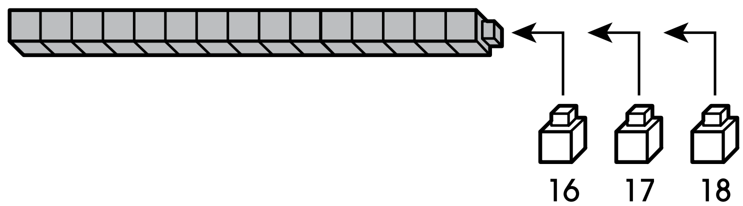
- Use manipulatives and a part-part-whole model to demonstrate how to add values with sums up to 20 without regrouping.



  Present the problem 8 + 1 = ________. Count 8 tokens and place them in the first circle labeled “part.” Place 1 token in the other circle labeled “part.” Move the tokens from the smaller circles that represent the parts to the large circle that represents the whole. Count the total number of tokens to determine the answer, 9. Complete the addition sentence, 8 + 1 = 9.

- Ask students to use the part-part-whole model and manipulatives to solve addition problems with sums up to 20.

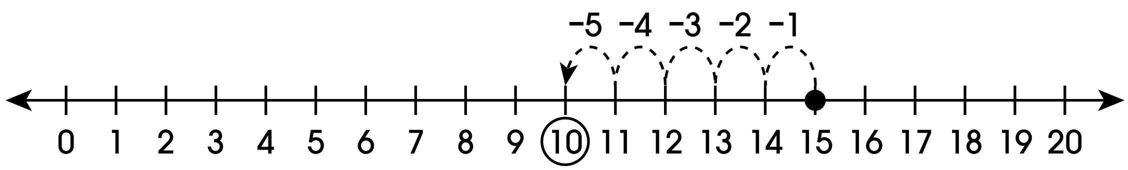
- Use snap-together blocks to demonstrate how to add numbers with sums up to 20 without regrouping. Present a group of snap-together blocks in two colors and the problem 15 + 3 = ______. Demonstrate counting out 15 blocks of one color. Then count out 3 blocks of another color and add them one by one to the stack of 15. Count forward from 15 as each block of the group of 3 is added to find the solution. Complete the addition sentence, 15 + 3 = 18.



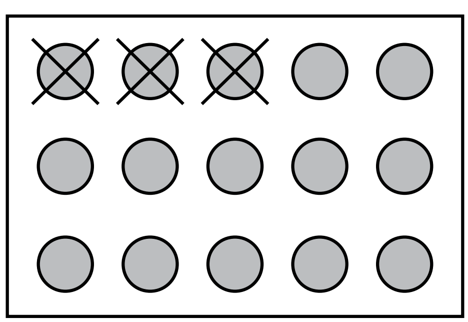
- Ask students to use blocks in two colors to solve addition problems with sums up to 20.

❑ **Subtract from 20 or less without regrouping.**

- Use a number line to demonstrate how to subtract numbers from 20 or less without regrouping. Present a number line and the problem 15 – 5 = ______. Demonstrate subtraction by first locating 15 on the number line. Then explain that subtracting means moving to the left on a number line to get a smaller number. Demonstrate moving to the left 5 units to arrive at the solution to the problem. Complete the subtraction sentence, 15 – 5 = 10.



- Ask students to use a number line from 0 to 20 to solve subtraction problems.
- Use manipulatives to demonstrate how to subtract numbers from 20 or less without regrouping. Present the problem 15 – 3 = ______. Count out 15 tokens. Explain that subtracting means to take away or remove some tokens. Remove 3 tokens. Count the number of tokens that are left, 12. Complete the subtraction sentence, 15 – 3 = 12.



- Ask students to use manipulatives and real-life objects or situations to solve subtraction problems.

3.A.1 Operations and Algebraic Thinking

Prerequisite Skills

- Demonstrate the relationship between whole numbers 1–20, by identifying that each sequential number refers to a quantity that is one larger.
- Use models to increase a quantity while using the terms “2 more,” “plus 1,” or “add 3.”
- Use models to decrease a quantity while using the terms “take away 2,” “minus 2,” or “subtract 3.”

Key Terms

add, addition, decrease, increase, less, more, subtract, subtraction, total

Additional Resources or Links

https://www.engageny.org/resource/grade-1-mathematics-module-2

https://www.insidemathematics.org/common-core-resources/1st-grade

3.A.1.b

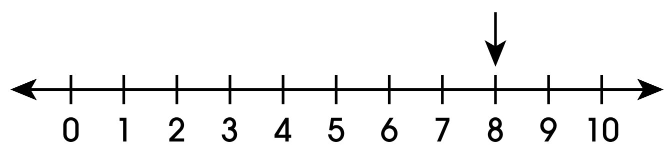
Determine the reasonableness of whole number sums and differences using estimations and number sense.

Extended: Round one- and two-digit whole numbers to the nearest ten, and estimate two-digit sums and differences to the nearest ten.

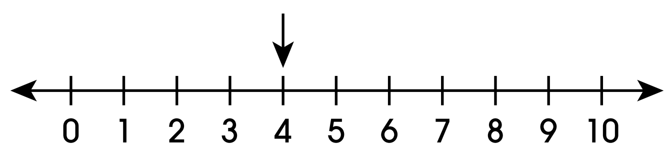
Scaffolding Activities for the Extended Indicator

❑ **Round a one-digit number to 0 or 10 using a number line.**

- Introduce the concept of rounding. Explain that sometimes an estimate, rather than an exact number, is all that is needed. Provide relevant examples of using an estimate or approximate number.
- Use a number line to demonstrate that the location of a one-digit number on a number line can either be closer to 0 or 10.

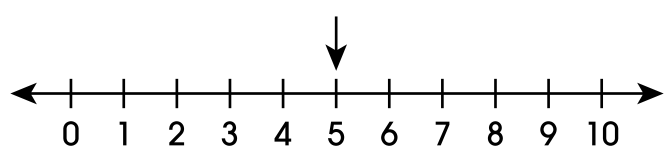


The number 8 is closer to 10 than it is to 0, so it rounds to 10.



The number 4 is closer to 0 than it is to 10, so it rounds to 0.

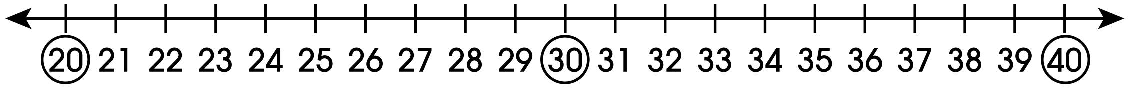
- Explain that the number 5 is the same distance from 0 as it is from 10, so in the case of 5, the number always rounds to 10.



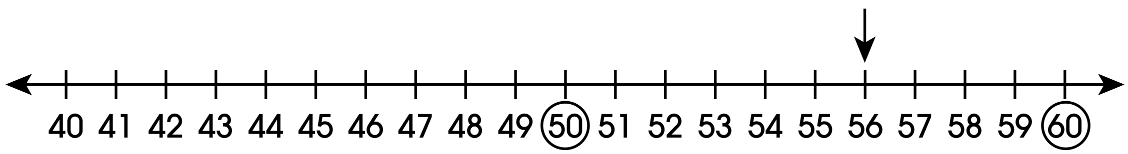
- Ask students to round a one-digit number to 0 or 10 by pointing to a given number on a number line.

- [ ] **Round a two-digit number to the nearest ten using a number line.**

- Demonstrate finding the tens on a number line. Any two-digit number that is a multiple of 10 will end with 0.

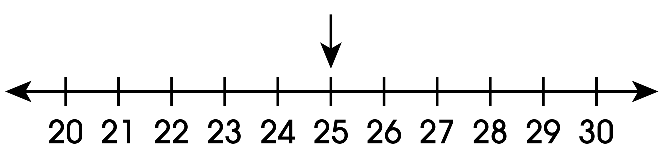


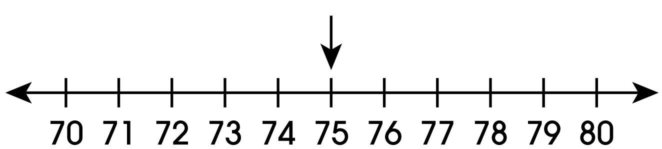
- For students who can skip count by 10, relate skip counting to finding the multiples of 10 on the number line.
- Ask students to identify all the multiples of 10 when provided a number line from 0 to 100 or sections of a number line.
- Demonstrate finding the number 56 on a number line and then finding the multiples of 10 that 56 is located between.



The number 56 is between 50 and 60, so those numbers can be circled or highlighted.

- Ask students to find the multiples of 10 that are on either side of various two-digit numbers when provided a number line from 0 to 100 or sections of a number line.
- Demonstrate that once the two multiples of 10 on either side of a number are found, the next step to rounding is to choose the multiple of 10 that is closer to that number on the number line. From the example above, 60 is closer to 56 than 50 is, so 56 rounds to 60.
- Explain that 5 rounds to 10, and any number ending in a 5 will round to the multiple of 10 to the right of that number on a number line or to the larger number.





- Ask students to round various two-digit numbers to the nearest ten using a number line from 0 to 100 or a section of a number line. Be sure to include numbers with 5 in the ones place.

❑ Estimate two-digit sums to the nearest ten.

- Use rounding of the addends to estimate a sum. Demonstrate that an addition problem can be estimated by first finding the multiples of 10 closest to the numbers given. Present the problem 27 + 58.

27	27 is closest to 30
+ 58	60 is closest to 60

- Explain that rounding of the addends helps to estimate the sum because 30 + 60 is easier to add. The estimate of 27 + 58 is about 30 + 60 = 90. Show students a variety of addition problems, keeping the sum at 100 or less. Continue to use number lines as needed for support to round numbers.
- Ask students to estimate addition results to the nearest 10 up to 100. For example, present the following figure.

41	41 is about _____
+ 33	60 is about _____

- Then give students these three equations for the possible estimate of the sum: 40 + 30 = 70, 50 + 30 = 80, and 50 + 40 = 90. Students should identify that 40 + 30 = 70 is the best estimate for this sum.

❑ Estimate two-digit differences to the nearest ten.

- Use rounding to estimate the difference of two numbers. Demonstrate that a subtraction problem can be estimated by first finding the multiples of 10 closest to the numbers given. For example, find the difference of 89 and 22.

89	89 is closest to 90
– 22	22 is closest to 20

- Explain that rounding numbers helps to estimate the difference because 90 – 20 is easier to subtract. The estimate of 89 – 22 is about 90 – 20 = 70. Show students a variety of subtraction problems, keeping the numbers at 100 or less. Continue to use number lines as needed for support to round numbers.
- Ask students to estimate subtraction results to the nearest 10 up to 100. For example, show the following figure.

54	54 is about _______
– 13	13 is about _______

- Then give students these three equations for the possible estimate of the difference: 50 – 10 = 40, 50 – 20 = 30, and 60 – 10 = 50. Students should identify that 50 – 10 = 40 is the best estimate for this difference.

Prerequisite Extended Indicators

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

addend, closer, difference, estimate, multiple, nearest, number line, round, sum

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/3/NBT/A/1/tasks/1805

https://im.kendallhunt.com/k5/teachers/grade-3/unit-3/lessons.html

3.A.1.c

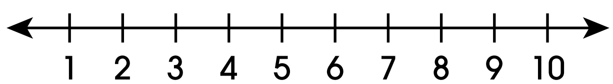
Solve and write one-step whole number equations to represent authentic problems using the four operations including equations with an unknown start, unknown change, or unknown result.

Extended: Solve one-step addition and subtraction equations using the digits 0–9, limited to equations with an unknown change or unknown result.

Scaffolding Activities for the Extended Indicator

❑ **Use a horizontal number line representing whole numbers 0 through 10 to solve subtraction equations.**

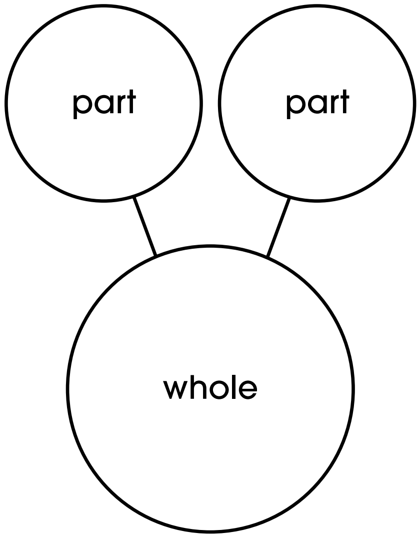
- Use a horizontal number line to demonstrate how to find the missing number in a subtraction equation. For example, model solving the subtraction equation 8 − 2 = ? on a number line. Indicate that the question mark answers the question “What is 8 minus 2?” Model the subtraction by starting at the 8 and counting 2 units to the left to show that 8 take away 2 is 6.
- Use the subtraction sentence 9 − ? = 8. Indicate that the question mark is asking “How much do you take away from 9 to get 8?” Model the subtraction problem by starting at the 9 and counting the number of units to get to 8. Use a variety of methods to model the subtraction problem (e.g., stand next to a number line and count the number of steps; count the number of arcs, sometimes referred to as hops, drawn above a number line while moving to a different number on the number line; use a created or purchased number line slider board).

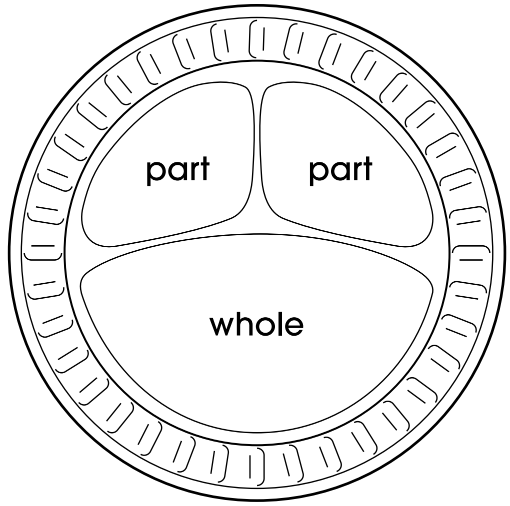


- Ask students to use a horizontal number line and one of the techniques demonstrated to solve for the difference in a subtraction equation, such as 5 − 3 = ? or 9 − 1 = ?.
- Ask students to use a horizontal number line and one of the techniques demonstrated to solve for a subtrahend in a subtraction equation, such as 5 − ? = 4 or 8 − ? = 5.

❑ **Use a part-part-whole method to solve single-digit addition equations.**

- Use a part-part-whole model to demonstrate how to find the missing number in an addition equation. For example, model 4 + 4 = ? by describing the process aloud. The process is to place 4 objects in one of the areas designated “part” and place another 4 objects in the other area designated “part.” Then, move all the objects from the areas designated “part” to the area designated “whole” and count all the objects to find the total.
- Model 6 + ? = 9 by describing the process aloud. The process is to place 9 objects in the area designated “whole.” Then, move 6 objects from the area designated “whole” to one of the areas designated “part.” Move the remaining pieces from the area designated “whole” to the other area designated “part” while counting the number of objects.





paper plate

- Ask students to use the part-part-whole model to solve for the sum in an addition equation, such as 6 + 2 = ? or 4 + 5 = ?.
- Ask students to use the part-part-whole model to solve for an addend in an addition equation, such as 6 + ? = 7 or ? + 1 = 2.

❑ **Identify how the position of an unknown changes the problem in an addition or subtraction sentence.**

- Explain that when solving number sentences, sometimes the number missing is at the end. In 5 + 3 = ____, the problem is asking for the total when adding 5 and 3. Model rephrasing a number sentence as a question.

6 − 4 = ☐	2 + 1 = ____
What is six take away four? Start with six and then remove four. How many are left?	What is the total of 2 and 1? Start with two and then add one more. How many altogether?

- Explain that the missing number can also be in the middle of a number sentence. In 4 + ____ = 7, the missing number is added to four to make seven. Model rephrasing a number sentence as a question.

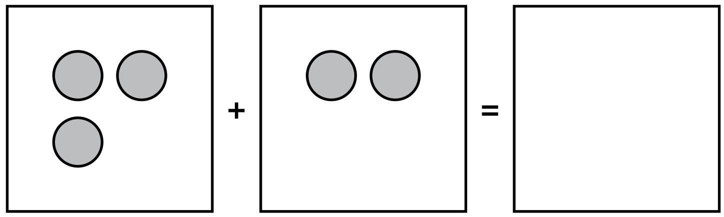
3 + ____ = 5	8 – ☐ = 1
What number is added to three to make five? When starting with three, how many more do we need to have five?	What number is taken away from eight to make one? When starting with eight, how many need to be removed so there is one left?

- Ask students to identify the missing information in a number sentence.

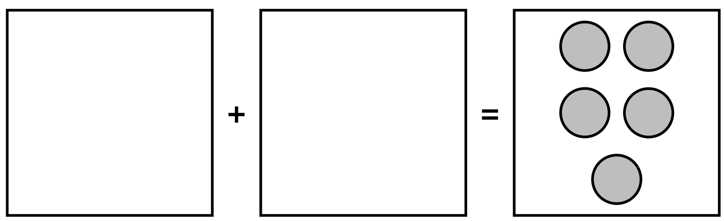
3.A.1 Operations and Algebraic Thinking

❑ **Solve addition equations with a missing addend or a missing sum.**

- Present the addition sentence 3 + 2 = ____. Reference a question model. Start with 3 and then add 2 more. How many altogether? Count out 3 tokens and place them in the first box. Add 2 more tokens and place them in the second box.

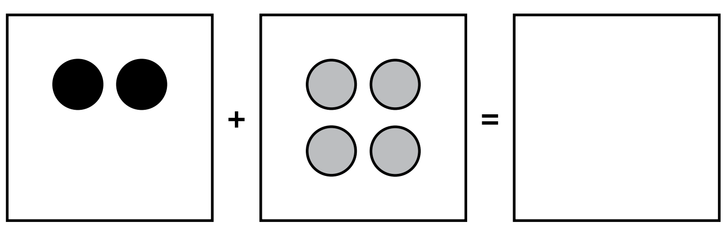


Next, demonstrate adding or combining the tokens by moving the tokens into the box after the equal sign. Count the tokens to find the total, 5. Complete the addition sentence, 3 + 2 = 5.

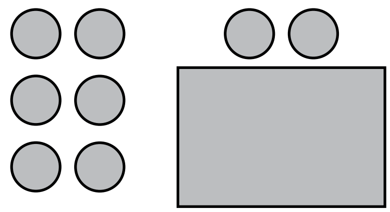


- Ask students to use manipulatives to solve addition equations with the sum as the missing number.

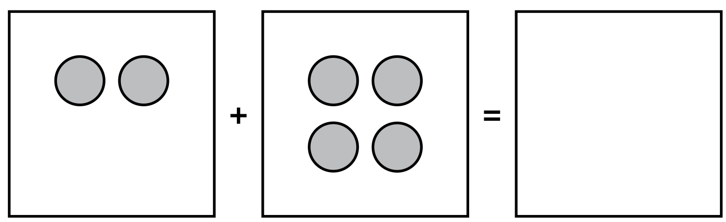
- Present the addition sentence 2 + ____ = 6. Reference a question model. When starting with 2, how many more do we need to have 6? Start with 2 and place in the first box. Count out 3, 4, 5, 6 tokens as you place 4 tokens in the second box. Count the number of tokens in the second box, 4. Complete the addition sentence, 2 + 4 = 6.



- Another option is to explain that the answer 6 in this problem represents the whole, or total, number and indicates that 6 tokens are needed to solve this problem. Count out 6 tokens. Next, cover the tokens so only the quantity for the first addend (2 tokens) is visible. Place those 2 tokens in the first box. Next, indicate that the number of tokens covered is the missing number in the addition sentence. Uncover the 4 tokens and place them in the second box.



Count the number of tokens in the second box, 4. Complete the math problem, 2 + 4 = 6.



- Ask students to use manipulatives to solve addition equations with the second added as the missing number.
- Model using manipulatives to solve addition equations with a missing addend or a missing sum.

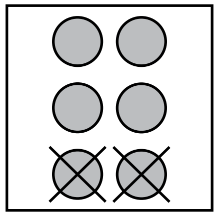
2 + □ = 9	4 + □ = 8	3 + 3 = □

- Ask students to use manipulatives to solve addition equations with a missing addend or a missing sum.

2 + 4 = □	6 + □ = 9	4 + 4 = □
5 + □ = 6	1 + 8 = □	0 + □ = 6

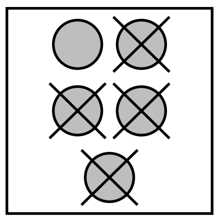
- ☐ **Solve one-step subtraction equations with a missing subtrahend or missing difference.**

  - Present the subtraction sentence 6 − 2 = ____. Reference a question model.
    Start with 6 and then take away 2. How many are left? Model solving the problem.
    Count out 6 tokens. Take away 2. Count the number of tokens that left, 4.
    Complete the subtraction sentence, 6 − 2 = 4.



  - Ask students to use manipulatives to solve subtraction equations with the difference as the missing number.

  - Present the number sentence 5 − ____ = 1. Reference a question model. When starting with 5, how many need to be removed so there is 1 left? Count out 5 tokens. Remove tokens until there is 1 left. Count the number of tokens that were removed, 4. Complete the number sentence, 5 − 4 = 1.



    Indicate that the number of tokens removed or crossed off, in this case 4, is the missing number in the equation. Four tokens were removed; therefore, 5 − 4 = 1.

  - Model using manipulatives to solve subtraction equations with a missing subtrahend or missing difference.

8 − ☐ = 5	9 − ☐ = 4	6 − 5 = ☐

  - Ask students to use manipulatives to solve subtraction equations with a missing subtrahend or missing difference.

7 − 3 = ☐	8 − ☐ = 2	6 − 1 = ☐
9 − ☐ = 2	4 − 2 = ☐	5 − ☐= 3

3.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicator

MAE 3.A.1.a— Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

add, altogether, equation, number sentence, remove, subtract, take away, total

Additional Resources or Links

https://www.engageny.org/resource/grade-1-mathematics-module-1-topic

http://nlvm.usu.edu/en/nav/frames_asid_161_g_1_t_1.html?from=search.html?qt=color+chips

(Note: Java required for website. Most recent version recommended, but not needed.)

3.A.1.d

Interpret and solve two-step authentic problems involving whole numbers and the four operations.

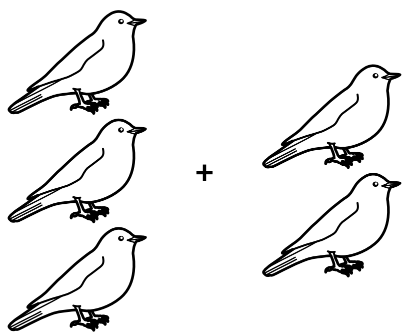
Extended: Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

Scaffolding Activities for the Extended Indicator

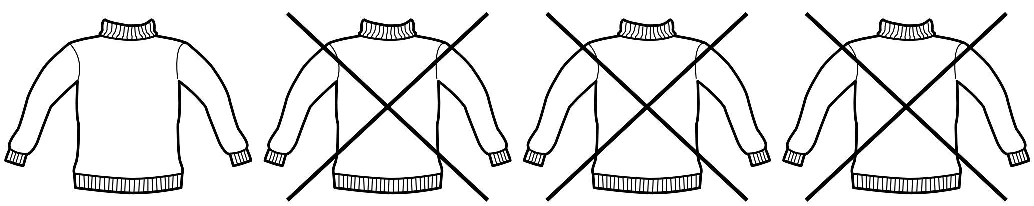
❑ **Solve a one-step addition or subtraction problem using a visual model and the digits 0–9.**

- Use manipulatives or drawings to demonstrate creating a visual model that represents a real-world problem. Present the real-world scenario and figure shown. Demonstrate counting or adding 3 + 2 to find the total number of birds.

  Alfonzo saw 3 birds at a bird feeder.
  Then 2 more birds came to the feeder.
  How many birds were at the bird feeder?



- Present the real-world scenario "There are 4 sweaters in a closet, and then 3 sweaters are removed from the closet. How many sweaters remain in the closet?" and a drawing of 4 sweaters. Demonstrate crossing off 3 sweaters to represent removing 3 sweaters from the closet and identifying 1 as the number of sweaters remaining.



  Continue to demonstrate solving a variety of addition and subtraction problems using visual models that represent real-world scenarios with the numbers 0–9.

- Ask students to identify the correct model for an authentic addition or subtraction problem when given two or more choices of models.

- Ask students to solve an authentic addition or subtraction problem using a model.

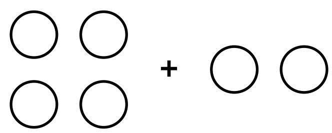
3.A.1 Operations and Algebraic Thinking

- ❑ **Solve an authentic one-step problem using addition or subtraction and the digits 0–9.**

  - Use equations to model authentic addition and subtraction problems. Present the scenario shown.

    Maurice saw 4 dogs on his walk to school.
    He saw 2 more dogs on his walk to the park.
    What was the total number of dogs Maurice saw?

    Explain that the numbers and words in the story can be used to create a drawing that will help solve the problem (or answer the question) in the story. Demonstrate drawing 4 circles to represent the 4 dogs Maurice saw on his walk to school. Explain that the clue word "more" indicates that it is an addition problem and demonstrate drawing the addition sign and 2 more circles to represent the dogs Maurice saw on his walk to the park. Demonstrate writing the expression 4 + 2, using an appropriate computation strategy (e.g., counting the circles, start with 4 and add on 2) to find the total number of dogs, and completing the equation 4 + 2 = 6.



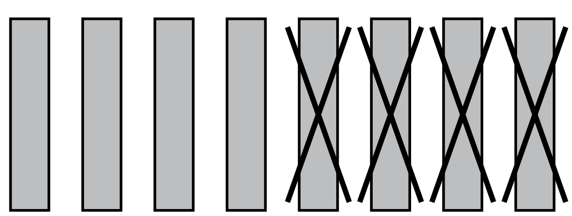
4 dogs + 2 dogs

4 + 2 = 6

  - Present the scenario shown and create a drawing to represent the scenario.

    Abigail has 8 markers.
    She gives 4 markers to her brother.
    How many markers does Abigail have remaining?

    Demonstrate drawing 8 lines or rectangles to represent Abigail's markers. Explain that the clue words "gives" and "remaining" indicate a subtraction problem and cross off 4 of the lines or rectangles to represent the markers Abigail gave to her brother. Demonstrate writing the expression 8 − 4, counting the remaining lines or rectangles that represent the remaining markers, and completing the equation 8 − 4 = 4.



8 markers – 4 markers

8 − 4 = 4

Continue to demonstrate solving a variety of addition and subtraction problems presented in a real-world scenario using equations and the numbers 0–9.

3.A.1 Operations and Algebraic Thinking

- Ask students to identify the correct equation that represents a real-world addition or subtraction problem when given two or more choices of equations.
- Ask students to solve a real-world addition or subtraction problem using an equation.

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations including visual models, standard forms, and word forms.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

add, difference, less, more, remaining, subtract, sum, total

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/perfect%20pair.pdf

http://tasks.illustrativemathematics.org/content-standards/K/OA/A/2/tasks/1151

3.A.1.f

Use drawings, words, arrays, symbols, repeated addition, equal groups, and number lines to interpret and explain the meaning of multiplication and division and their relationship.

Extended: Identify multiplication equations and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to size 20.

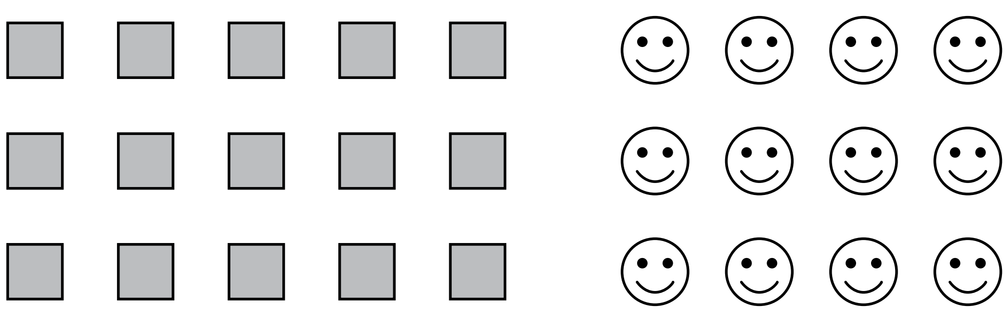
Scaffolding Activities for the Extended Indicator

❑ **Use common objects with multiple parts (for example, toy cars that each have four wheels or toy figures that have two eyes and two ears) to represent multiplication as repeated addition.**

- Use real-life objects to demonstrate that multiple copies of the same object type represent repeated addition. For example, provide multiple toy cars to students. Count each of the 4 wheels on the first car. Then, pick up another car and repeat counting the 4 wheels. Indicate that there are 8 wheels in total on the two cars. Write the number sentences $4 + 4 = 8$ and $4 \times 2 = 8$. Indicate that the sentences represent the same thing, the total number of wheels on the cars. Repeat the activity with 3, 4, and 5 cars. Indicate that each time a car is added to the group and the wheels are counted, there is another 4 added to the addition sentence and the number that 4 is multiplied by increases by one in the multiplication sentence.
- Ask students to identify addition and multiplication sentences to represent multiple copies of the same parts in common objects.

❑ **Use arrays consisting of objects or pictures to illustrate multiplication as repeated addition.**

- Use an array such as three rows of five squares or three rows of four happy faces to demonstrate how to count the objects in the array.



Count the squares as $5 + 5 + 5 = 15$ and as $5 \times 3 = 15$. Additionally, count each square to show that there are 15. Repeat the same process for the happy faces: $4 + 4 + 4 = 12$ and $4 \times 3 = 12$. Continue with different arrays.

- Ask students to complete tables that represent multiplication three ways: as a multiplication sentence, as an addition sentence, and as an array. Present students with a multiplication sentence and ask them to select or create an array and addition sentence that match.

Multiplication Sentence	Addition Sentence	Array
3 × 4		

Present students with a repeated-number addition sentence and ask them to select or create an array and multiplication sentence that match.

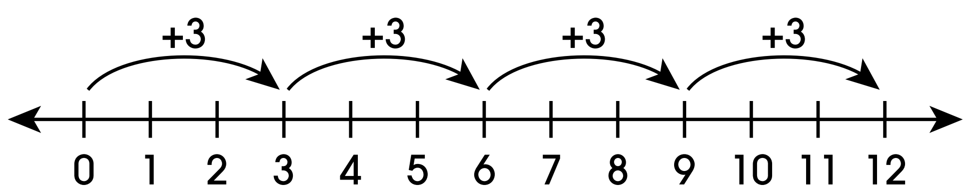
Multiplication Sentence	Addition Sentence	Array
	5 + 5 + 5	

Present students with an array and ask them to select or create a multiplication sentence and addition sentence that match.

Multiplication Sentence	Addition Sentence	Array
		* * * * * * * * * * * * * * * *

❑ **Use number lines to demonstrate multiplication as repeated addition.**

- Use a number line to show the addition sentence 3 + 3 + 3 + 3 = 12 as four equal size jumps of 3.

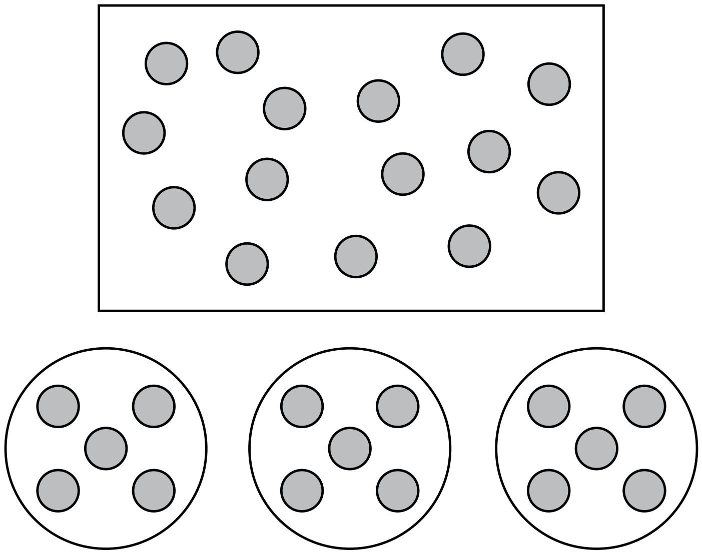


Explain that 4 equal jumps of 3 is the same as 4 groups of 3, which can be written as the multiplication sentence 4 × 3 = 12.

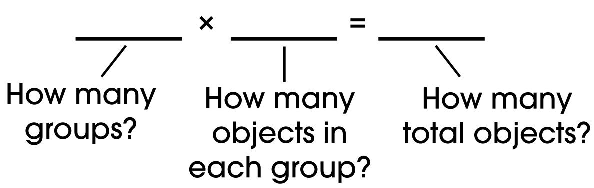
- Continue demonstrating other repeated addition sentences as equal jumps on a number line and then writing the equivalent multiplication sentence.
- Ask students to represent repeated addition problems as equal jumps on a number line.
- Ask students to represent multiplication problems as equal jumps on a number line.

- **Identify a multiplication equation that represents a set of objects sorted into groups of equal size.**

  - Use manipulatives to show a group of 15 objects. Also show 15 objects in smaller groups of equal size. Indicate that both examples have the same number of objects and that having groups of equal size can help students find the total number of objects quickly.

    Refer to the set of 15 objects that is arranged in 3 equal groups of 5 and ask the following questions. How many groups are there? How many objects are in each group? How many total objects are there? Demonstrate how to use this information to fill in the template for the multiplication equation.

    Repeat the same process with other examples of up to 20 objects arranged in groups of equal size.

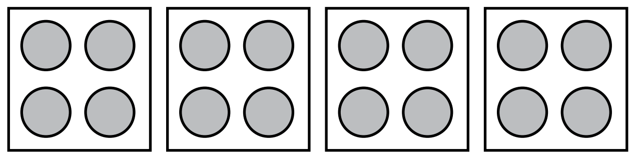
  - Ask students to identify the multiplication equation for images of objects in equal groups. Be sure to practice all multiplication equations of groups up to 20. For example, use 2 groups of 6 objects to show $2 \times 6 = 12$; also use 6 groups of 2 objects to show $6 \times 2 = 12$.

- ❑ **Use groups of equal size to represent a multiplication equation.**

  - Write the equation 4 × 4 = 16 on the template indicating the number of groups, the number of objects in each group, and the total number of objects.

    4 × 4 = 16

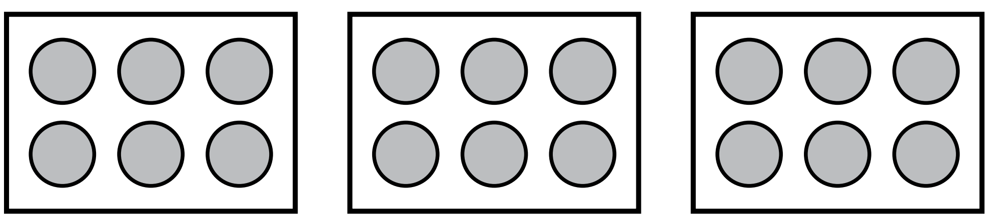
    How many groups? | How many objects in each group? | How many total objects?

    Do this for several equations, with up to 20 objects total. Then progress to creating groups of equal size without using the template. Demonstrate this process with objects and drawings of objects.

  - Ask students to create groups of equal size that represent a multiplication equation that is written on a template. Then progress to a multiplication equation that is not written on the template.

  - Ask students to identify a multiplication equation that matches a given picture. For example, give students the picture shown, with three possible equations to choose from.

    A. 2 × 6 = 12

    B. 3 × 6 = 18

    C. 3 × 6 = 18

    Students should choose the equation 3 × 6 = 18.

3.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicators

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

add, addition, addition sentence, array, equal, equation, multiply, multiplication, multiplication sentence, sum, times

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-lesson-1

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-lesson-2

3.A.1.h

Multiply one-digit whole numbers by multiples of 10 in the range of 10 to 90 using strategies based on place value and properties of operations.

Extended: Multiply 1 and 2 by multiples of 10 with a maximum product of 100.

Scaffolding Activities for the Extended Indicator

❑ **Multiply 1 by multiples of 10 with a maximum product of 100.**

- Explain that multiplying any number by 1 will result in a product (answer) that is the same as the other factor (number). Since multiplying 1 by 10 is the same as repeating the 10 one time, the product is 10.

  $$1 \times 10 = 10$$

  Another way to interpret 1 × 10 is one group of ten. Use real-life objects, base ten blocks, other manipulatives, or drawings to demonstrate making one group of ten, one group of twenty, and one group of thirty up to one group of 100 to find the products for 1 × 10, 1 × 20, and 1 × 30 up to 100.

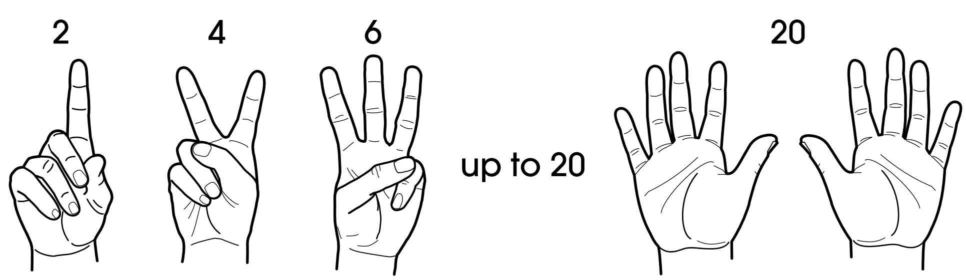
  Emphasize that the order of the factors does not change the product (or answer): 10 × 1 = 10 and 1 × 10 = 10. Use base ten blocks, other manipulatives, or drawings to compare one group of ten to ten groups of one. Repeat with the other factor combinations with a maximum product of 100.

- Ask students to multiply and compare the products of 1 × 10 and 10 × 1. Repeat with 1 × different multiples of 10 up to 100.

- Ask students to multiply 10 × 1, 20 × 1, 30 × 1, 40 × 1, 50 × 1, 60 × 1, 70 × 1, 80 × 1, 90 × 1, and 100 × 1 using all factor combinations.

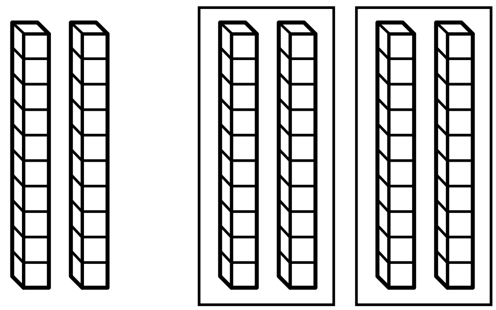
❑ **Multiply 2 by multiples of 10 with a maximum product of 100.**

- Use skip counting by two to find the product of two times ten. For example, skip count aloud (2, 4, 6, 8, . . .), using fingers to count how many twos are counted.

  Show how to skip count by two involving another person's hands to get to forty, which would be twenty times, and then with a third person's hands to get to sixty, which would be skip counting by two thirty times. Repeat for two groups of forty and fifty.

Use base ten blocks or other manipulatives to show 2 × 10 as two groups of ten and 2 × 20 as two groups of twenty. Repeat for two groups of thirty, forty, and fifty.



- Use repeated addition to demonstrate multiplying two by ten. This can be done using manipulatives or the number sentences shown. Repeat for 2 × 20, 2 × 30, 2 × 40, and 2 × 50.

$$2 \times 10$$
$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20$$

Demonstrate creating picture models that represent the multiplication problems that can then be used to count to find the product (e.g., tally marks or circles in sets of two). When appropriate, introduce using the standard algorithm to multiply two by ten, twenty, and thirty. Be sure to emphasize that the order of the factors does not change the product.

- Ask students to multiply two by ten, twenty, thirty, forty, and fifty when given picture models or manipulatives.
- Ask students to multiply two by ten, twenty, thirty, forty, and fifty when not given picture models or manipulatives.

Prerequisite Extended Indicators

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

factor, multiply, product, repeated addition, skip count

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-3-topic-f-lesson-19

https://www.engageny.org/resource/grade-3-mathematics-module-3-topic-f-lesson-20

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Mathematics—Grade 3
Geometry

3.G.1 Shapes and Their Attributes

3.G.1.a

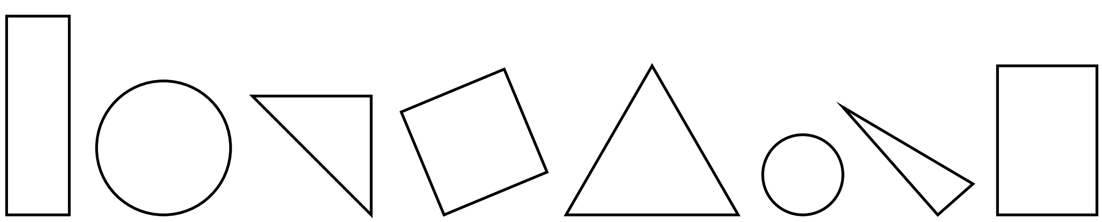
Sort quadrilaterals into categories according to their attributes.

Extended: Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Scaffolding Activities for the Extended Indicator

❑ **Identify circles and triangles.**

- Explain that two-dimensional shapes are flat figures. Compare and contrast a circle to a sphere (ball) and a square to a cube (block or die).
- Explain that two-dimensional shapes have names. Circles are round shapes without corners or edges. Reference examples of circles around the classroom. Explain that triangles are two-dimensional shapes that have three sides and three angles. Sides are the straight lines that form the shape. An angle is where two sides meet. Reference examples of triangles around the classroom.
- Model sorting cutout circles and cutout triangles in a variety of sizes into two categories: circles and triangles.
- Ask students to sort cutout circles and cutout triangles in a variety of sizes into two categories: circles and triangles.
- Model identifying circles and triangles by coloring all circles green and all triangles yellow, placing emphasis on the round shape of the circles and the three straight lines in the triangles.



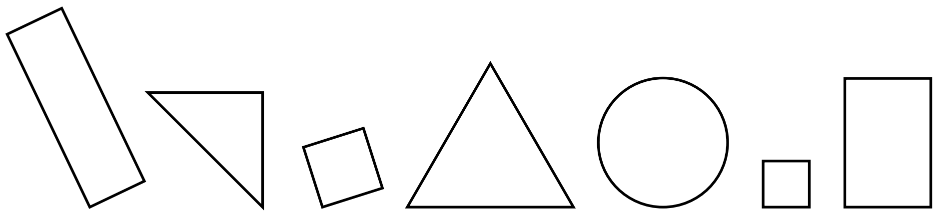
- Ask students to identify circles and triangles.

❑ **Identify circles, triangles, rectangles, and squares.**

- Explain that squares and rectangles are shapes with four sides and four angles. All four sides of a square are the same length. In a rectangle, the opposite sides are the same length. Reference real-world objects and math manipulatives that are in the shape of a square or a rectangle. Demonstrate sorting the objects and manipulatives into the two categories: squares and rectangles.

3.G.1 Shapes and Their Attributes

- Ask students to sort real-world objects and math manipulatives into two categories: squares and rectangles.
- Model creating squares using four manipulatives that are the same length (e.g., straws, pipe cleaners, pencils). Model creating rectangles using manipulatives of two different lengths. Reference the four equal sides in squares and opposite equal sides in rectangles.
- Ask students to create squares and rectangles using manipulatives of different lengths.
- Model identifying squares and rectangles by coloring all the squares red and all the rectangles blue.



- Ask students to identify squares and rectangles.
- Model identifying circles, triangles, rectangles, and squares.
- Ask students to identify circles, triangles, rectangles, and squares.

Prerequisite Skills

- Differentiate between flat surfaces and curved surfaces, and sort objects by whether they slide, stack, or roll.
- Differentiate between a curved line and a straight line.
- Determine defining attributes (sides and corners) and nondefining attributes (size, color, and orientation) of two-dimensional shapes.

Key Terms

angle, circle, opposite, rectangle, side, square, triangle, two-dimensional shape

Additional Resources or Links

https://www.engageny.org/resource/kindergarten-mathematics-module-2-topic-lesson-1

http://tasks.illustrativemathematics.org/content-standards/K/G/B/4/tasks/515

3.G.2 Area and Perimeter

3.G.2.a

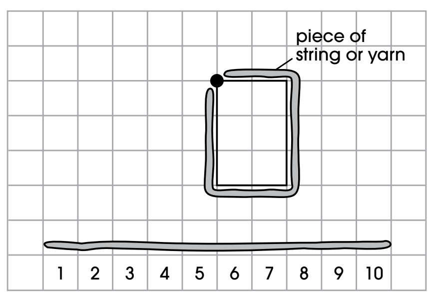
Solve authentic problems involving perimeters of polygons when given the side lengths or when given the perimeter and unknown side length(s).

Extended: Find the perimeter of a square or rectangle given the side lengths and a visual model.

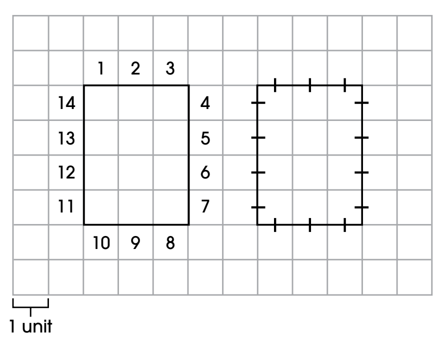
Scaffolding Activities for the Extended Indicator

❑ **Find the perimeter of a square or rectangle drawn on grid paper.**

- Explain that perimeter is the distance along the outside edge of a shape. Place or draw a square or rectangle on a piece of grid paper. Trace or highlight the lengths of the sides and explain that the perimeter is the total length of all the sides. Make a mark on one corner of the square or rectangle to represent a starting point. Then use a piece of yarn or string to go around all sides. Cut the yarn or string at the length that represents the perimeter. Place the length of the string on the coordinate grid to find the total length of all the sides. Another method is to use painter's tape to tape out the perimeter of a square or rectangle on a tiled floor.



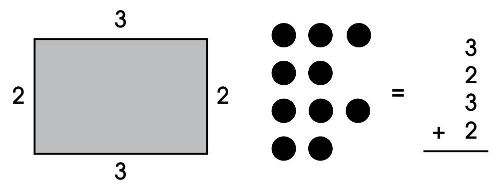
- Demonstrate counting strategies to find the perimeter of a square or rectangle drawn on grid paper. Count each unit along each edge to find the perimeter. Be sure to explain that the perimeter is found by counting the units on each side, not the squares inside the rectangle. Use tick marks to count each unit along the edge of the shape to find the perimeter, as shown.



- Ask students to identify the sides of a square or rectangle by highlighting or tracing.

3.G.2 Area and Perimeter

- Ask students to use a counting strategy to find the perimeter of a square or rectangle placed or drawn on grid paper.
- Find the perimeter of a square or rectangle given side lengths and a visual model.
- Use a square or rectangle with the side lengths labeled to demonstrate how to add the side lengths to find the perimeter. Model strategies to find the total length of the four sides. For example, use tokens to represent each side length and then add them to find the perimeter. Continue to demonstrate finding the perimeter of squares or rectangles using a familiar computation method (e.g., manipulatives, number lines, calculator).



- Ask students to determine the perimeter of a square or rectangle when given the side lengths.

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

add, edge, perimeter, side, side length, sum, total

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-c-lesson-12

http://nlvm.usu.edu/en/nav/frames_asid_166_g_2_t_3.html?open=activities&from=search.html?qt=perimeter

(Note: Java required for website. Most recent version recommended, but not needed.)

3.G.2.b

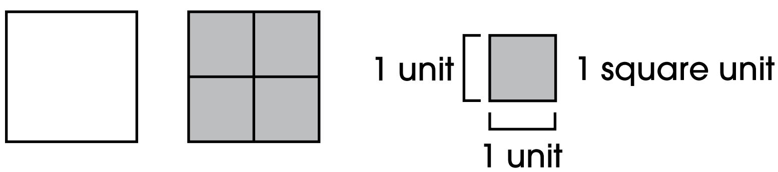
Use concrete and pictorial models to measure areas in square units by counting square units.

Extended: Find the area of a square or rectangle by counting whole-number unit squares.

Scaffolding Activities for the Extended Indicator

❑ **Identify the unit squares included in the area of a square.**

- Use manipulatives to teach area as a count of the number of unit squares that cover a shape. Unit squares are used for covering a shape and have a length of 1 unit on each side. Use a 2-by-2 square and unit square tiles to show that the unit square tiles can be placed on the square to cover it.

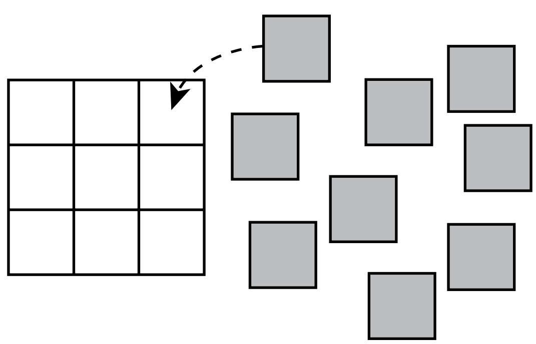


Use the unit square tiles to cover the entire inside of the 2-by-2 square. Note that the unit square tiles will completely cover the 2-by-2 square.

- Show other examples of a square, for example, a 4-by-4 square, marked with unit square lines and ask students to identify a unit square.

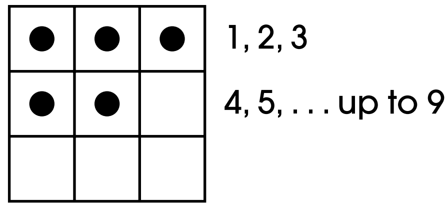
❑ **Find the area of a square by counting whole number unit squares.**

- Use manipulatives to calculate area. For example, use a 3-by-3 square with 9 unit squares marked on the 3-by-3 square. Gather 9 tiles that are the same size as the unit squares. Place one tile over each unit square to make a connection between a tile and a unit square.



Find the area of the square by counting each tile as it is placed on the square to cover a unit square. The count of the tiles, which is the area, is 9 unit squares.

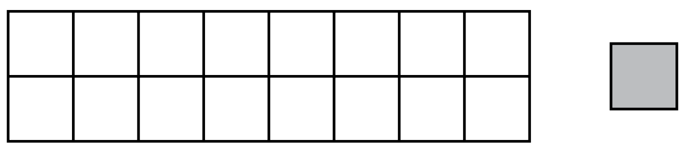
Use a counting strategy to determine the area of the square by counting the number of unit squares from left to right. Make a small dot in each unit square to keep track of the unit squares that have been counted.



- Ask students to identify the number of unit squares in a given gridded square by counting.

❑ **Identify the unit squares included in the area of a rectangle.**

- Use a rectangle marked with 1-unit grid lines to show area as the count of unit squares that cover the rectangle. For example, using the rectangle and tiles, show that tiles can be placed to cover the rectangle. Describe the tile as a unit square that measures 1 unit on each side.

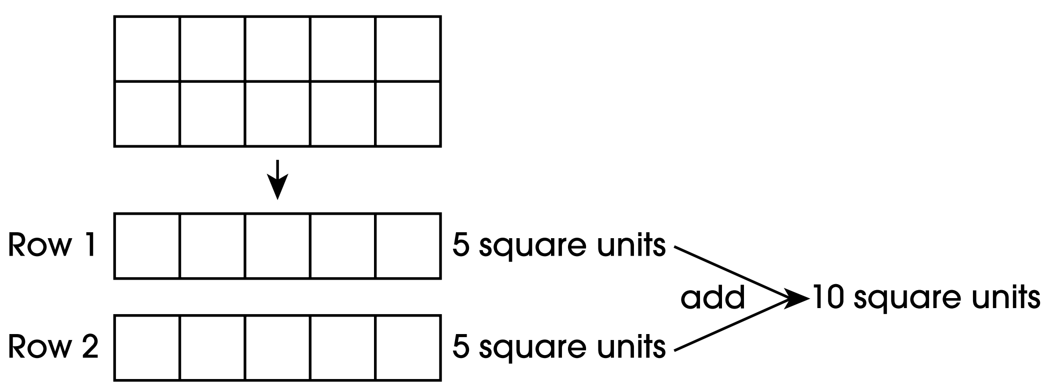


Demonstrate that 16 tiles cover the rectangle and that each tile corresponds to one of the unit squares marked on the rectangle. Reinforce the idea that the area of the rectangle is 16 square units.

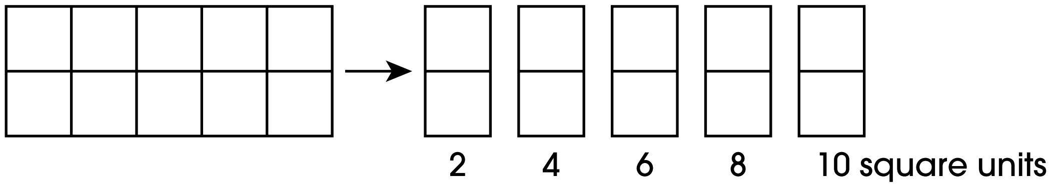
- Ask students to identify a unit square within a rectangle shown with unit squares inside.

❑ **Find the area of a rectangle by counting whole numbers of unit squares.**

- Use a rectangle set on a grid to teach counting strategies for calculating area, such as grouping row and column counts. For example, given a 2-by-5 rectangle marked with unit squares, group rows of unit squares to determine the area of the rectangle.



Alternately, use groupings of the 2 columns of unit squares to determine the area of the rectangle.



- Ask students to use grouping strategies to determine the area of a rectangle marked with unit squares.

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Key Terms

area, column, grouping, rectangle, row, square, square unit, tile, unit square

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-4-topic-lesson-3

https://www.engageny.org/resource/grade-3-mathematics-module-6-topic-lesson-2

http://nlvm.usu.edu/en/nav/frames_asid_192_g_1_t_1.html?from=topic_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

3.G.2.c

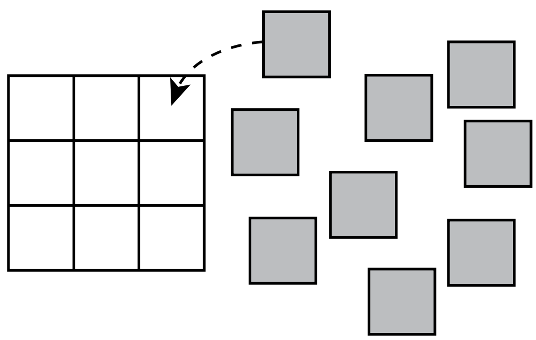
Find the area of a rectangle with whole-number side lengths by modeling with unit squares; show that area can be additive and is the same as it would be found by multiplying the side lengths.

Extended: Find the area of a square or rectangle with whole-number side lengths by counting unit squares and showing that repeated addition is the same as multiplying the side lengths.

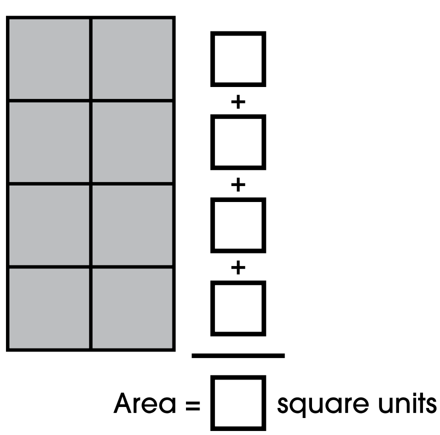
Scaffolding Activities for the Extended Indicator

❑ **Find the area of a square or rectangle using repeated addition.**

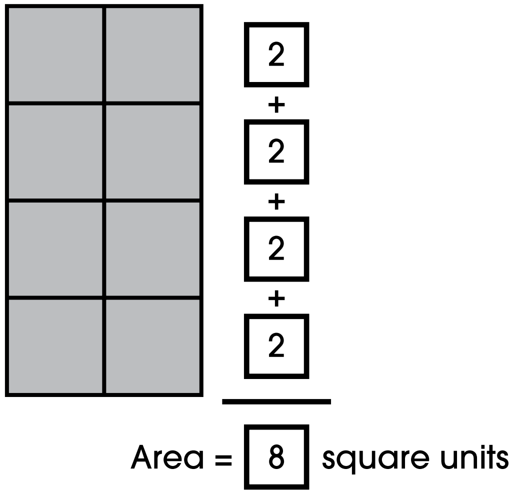
- Use manipulative tiles to help find the area of the square. Start by placing one unit tile onto each unit of the square and explain to students that the shaded area inside the square represents the area in square units.



- Explain that you can find the area of the square by using repeated addition. Model counting the manipulative tiles in the first row and identifying that there are 3. Continue this for the second and third rows. Since there are 3 tiles in each row, the area can be found by adding 3+3+3, totaling 9. Reiterate that the area of this square is 9 square units.
- Provide students with a rectangle as shown. Indicate the blank boxes next to each row. Inform students that each unit square in each row should be counted, and the number can go in the box.



Once all boxes next to the rectangle are filled in, model using repeated addition to find the area of the rectangle.

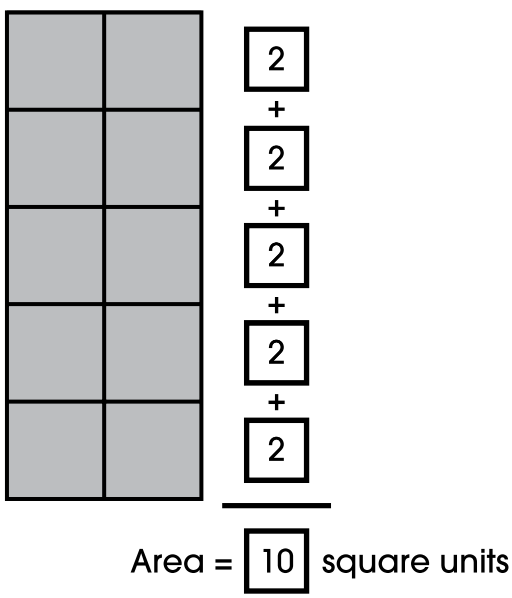


Repeat this process with various size squares and rectangles.

- Ask students to find the area of various squares and rectangles using repeated addition.

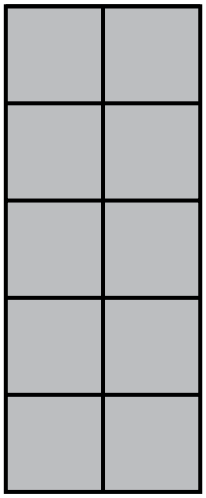
❑ **Find the area of a square or rectangle using repeated addition and by multiplying the side lengths to show that both methods result in the same answer.**

- Present a rectangle model, as shown. Model finding the area using repeated addition. Write a 2 in each box next to the rectangle. Demonstrate adding 2+2+2+2+2 to get a total of 10 square units.



Explain that repeated addition is one way to find the area of this rectangle.

- Use the same rectangle model to demonstrate using multiplication to find the area of the same rectangle. Point to each of the sides of the rectangle. Model counting the unit squares in the top side length and writing 2. Next, model counting the unit squares in the other side length and writing 5. Explain that when both side lengths of the rectangle are multiplied together, it will give you the area.



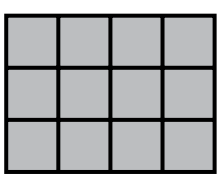
$\boxed{} \times \boxed{} = \boxed{}$ square units

$\boxed{2} \times \boxed{5} = \boxed{10}$ square units

Point to the area of 10 in the first model (repeated addition) and then point to the area of 10 in the second model (multiplication). Ask students if 10 is equal to 10. Explain that adding 2 together 5 times is the same as multiplying 2 x 5, which shows that repeated addition and multiplying side lengths result in the same answer.

Repeat this process with various squares and rectangles of different sizes.

- Ask students to find the area of a square or rectangle using repeated addition.
- Ask students to find the area of the same square or rectangle using multiplication.
- Ask students if both methods resulted in the same answer.

	+	×	**Same Area?**
	4 + 4 + 4 = 12	4 × 3 = 12	Yes No

Prerequisite Extended Indicators

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.G.2.b—Find the area of a square or rectangle by counting whole-number unit squares.

Key Terms

area, column, length, rectangle, row, side, square, square unit, tile, unit square, width

Additional Resources or Links

https://www.insidemathematics.org/classroom-videos/public-lessons/area-and-perimeter-pre-lesson

http://nlvm.usu.edu/en/nav/frames_asid_281_g_2_t_4.html?open=activities&from=grade_g_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

3.G.3 Measurement

3.G.3.a

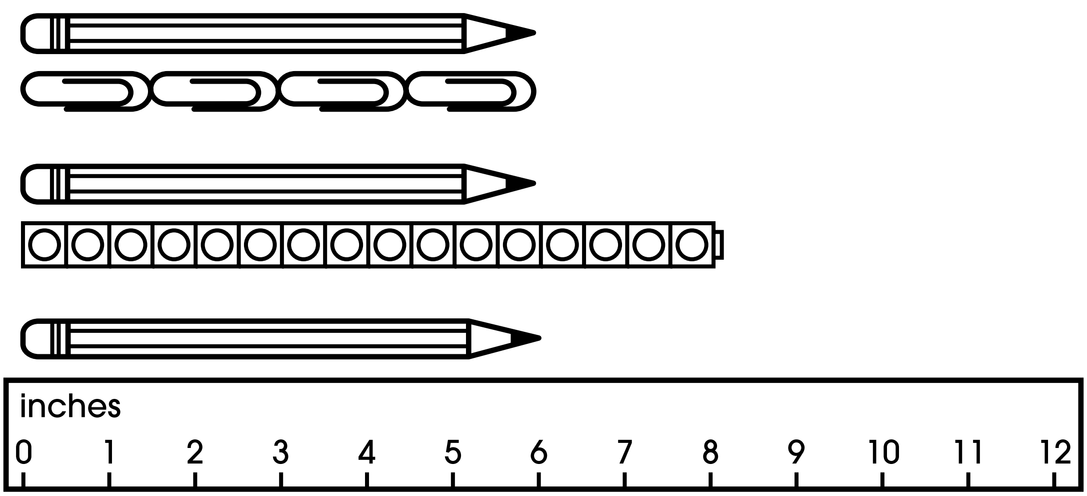
Identify and use the appropriate tools and units of measurement, both customary and metric, to solve authentic problems involving length, weight, mass, liquid volume, and capacity (within the same system and unit).

Extended: Identify the appropriate tool to use to solve authentic problems involving length, weight, and liquid volume.

Scaffolding Activities for the Extended Indicator

❑ **Identify tools used for measuring.**

- Demonstrate examples of simple things that are measured in daily life by using questions. Examples: How long is this pencil? How much does this rock weigh? How heavy is this rock? How much water is in this glass?
- Explain that there are many ways to measure the length of things. Using manipulatives, demonstrate measuring a pencil using paper clips laid out in a row, cubes, and a ruler.



Demonstrate similar measurements for weight by using a balance with a rock on one side and cubes/marbles/etc. on the other and also a scale.

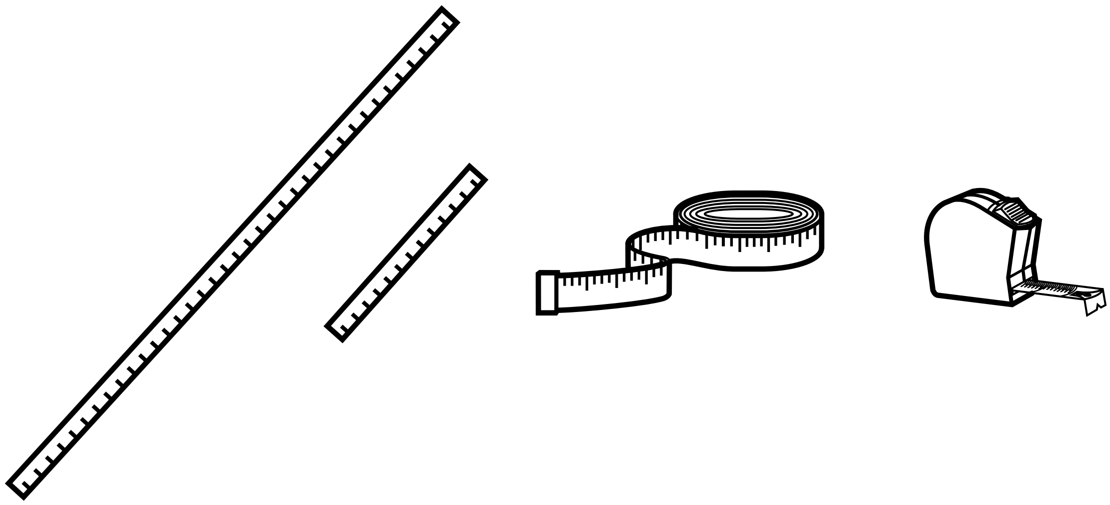
Demonstrate similar measurements for liquid volume using a given amount of water in a clear glass container and measuring it out with a small paper cup, a small plastic bottle, and then with a measuring cup.

Compare and contrast nonstandard tools with standard tools of measurement. Explain that using standard tools means that everyone understands how much we have measured.

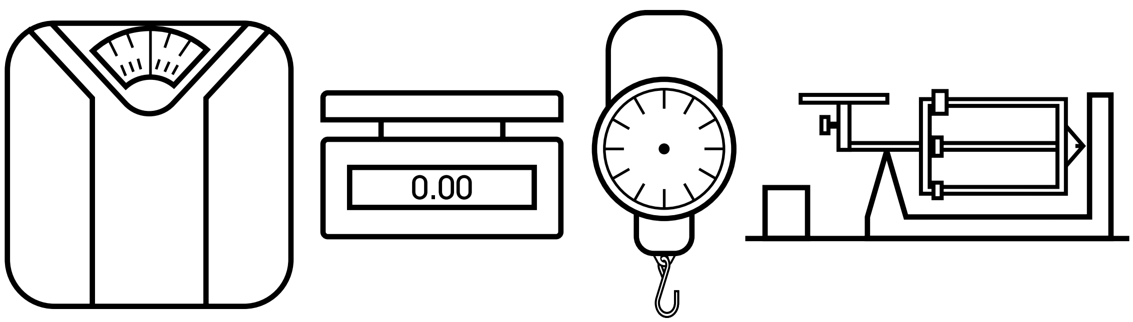
- Ask students to identify standard tools for measuring length (a ruler), weight (a scale), and liquid volume (a measuring cup).
- Ask students to identify the use for a ruler, a scale, and a measuring cup.

❑ **Identify appropriate tools to measure length, weight, and liquid volume.**

- Explain that the standard tools for measuring length are a yardstick, a ruler, a rigid tape measure, and a flexible tape measure. Demonstrate using standard tools appropriately to measure length: measure the length of a table using a yardstick and a tape measure, measure the length of a book using a ruler, measure the length around a can using a flexible tape measure.

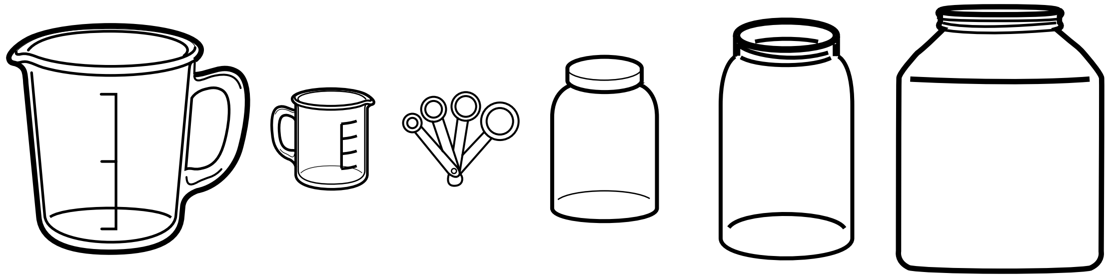


- Demonstrate using standard tools to measure weight: bathroom scale, kitchen scale, luggage scale, triple beam scale (science).



Explain that specific scales should be used based on what is being measured. For example, a bathroom scale is used to find out how much you weigh, and a kitchen scale is used to find out how much two apples weigh.

- Demonstrate using standard tools to measure liquid volume: cups; measuring spoons; and pint, quart, and gallon containers.



Explain that specific tools to measure liquid volume should be used based on how much is being measured. For example, a measuring cup is used when measuring larger cups of liquid for a recipe, and measuring spoons are used when measuring small amounts of liquid like honey in a recipe.

- Ask students to select the appropriate tool for measurement, given a measuring situation of length, weight, or liquid volume.
- Ask students to describe a situation where a given measurement tool would be used.

Prerequisite Skills

- Identify various tools of measurement in the natural environment by name.

Key Terms

measurement, length, weight, liquid volume, standard measurement, yardstick, ruler, tape measure, scale, measuring spoons, cups, pint, quart, gallon

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/documents/sample_materials/br3-tg-u4-m2.pdf

https://www.engageny.org/resource/grade-2-mathematics-module-2

3.G.3.b

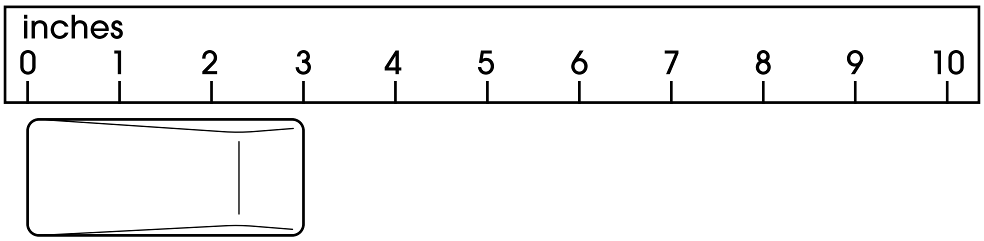
Estimate and measure length to the nearest half inch, fourth inch, and centimeter.

Extended: Measure the length of an object to the nearest inch.

Scaffolding Activities for the Extended Indicator

❑ **Align an object on a ruler for measuring.**

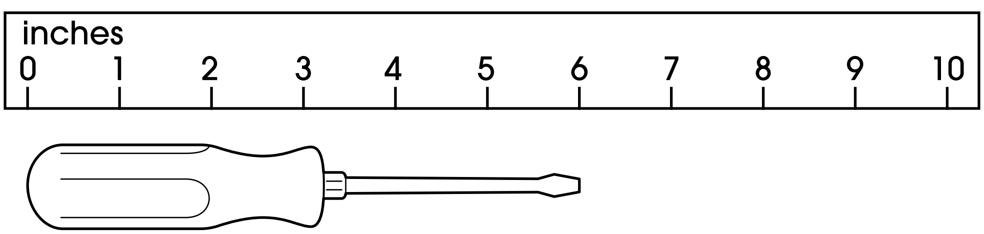
- Use a ruler to demonstrate how to align an object for measuring. For example, present a ruler and an object and show how to align the left end or edge of the object to the 0 on a ruler.



- Provide examples of objects that are correctly and incorrectly aligned. Ask students to indicate which objects are correctly aligned.
- Ask students to align the left end of an object to 0 on a ruler.

❑ **Measure length to the nearest inch using a model of an object.**

- Explain that the distance between each number on the ruler represents a length of 1 inch. Demonstrate placing the left edge of an object at 0 inches. Using the following example, explain that since the right end aligns with 6, the object is 6 inches long.



- Ask students to identify the length of an object when it is placed next to a ruler.
- Ask students to use a ruler to measure an object to the nearest inch.

Prerequisite Extended Indicators

3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

3.G.3.a—Identify the appropriate tool to use to solve authentic problems involving length, weight, and liquid volume.

Key Terms

edge, end, inch, length, long, measure, ruler

Additional Resources or Links

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-c-lesson-14

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-c-lesson-15

3.G.4 Time

3.G.4.a

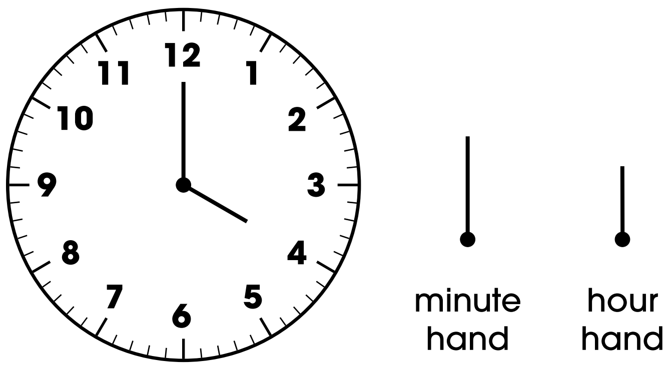
Tell and write time to the minute using both analog and digital clocks.

Extended: Tell time to the hour using both analog and digital clocks.

Scaffolding Activities for the Extended Indicator

❑ **Identify the hour hand, the minute hand, and the numbers on an analog clock.**

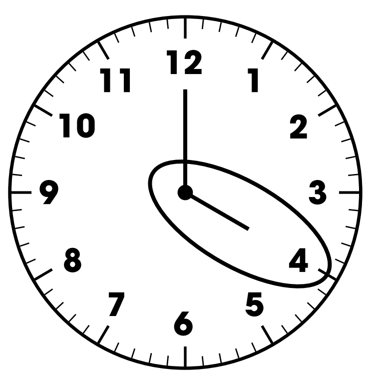
- Use an analog clock to demonstrate the difference between the hour hand and the minute hand. Compare the length of the hour hand and the length of the minute hand. Indicate that both the hour hand and the minute hand move around a clock to show the time. Use real analog clocks and pictures of analog clocks to identify the hour hand, the minute hand, and the numbers on the clockface that indicate the time.



- Ask students to identify the hour hand, the minute hand, and the numbers on an analog clock.

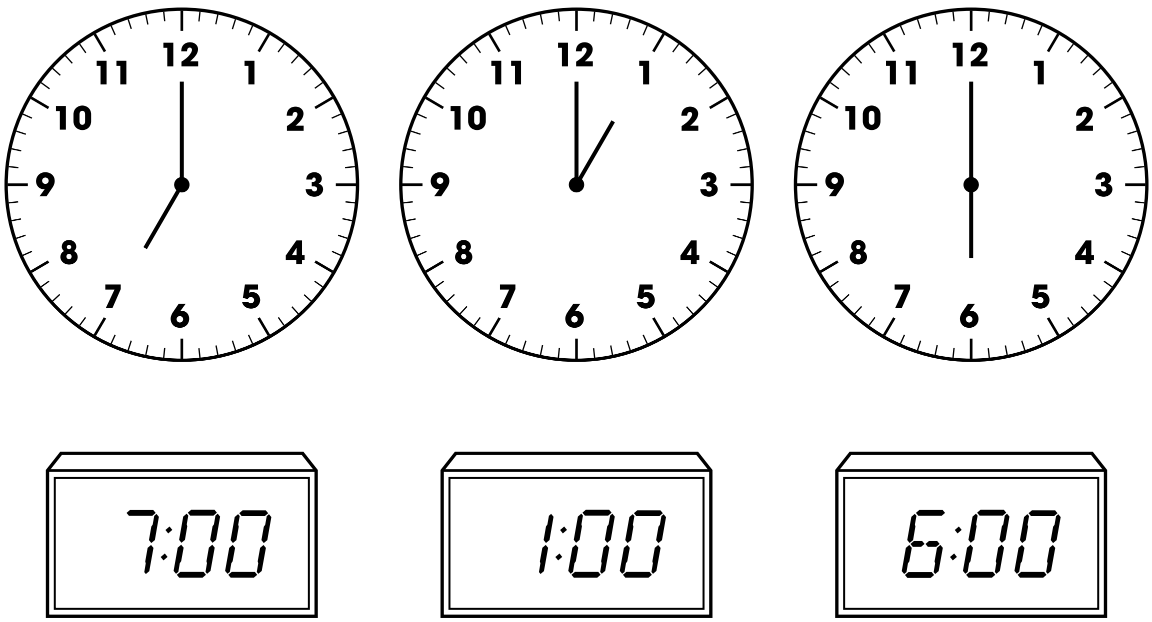
❑ **Tell time to the hour on a digital clock and an analog clock.**

- Demonstrate finding the hour hand on an analog clock. Circle the hour hand and the number the hour hand points to. Emphasize saying the name of the number circled when stating the time. For example, four is circled, so it is “four o’clock.” Be sure to make a connection between the minute hand pointing to twelve and “o’clock” when it is the exact hour. Show clocks in which the minute hand is not pointing to twelve as nonexamples.



- Demonstrate reading the time on a digital clock by saying the name of the first number. For example, the first number is 4, so it is “four o’clock.” Be sure to make a connection between the numbers after the colon both being zero and “o’clock” when it is the exact hour. Show clocks in which it is not the exact hour as nonexamples.

- Ask students to identify the time to the hour on both analog and digital clocks. Ask students to match analog and digital clocks that have the same time.



Prerequisite Skills

- Describe the relative positions of objects (e.g., above, below, beside, in front of, behind, next to, between).
- Compare the length of two objects using the words *longer* and *shorter*.
- Read and write whole numbers up to 12.

Key Terms

analog, digital, hour, longer, minute, o’clock, shorter, time

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/once%20upon%20a%20time.pdf

https://www.engageny.org/resource/grade-2-mathematics-module-8-topic-d

3.G.4.b

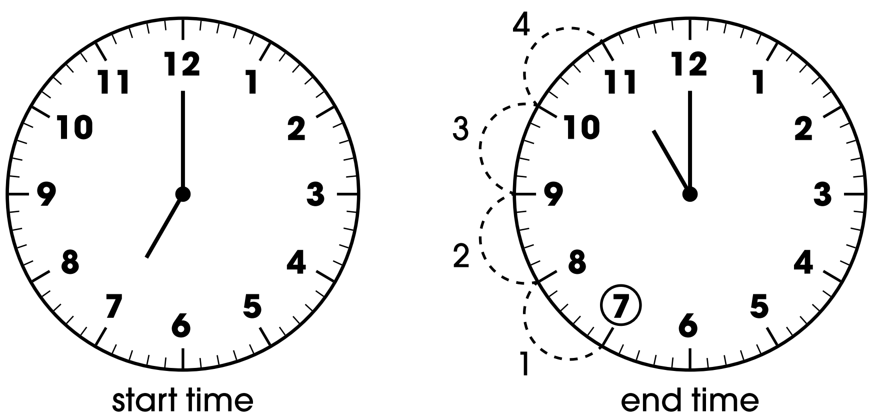
Solve authentic problems involving addition and subtraction of time intervals and find elapsed time.

Extended: Solve authentic problems involving addition and subtraction of time intervals to find elapsed time, limited to whole-number hours.

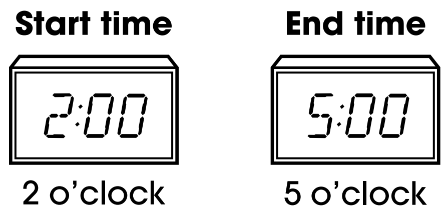
Scaffolding Activities for the Extended Indicator

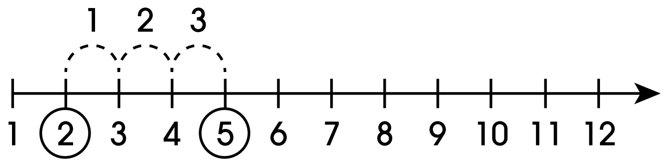
❑ **Find the elapsed time when given the start time and end time.**

- Introduce the concept of elapsed time. Explain that elapsed time is the amount of time that passes from the time something starts to the time something ends. Provide examples of elapsed time using relevant real-world scenarios.
- Demonstrate finding the elapsed time when given the start time and end time on analog clocks. Show a start time of seven o'clock and an end time of eleven o'clock. On the second clock, circle the start time and then demonstrate counting the intervals from 7 to 11 to get the elapsed time of 4 hours. Continue demonstrating with other examples of start and end times shown on analog clocks.



- Demonstrate finding the elapsed time when given the start time and the end time on digital clocks. Show a digital clock that displays two o'clock and represents the start time. Show a second digital clock that displays five o'clock and represents the end time. Circle the number 2 for the start time and the number 5 for the end time on a number line. Then count the intervals between the two numbers to get the elapsed time of 3 hours. Continue demonstrating with other examples of start and end times shown on digital clocks.





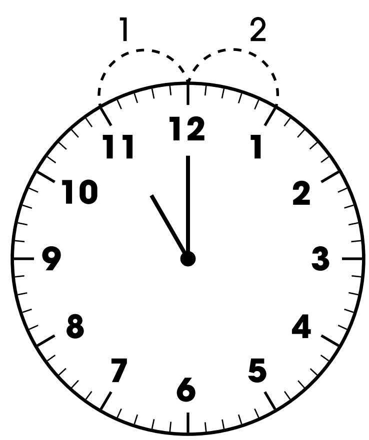
- Demonstrate other counting strategies to find the elapsed time in authentic problems. For example, say the start time of a movie, and then count up with tally marks or fingers to the end time of the movie.
- Ask students to use a counting strategy to find the elapsed time when start and end times are given in digital or analog forms for various authentic problems.

❑ **Find the end time when given the start time and the elapsed time.**

- Demonstrate using addition to find the end time when given a start time and the elapsed time. Show the start time of six o'clock on an analog clock. Count up two intervals to find the end time of eight o'clock.
- Demonstrate other counting strategies to find the end time. For example, say the start time and then count up with tally marks or fingers to the end time. Continue with other examples showing the start time on both analog and digital clocks.

Start time		Elapsed time		End time
6:00	+	2 hours	=	?

Be sure to include examples where the elapsed time passes twelve o'clock: for example, a start time of 11:00 a.m. and an elapsed time of 2 hours.



Be sure to include authentic examples and situations of elapsed time: for example, Sarah started piano lessons at 2:00 p.m. Her lessons lasted 2 hours. What was the end time of her piano lesson?

3.G.4 Time

- Ask students to solve an authentic problem by identifying the end time on both analog and digital clocks when given a start time and the elapsed time.

Prerequisite Extended Indicators

MAE 3.G.4.a—Tell time to the hour using both analog and digital clocks.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

elapsed time, end time, hour, o'clock, start time

Additional Resources or Links

https://www.engageny.org/resource/grade-2-mathematics-module-8-topic-d-lesson-16

http://nlvm.usu.edu/en/nav/frames_asid_318_g_1_t_4.html?from=search.html?qt=nnumber%20line

(Note: Java required for website. Most recent version recommended, but not needed.)

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3.D.1 Data Collection

3.D.1.a

Create scaled picture graphs and scaled bar graphs to represent a data set with more than four categories, including data collected through observations, surveys, and experiments.

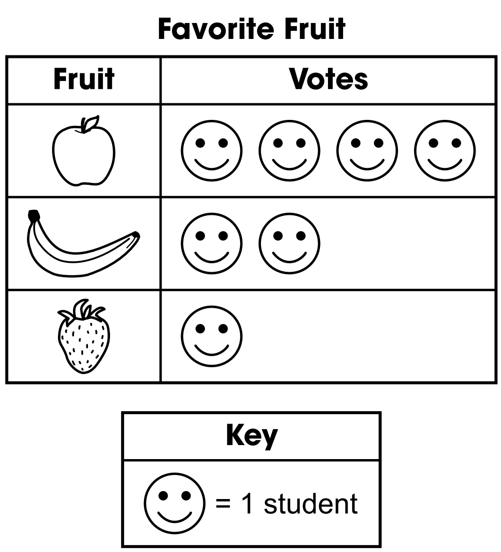
Extended: Identify characteristics (e.g., title, labels, key, scale, quantities, categories) on a bar graph, pictograph, and circle graph.

Scaffolding Activities for the Extended Indicator

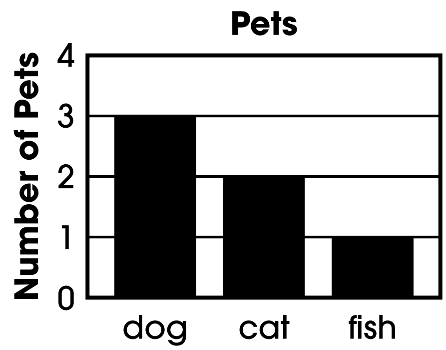
❑ **Recognize bar graphs, pictographs, and circle graphs.**

- Recognize bar graphs, pictographs, and circle graphs as distinct from other data displays such as tables and line plots. Show examples of bar graphs, pictographs, circle graphs, tables, and line plots and describe the characteristics of each type of data display.

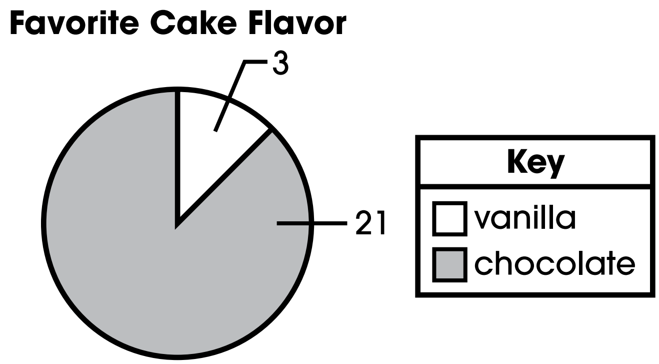
  Present a pictograph and identify it as a graph that uses symbols or pictures to represent data.



  Present a bar graph and identify it as a graph that uses bars of different lengths to represent data.



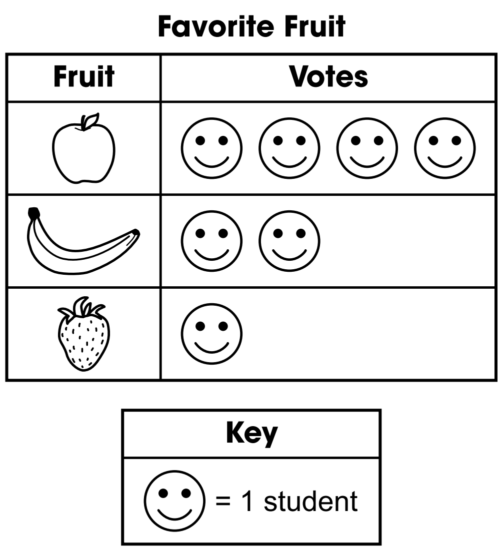
Present a circle graph and identify it as a visual representation of data, made by dividing a circle into sections that each represent parts of a whole.



- Ask students to identify a pictograph, bar graph, and circle graph from a group of data representations.

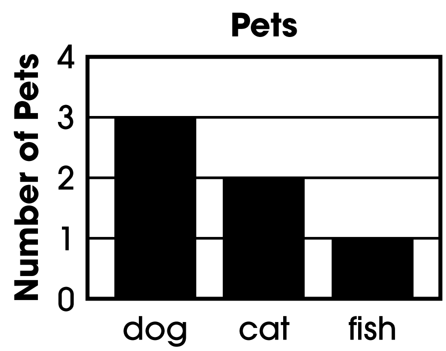
❑ **Identify a characteristic of a bar graph, pictograph, or circle graph.**

- Point to and name the title, axis labels, and categories for bar graphs, pictographs, and circle graphs. Present a pictograph showing data collected about which fruit students prefer.



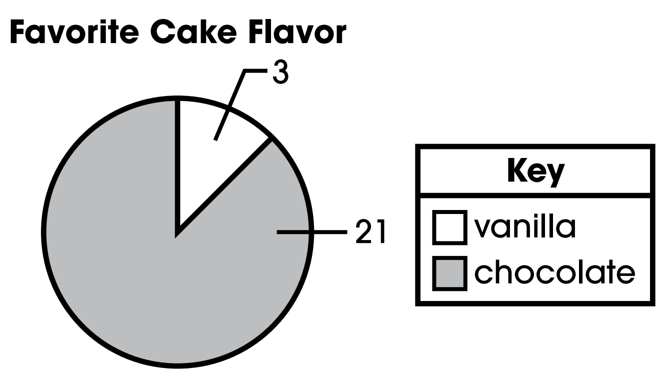
Identify the title of the pictograph and reinforce that the title tells what the pictograph is about. Explain that the picture graph has two columns where the first column displays the three different fruits students voted on and the second column displays the number of votes for each fruit. Indicate to each row and column.

Present a bar graph showing data collected about which pets students have at home.



Identify the title of the bar graph and reinforce that the title tells what the bar graph is about. Identify the categories given at the horizontal axis of the bar graph. Identify the label on the vertical axis.

Present a circle graph showing data collected about the favorite cake flavor of students.



Identify the title of the circle graph and reinforce that the title tells what the circle graph is about. Identify the categories of vanilla and chocolate. Identify the numerical labels that tell how many students voted for vanilla and how many students voted for chocolate.

- Ask students to point to the title, axis labels, and categories of a given bar graph, pictograph, and circle graph.

- Ask students to complete a bar graph by placing the missing axis labels and title in the appropriate places on a bar graph. For example, present students with the gridlines and bars for a bar graph as shown and three cards with the following information: the title, the labels for the categories on the horizontal axis, and the label for the scale on the vertical axis. Ask students to place the three cards to complete the bar graph. Repeat the process with pictographs and circle graphs.

❑ **Recognize the scale of bar graphs and the key of pictographs and circle graphs.**

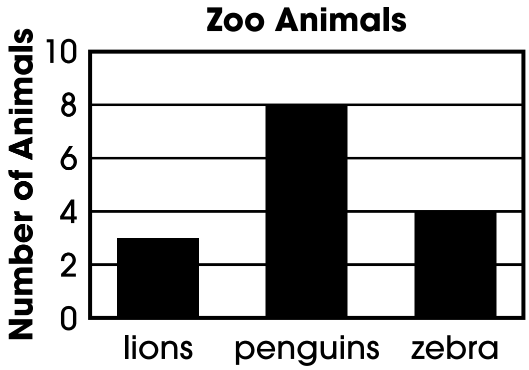
- Present a pictograph with a key. Identify the key and explain that the key indicates what each picture symbol in the pictograph represents. Show a variety of pictographs to emphasize that many different picture symbols are used in pictographs and to provide practice finding the key in different locations above, below, or beside the graph. Repeat the process with circle graphs as well.

3rd Grade Book Sale

Day	Books Sold
Tuesday	Book Book Book
Wednesday	Book Book Book Book
Thursday	Book Book

Key
Book = 2 books

Present a bar graph with a clearly marked scale.



Identify the scale as the numbers along the left side of the bar graph and explain that the scale is needed to determine the number in each category.

- Ask students to point to the scale on a bar graph and the key on a pictograph or circle graph.

❑ **Identify the scale of a bar graph and/or the key of a pictograph or circle graph.**

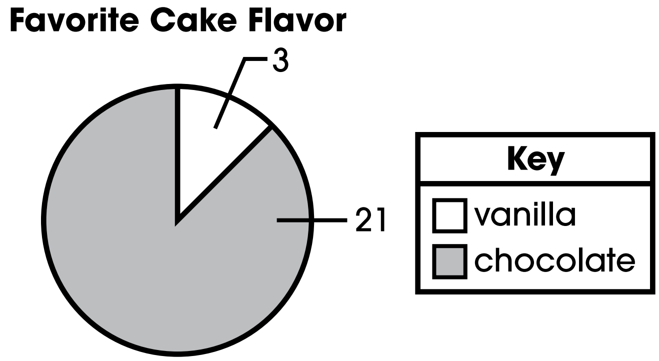
- Present the pictograph and locate the key. Explain that the key indicates that each book in a category stands for 2 books in the book sale. Use other examples to demonstrate keys where the picture symbol represents other amounts.

3rd Grade Book Sale

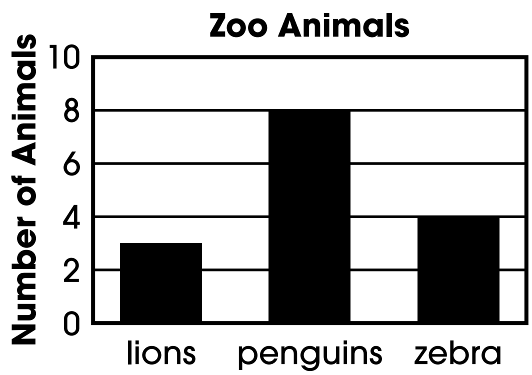
Day	Books Sold
Tuesday	Book Book Book
Wednesday	Book Book Book Book
Thursday	Book Book

Key
Book = 2 books

- Present the circle graph and locate the key. Explain that the key indicates that the circle graph represents two different flavors. Explain that the labels of numbers and the key tell us that 3 students voted for vanilla cake and 21 students voted for chocolate cake. Use other examples to demonstrate how the key helps us read the circle graph.



Present the bar graph above and locate the scale. Explain that the scale shows that the space between the lines represents a value of 2. Demonstrate that the height of the bars will end either at a scale line or between two lines. Use other examples to demonstrate bar graphs with different values and scales.



- Ask students to identify the key of a pictograph and to determine the value of an individual symbol used in the pictograph.
- Ask students to identify the key of a circle graph and to determine the value of an individual section in the circle graph.
- Ask students to identify the scale of a bar graph and to determine the scale used (i.e., the number value between each line).

Prerequisite Skills

- Recognize different parts of an illustration or image.
- Identify objects that do not belong to a particular group and the reason why objects do not belong to a particular group.
- Identify, sort, and classify objects by size, shape, color, and other attributes.

Key Terms

axis, bar graph, category, circle graph, data, key, label, pictograph, scale, title

Additional Resources or Links

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-2

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-3

3.D.1.b

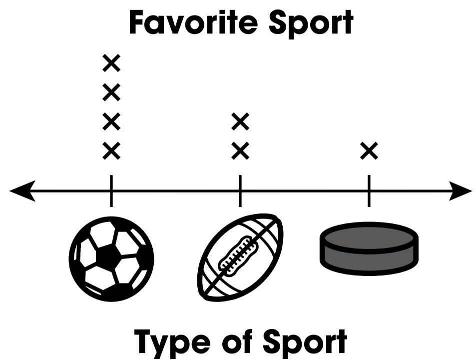
Generate and represent data using line plots where the horizontal scale is marked off in halves and whole-number units.

Extended: Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Scaffolding Activities for the Extended Indicator

❑ **Identify the location of the title and labels for line plots.**

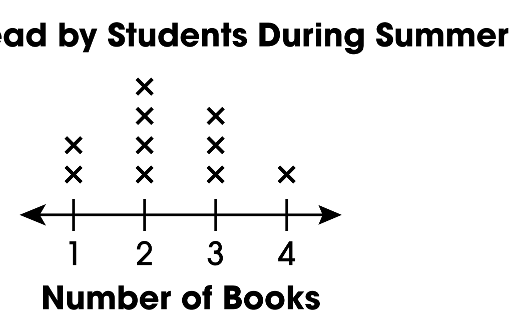
- Present a line plot as shown and explain that a line plot is a way to organize data using a number line. Point to and identify the title of the line plot. Explain that line plots need titles to tell us what the graph is about. Point to the label of the line plot. Explain that line plots need labels to provide additional information about the type of data shown. Model identifying the title and labels for a variety of line plots.



- Ask students to identify the title of a line plot.
- Ask students to identify the label(s) of a line plot.

❑ **Identify the horizontal axis of a line plot.**

- Present a line plot as shown. Point to the horizontal axis. Explain that the number line that goes from left to right is the horizontal axis of a line plot. Explain that the numbers below the horizontal axis are necessary to organize the data. Model identifying the horizontal axis for a variety of line plots.

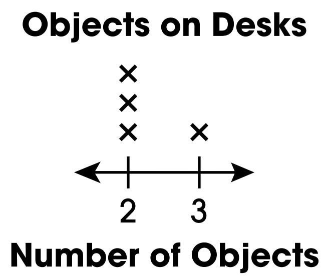


- Ask students to identify the horizontal axis of a line plot.

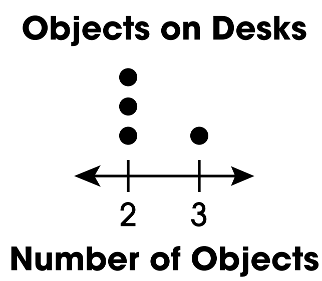
3.D.1 Data Collection

- ❑ **Recognize that x's or dots on a line plot represent quantities.**

  - Present the line plot shown and explain that the line plot is about the number of objects on different desks. Point to the numbers below the number line and explain that they show how many objects were on the desks. Point to the x's and explain that each x represents one, and in this case, one desk. Explain that line plots have x's or dots above the number line to represent quantities or how many of each. In this occurrence, the x's show that there was a quantity of three desks that had two objects and a quantity of one desk that had 3 objects.



  Inform students that data on line plots can be shown with x's or dots. Present the line plot with dots. Point to the dots and explain that the quantities displayed by these dots are the same quantities shown by the line plot with x's. Both line plots show the same quantities of objects on desks.



  Pose the question "How can I find out how many desks had two objects?" Model pointing to the three x's on the first line plot and then the three dots on the second line plot. Explain that each x or each dot represents the quantity of one desk. Continue to model identifying x's and dots on various line plots as representations of quantities.

  - Ask students to point to the part of the line plot that represents the quantity, or how many of each, of data on various line plots.
  - Ask students "How many . . ." questions about the quantities shown when provided with various line plots.

Prerequisite Extended Indicators

MAE 3.D.1.a— Identify characteristics (e.g., title, labels, key, scale, quantities, categories) on a bar graph, pictograph, and circle graph

Key Terms

data, horizontal axis, label, line plot, number line, title, quantity

Additional Resources or Links

https://tasks.illustrativemathematics.org/content-standards/2/MD/D/9/tasks/485

https://www.mathlearningcenter.org/educators/free-resources/lessons-publications/bridges-1st-edition-activities/3 (Data Analysis, Set E3: Line Plots, pdf)

3.D.2 Analyze Data and Interpret Results

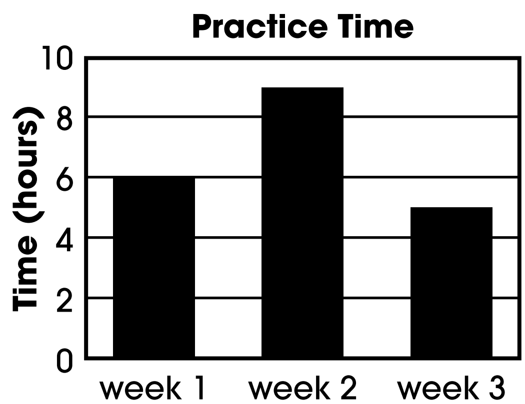
3.D.2.a

Analyze data and make simple statements using information represented in picture graphs, line plots, and bar graphs.

Extended: Identify and compare quantities in pictographs and bar graphs.

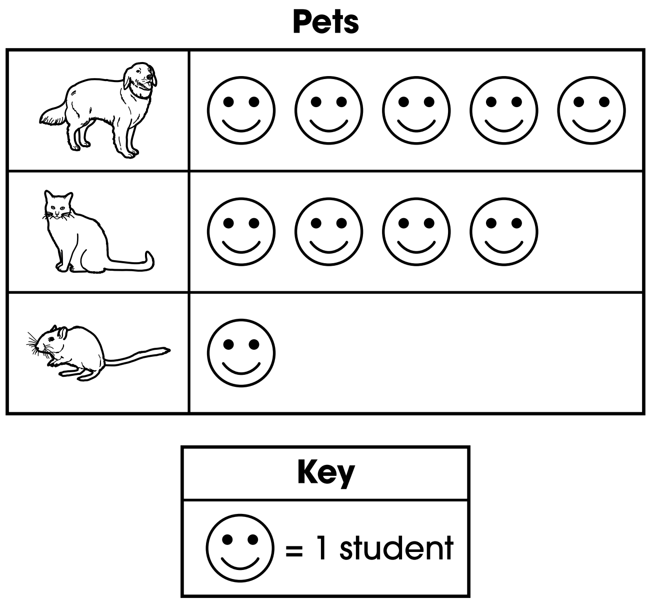
Scaffolding Activities for the Extended Indicator

- ❑ **Identify the categories, frequencies, and scale of a bar graph or the categories, frequencies, and key of a pictograph.**
  - Present the bar graph shown. Explain that this bar graph represents the number of hours a student spent practicing piano lessons each week. Demonstrate locating the title, categories, and scale.



Demonstrate using the scale of 2 to determine the number of hours in each category.

- Present the pictograph shown. Explain that this pictograph shows the data gathered when 10 students were asked what pet they have at home. Demonstrate locating the title, categories, and key.

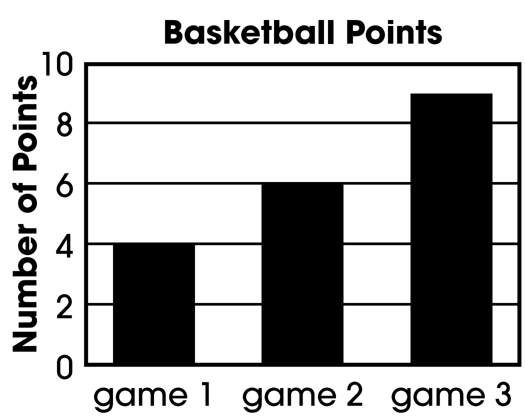


Demonstrate using the key to determine the number of students in each category.

- Ask students to compare a bar graph with a scale of 1 to a bar graph with a scale **not** equal to 1. Ask students to select the bar graph with the scale that is **not** equal to 1.
- Ask students to compare a pictograph with a key of 1 to a pictograph with a key **not** equal to 1. Ask students to select the pictograph with the key that is **not** equal to 1.

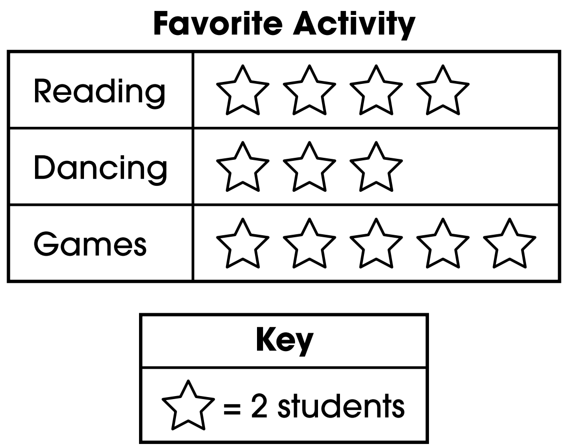
❑ **Interpret and compare information in a bar graph or a pictograph using at least two data points.**

- Present the bar graph shown. Explain that this bar graph represents the number of points a student scored during three basketball games. Demonstrate locating the title, categories, and scale.



Demonstrate using the scale to determine the number of points in each category. Explain how to identify a number that is between the labeled lines. Ask questions about the bar graph data, such as “Which game had the least number of points scored?” Avoid problems that require addition or subtraction between categories for this standard.

- Present the pictograph shown. Explain that this pictograph represents the data collected when 24 students were asked what they liked to do for fun. Demonstrate locating the title, categories, and key.



Demonstrate using the key to determine the number of students in each category. Demonstrate skip counting to determine 6 students chose dancing. Or draw 2 tally marks by each star and then count the tally marks to determine the total number of students. Ask questions about the pictograph data, such as “Which activity is the most popular?” and “Did more students like dancing or games?” Avoid problems that require addition or subtraction between categories for this standard.

- Ask students to interpret and compare data between categories on a bar graph with a scale of 1 and a pictograph with a key of 1.
- Ask students to interpret and compare data between categories on a bar graph with a scale that is not equal to 1 and on a pictograph with a key that is not equal to 1.

Prerequisite Extended Indicators

MAE 3.D.1.a—Identify characteristics (e.g., title, labels, key, scale, quantities, categories) on a bar graph, pictograph, and circle graph.

MAE 3.N.1.b—Compare and order whole numbers 1–20, using number lines or quantities of objects.

Key Terms

bar graph, compare, category, data, greatest, key, least, most, pictograph, scale

Additional Resources or Links

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-1

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-3

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