NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators High School

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade HS
Number

HS.N.1 Estimation and Technology

HS.N.1.a

Select, apply, and explain the method of computation when problem solving using real numbers (e.g., models, mental computation, paper-pencil, technology).

Extended: Identify an operation that leads to a solution in a word problem.

Scaffolding Activities for the Extended Indicator

❑ **Identify an operation that leads to the solution of an authentic problem.**

- Use story problems to demonstrate how to look for key words or other clues about the operation needed to solve the problem. Present the story problem shown below. Explain that words like “altogether,” “total,” “more,” and “sum” all indicate addition. The question in this problem needs addition to solve the problem.

  The soccer team scored 1 point in the first half of the game.

  In the second half of the game, the soccer team scored 5 points.

  How many points did the soccer team score altogether?

  Present the story problem shown below. Explain that terms like “remain,” “left over,” “less,” and “difference” all indicate subtraction. This problem about moths needs subtraction to solve.

  There are 10 moths flying around a light bulb at night.

  Then 3 moths fly away.

  How many moths remain?

  Present the story problem shown below. Explain that terms like “each,” “in a row,” and “product” indicate multiplication. This problem about reading needs multiplication to solve.

  Micah reads 20 pages of a book each day.

  Micah reads for 5 days in a row.

  How many pages of the book does Micah read?

  Present the story problem shown below. Explain that terms like “each time,” “per,” and “quotient” indicate division. This problem about swimming needs division to solve.

  Rachel is swimming laps and wants to swim 40 laps this week.

  She swims 4 times this week and swims an equal number of laps each time.

  How many laps should Rachel swim each time to complete 40 laps?

Continue to demonstrate identifying the correct operation (+, –, ×, or ÷) in a variety of story problems. It might be helpful to create a chart with the four operations and clue words as a reference. Provide drawings as necessary to reinforce the operation needed to solve each problem.

- Ask students to identify the operation needed to solve a story problem when given a drawing or chart with clue words to reference.
- Ask students to identify the operation that leads to the solution of an authentic problem.

❑ **Identify an expression or equation that leads to the solution of an authentic problem.**

- Use story problems to demonstrate how to identify an expression or equation that leads to the solution. For example, when a problem indicates that there will be **more** of something (or the answer will be a greater number than what is presented in the problem), it typically means it will either be an addition or a multiplication expression.

  Present the following story problem about a basketball game.

  Stef made 4 shots in her basketball game.

  Each shot Stef made was worth 3 points.

  How many points did Stef score?

  Demonstrate identifying the expression or equation by first finding the numbers in the story problem that will be needed for the expression. For this example, explain that the numbers are 4 and 3, representing the shots and the point value of each shot. Next, look for the clue words that indicate which operation to use with the 4 and the 3. In this case, multiplication will be used, so the expression can be 4 × 3 or 3 × 4. Addition can be ruled out as an option because 4 + 3 in this context would be adding the number of shots to the points, so the answer would not match the question, which asks only about the total points. Another expression could be 3 + 3 + 3 + 3, but 4 × 3 is the simpler option.

  Present the following story problem about apples.

  There are 30 slices of apples for a snack.

  The slices of apples will be shared equally with 5 friends.

  How many apple slices will each friend get?

  For this example, first identify that the numbers in the problem are 30 and 5. Next, explain that the answer to the problem will be less than the original number of 30 slices, so the problem involves either subtraction or division. Reference the clue words “shared equally” in this context. It may help to demonstrate with manipulatives. The expression to best fit this example is 30 ÷ 5.

Continue to demonstrate identifying the expression and the equation that leads to a solution in a variety of one-step addition, subtraction, multiplication, and division story problems.

- Ask students to identify an expression that leads to the solution of an authentic problem when given a choice of two or more expressions.
- Ask students to identify an equation that leads to the solution of an authentic problem when given a choice of two or more equations.

Prerequisite Extended Indicators

MAE 6.A.2.a—Match a simple word phrase with an input-output box.

MAE 6.N.2.a—Divide a two-digit number by a one-digit number with a remainder.

MAE 3.A.1.d—Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

Key Terms

add, divide, multiply, operation, solution, subtract

Additional Resources or Links

https://nysed-prod.engageny.org/resource/grade-2-mathematics-module-4-topic-c-lesson-16

https://nysed-prod.engageny.org/resource/grade-6-mathematics-module-4-topic-f-lesson-18

HS.N.1.b

Determine if the context of a problem calls for an approximation or an exact value.

Extended: Determine if the context of a problem calls for an approximation or an exact value.

Scaffolding Activities for the Extended Indicator

❑ **Use context clues given in word problems to determine the level of accuracy needed.**

- Explain to students that each mathematical situation can require different levels of accuracy. When calculating a value that is a solution to a problem or determining a measurement, there are times when an approximation will work and other times when an exact value is necessary. An approximation refers to a measurement or value that is close to the actual value. It is an estimation that is not precise. An exact level of accuracy refers to a measurement or value that is accurate or precise without any error.

  Emphasize to students that some problems or situations require an exact or more precise answer compared to other problems or situations that require only an approximate value. Use the examples below to discuss with students why exact values would be needed or why approximate values would be good enough. Model identifying the context clues in each sentence that provide information that will determine the level of accuracy needed. For example, when discussing how long it will take to complete a hike, it is not possible to calculate the time without a level of error, so that value can be measured with an approximation. On the other hand, during a football game, each team has the same number of players on the field, so figuring out how many players could be an exact measurement.

  Some examples of situations that require an exact or a very precise value are:

  - solving mathematical equations,
  - cutting pieces of wood to build a shed,
  - using the correct number of eggs for a recipe,
  - calculating the cost of an item after a discount is applies, and
  - determining the number of football players allowed on the field.

  Some examples of situations that require only an approximation for a value are:

  - estimating the weight of a very large object,
  - estimating the length of time needed to complete a long hike,
  - estimating the number of nails needed for a construction project,
  - estimating the number of hours it takes to drive halfway across the country, and
  - estimating without counting the number of attendees at a meeting where only some of a fixed number of chairs are being used.

- Ask students to use context clues to determine the level of accuracy needed for measuring when provided with a word problem.

- ❑ **Determine whether the context of a problem calls for an approximation or an exact value.**

  - Provide students with two problems, one with a context that can be measured by an approximate value and one that can be measured by an exact value.

    Approximate Value Context: Many statements about time are approximate in nature. For example, saying “in a few minutes” or “in a couple of hours” are statements used to communicate an approximate time.

    Statement: I will finish my drink soon.

    Explain to students that the statement provides no numbers or quantities. This provides context that it will be difficult to determine an exact measurement. The word “soon” also lets us know that the exact time when the drink will be finished is unknown. The lack of exact information lets us know that this can be measured with an approximate value.

    Exact Value Context: To solve any type of mathematical equation, exact values are necessary in each step to get to a final answer that is correct.

    Statement: I started drinking my water at 2:00 p.m. and finished at 2:07 p.m.

    Explain to students that when looking at the context of this statement, the first things to notice are the numbers. It provides a start time, an end time, and additional context that something was started and then finished. This context provides us with a lot of information that can be measured by using the term “exact.” For example, she started drinking at exactly 2:00 p.m.

  - Ask students to complete each sentence by filling in the blank with either the word “exact” or “approximate.”

    1. The _____ number of people who attended the soccer game was 132.

    2. The _____ time it will take to travel to my sister’s house is about 2 hours.

    3. The recipe requires an _____ number of eggs.

    4. The _____ time for the rain to start is sometime this afternoon.

    5. My math class begins at _____ 1:15 p.m.

Prerequisite Extended Indicator

MAE HS.N.1.a—Identify an operation that leads to a solution in a word problem.

Key Terms

approximation, approximate value, context, exact value, precise

Additional Resources or Links

https://www.insidemathematics.org/classroom-videos/public-lessons/2nd-grade-math-adding-and-subtracting-money-lesson-2a

https://curriculum.illustrativemathematics.org/k5/teachers/grade-3/unit-6/lesson-2/lesson.html

HS.N.1.f

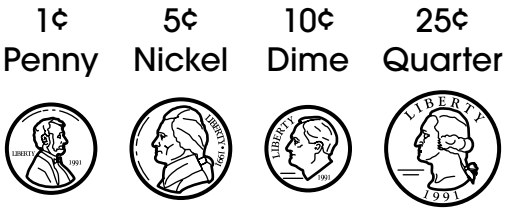
Convert equivalent rates (e.g., miles per hour to feet per second).

Extended: Use knowledge of equivalent rates to convert equivalent values with money (e.g., two dimes and a nickel is equal to one quarter).

Scaffolding Activities for the Extended Indicator

❑ **Convert equivalent rates for single coins or one bill up to $1.00.**

- Describe coins as each having a particular value. For example, a penny is worth 1¢, a nickel is worth 5¢, a dime is worth 10¢, and a quarter is worth 25¢. Use coins or coin manipulatives to show examples.



- Explain that different coins may be used to create the same value as another coin. Present the table shown and demonstrate making conversions between coins and 1 dollar.

	penny	**nickel**	**dime**	**quarter**	**dollar**
penny	1 penny				
nickel	5 pennies	1 nickel			
dime	10 pennies	2 nickels	1 dime		
quarter	25 pennies	5 nickels		1 quarter	
dollar	100 pennies	20 nickels	10 dimes	4 quarters	1 dollar

Use various examples to show the equivalencies and use skip counting as a strategy.

- Ask students to convert a single larger monetary value into coins with smaller values. For example, ask students to answer the following questions.

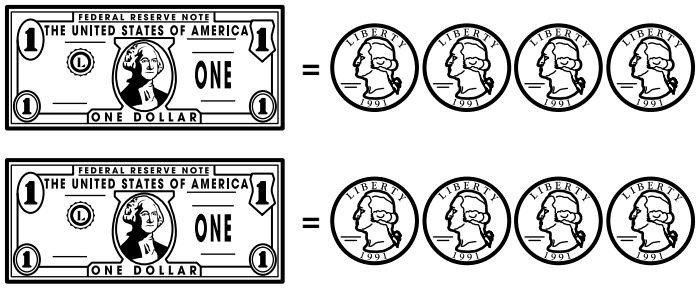
How many dimes are there in 1 dollar?	How many nickels are there in 1 quarter?
How many pennies are there in 1 dime?	How many quarters are there in 1 dollar?

- Ask students to convert coins with smaller values into a single coin with a larger value. For example, ask students to answer the following questions.

Which single coin is equal to 5 nickels?	Which single coin is equal to 10 pennies?
Which single coin is equal to 25 pennies?	Which single coin is equal to 2 nickels?

❑ **Convert equivalent rates for multiple coins or bills up to $2.00.**

- Explain that the value of multiple coins or bills may be found by using the value of single coins or bills. For example, quarters can be used to make 2 dollars because if it is known that 4 quarters are equal to 1 dollar, then 8 quarters must be equal to 2 dollars.



Demonstrate the same process to find the value of multiple coins. For example, if there are 5 nickels in 1 quarter, then there are 10 nickels in 2 quarters.

- Ask students to convert multiple coins or bills. For example, ask students to answer the following questions.

How many nickels are there in 2 dollars?	How many pennies are there in 2 quarters?
How many nickels are there in 2 dimes?	How many dimes are there in 2 dollars?

- Ask students to convert coins or bills in multiple ways. For example, ask students, “What are some coins that are equal to 2 dollars?” Students may respond with 20 dimes, 8 quarters, 40 nickels, etc.

Prerequisite Extended Indicators

MAE 8.A.2.b—Given a table, determine the rate of change of a proportional relationship.

MAE 7.R.1.b—Given a proportional relationship that represents an authentic situation, determine the missing quantity.

MAE 6.R.1.e—Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

Key Terms

convert, dime, dollar bill, exchange, nickel, penny, quarter

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_325_g_3_t_1.html?from=search.html?qt=money

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-b-lesson-6

HS.N.2 Sets and Operations

HS.N.2.a

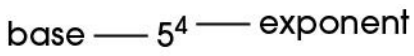
Extend the properties of exponents to rational numbers.

Extended: Rewrite a repeated multiplication problem as an exponential expression with a whole number base and a whole number exponent (e.g., 3 × 3 × 3 × 3 = 3^4).

Scaffolding Activities for the Extended Indicator

❑ **Recognize the vocabulary used to name exponents.**

- Use an expression to explain how to read and write exponents. For example, in the expression shown, the number 5 represents the base and the number 4 represents the exponent. It is read as "five to the fourth power."



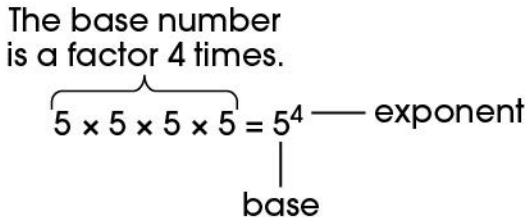
  Continue to read examples of exponential expressions, like 6^3 and 4^2. Indicate that the base is read first, then the exponent: "six to the third power" or "six cubed" and "four to the second power" or "four squared."

- Ask students to select the correct exponential expression when given a set of three expressions with the same exponents but different bases. For example, present the choices 4^2, 2^2, and 3^2 and ask which expression shows two squared or which expression shows three to the second power.

- Ask students to select the correct exponential expression when given a set of three expressions with different exponents but the same bases. For example, present the choices 4^3, 4^2, and 4^4 and ask which expression shows four cubed or which expression shows four to the second power.

❑ **Rewrite a repeated multiplication problem as an exponential expression.**

- Demonstrate that when a number is multiplied by itself several times, it may be written as an exponential expression. For example, 5 can be multiplied by itself 4 times. Represent the base and the exponent as shown.



  Continue to demonstrate with other examples by writing and describing correct usage of the base and exponent.

- Ask students to select the exponential expression represented by a given multiplication problem when presented options with the same base and different exponents. For example, ask students, “Which expression is equivalent to 7 × 7 × 7?” Present 7^2, 7^3, and 7^4 as the answer options. Students should select 7^3 as the correct answer.
- Ask students to select the exponential expression represented by a given multiplication problem when presented options with different bases and different exponents. For example, ask students, “Which expression is equivalent to 7 × 7 × 7?” Present 5^7, 7^3, and 3^7 as the answer options. Students should select 7^3 as the correct answer.

Prerequisite Extended Indicators

MAE 8.N.1.b—Represent numbers with the bases of 2, 3, 4, or 5 and positive exponents of 2 and 3 in expanded form (e.g., 4^3 = 4 × 4 × 4).

MAE 8.N.2.a—Identify the squares of whole numbers up to 10.

MAE 5.N.1.c—Represent 10, 100, 1,000, or 10,000 as a power of 10.

Key Terms

base, cubed, exponent, power, squared

Additional Resources or Links

https://curriculum.illustrativemathematics.org/MS/teachers/3/7/1/preparation.html

https://www.engageny.org/resource/grade-8-mathematics-module-1-topic-lesson-1/file/46091

HS.N.2.d

Compute with subsets of the complex number system including imaginary, rational, irrational, integers, whole, and natural numbers.

Extended: Add and subtract two-digit numbers with regrouping.

Scaffolding Activities for the Extended Indicator

❑ **Add two-digit numbers with regrouping.**

- Present an addition problem to students using two-digit addends.

  17 + 56 = ____

  Explain that when adding two-digit numbers, the numbers in the ones place are added together first. If the sum of the numbers in the ones place is 10 or more, it must be regrouped. Explain that regrouping with two-digit numbers is the process of making groups of ten. Regrouping with the use of a place-value chart is helpful to show how the groups of ten are rearranged. Refer to MAE 4.A.1.a as needed for additional scaffolding using base ten blocks.

  Model setting up the equation within a place-value chart.

	Tens	Ones
	1	7
+	5	6

  Point to 17 in the equation and emphasize that the 7 is in the ones place. Point to 56 in the equation and emphasize that the 6 is also in the ones place. Model adding 7 and 6 and state that the answer is 13. Explain to students that since the sum of the numbers in the ones place is more than 10, it will need to be regrouped. Explain that the number 13 has 1 group of 10, and 3 more ones. Demonstrate this by adding a 1 to the top of the tens column, and a 3 below the equation in the ones column.

	Tens	Ones
	1	
	1	7
+	5	6
		3

Next, model adding together all numbers in the tens column. Explain that 5 + 1 + 1 is 7. Indicate that the place-value chart shows there are 7 groups of ten and 3 ones, which is equal to 73.

	Tens	Ones
	1	
	1	7
+	5	6
	7	**3**

Repeat this process using different two-digit numbers.

- Provide students with an addition equation using two-digit numbers as shown. Provide students with number cards and a blank place-value chart. Model putting the number cards in the correct place values to create the equation. Model regrouping by adding the number card with a 1 to the top of the tens column.

  27 + 35 = ___

	Tens	Ones
+		

1	**2**	**2**	**3**	**5**	**6**	**7**

Repeat this process with various addition problems.

- Ask students to add two-digit numbers with regrouping. For example, provide a blank place-value chart and an addition problem as shown.

  49 + 66 = ____

	Tens	Ones
+		

❑ **Subtract a two-digit number from a two-digit number with regrouping.**

- Present a subtraction problem to students using two-digit numbers.

  65 – 29 = ____

  Indicate to the place-value chart. Explain that using a place-value chart is helpful when subtracting. Model writing the equation in the place-value chart as shown.

	Tens	Ones
	6	5
–	2	9

  Explain that the bottom number is being subtracted from the top number. Emphasize that the first step is to subtract the bottom number in the ones place from the top number in the ones place. Since the top number is smaller than the bottom number, the first step will be to regroup. Explain that regrouping with two-digit numbers is the process of making groups of ten. Regrouping with the use of a place-value chart is helpful to show how the groups of ten are rearranged. Explain that since 9 ones cannot be taken from 5 ones, a group of ten must be exchanged for 10 ones. Demonstrate this by crossing out the 6 in the tens place and making it a 5. Next, cross out the 5 in the ones place and write 15 above it.

	Tens	Ones
	5	15
	~~6~~	~~5~~
–	2	9

Indicate the 15 and the 9 in the ones place. Emphasize that 15 is greater than 9, so now the subtraction can begin. Model subtracting 9 from 15 and writing 6 in the ones column. Model subtracting 2 from 5 and writing 3 in the tens column. Emphasize that the place-value chart shows that there are 3 groups of ten and 6 ones, indicating that 65 – 29 = 36.

	Tens	Ones
	5	15
	~~6~~	~~5~~
–	2	9
	3	6

Repeat this process using different two-digit numbers

- Provide students with a subtraction equation using two-digit numbers as shown. Provide students with number cards and a blank place-value chart. Model putting the number cards in the correct place values to create the equation. Model the process of taking a group of 10 from the tens place by swapping out the number card with a 3 on it for a number card with a 2. Model the process of adding 10 ones to the ones column by taking away the 3 from the ones place and replacing it with the 13. Model solving the problem.

  33 – 17 = ___

	Tens	Ones
–		

1	**1**	**2**	**3**	**3**	**6**	**7**	**13**

Repeat this process by using different two-digit numbers.

- Ask students to subtract a two-digit number from a two-digit number with regrouping. For example, provide a blank place-value chart and a subtraction problem as shown.

  71 – 39 = ____

	Tens	Ones
–		

Prerequisite Extended Indicators

MAE 4.N.1.a—Identify representations of whole numbers up to 100.

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

addition, exchange, regrouping, subtraction

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-1-topic-d-lesson-11

https://www.engageny.org/resource/grade-4-mathematics-module-1-topic-e-lesson-13

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Mathematics—Grade HS
Algebra

HS.A.1 Algebraic Relationships

HS.A.1.b

Analyze a relation to determine if it is a function given mapping diagrams, function notation (e.g., f(x)=x^2), a table, or a graph.

Extended: Given a linear function represented with a table, identify the graph of the linear function.

Scaffolding Activities for the Extended Indicator

❑ **Identify ordered pairs from a table.**

- Use a table to show how to form ordered pairs. For example, the input values represent the *x*-coordinates and the output values represent the *y*-coordinates in the ordered pair (*x*, *y*). So, the ordered pairs for the relationship from the table are (1, 7), (2, 8), and (3, 9).

Input	Output
1	7
2	8
3	9

- Ask students to identify ordered pairs from a given table. Students should identify the ordered pairs as (2, 4), (4, 3), and (6, 2).

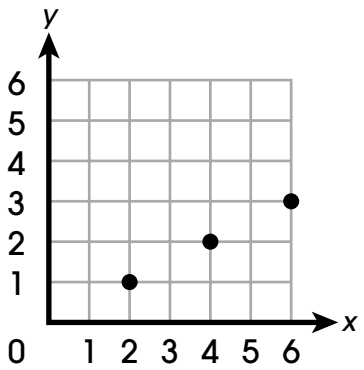
x	***y***
2	4
4	3
6	2

❑ **Identify a graph that represents a linear function when given a liner function represented with a table.**

- Explain the connection between ordered pairs from a table and a graph on a coordinate plane. For example, the table shown has the ordered pairs (2, 1), (4, 2), and (6, 3).

x	***y***
2	1
4	2
6	3

Since the ordered pairs indicate the *x*- and *y*-coordinates of three points on a graph, show the connection by graphing those three points.

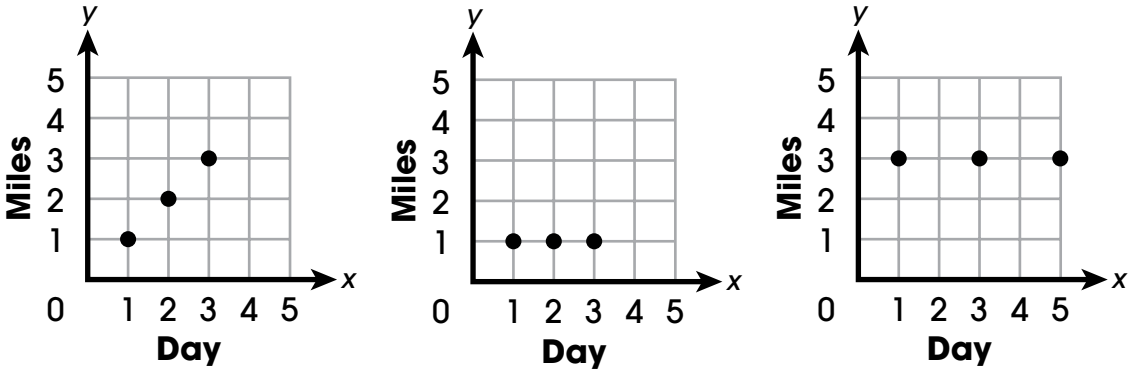


Show other tables with at least three points on a line, and then show how to identify the graph that corresponds to each table.

- Ask students to identify a graph that represents a given linear function from a table. For example, present the following table.

Day	Miles
1	1
2	1
3	1

Give students three possible graphs to choose from.



Students should identify that the points in the middle graph represent the same ordered pairs that are in the table.

Prerequisite Extended Indicators

MAE 5.G.2.b—Identify the x- or y-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

coordinate plane, function, graph, linear, ordered pair, relationship, table

Additional Resources or Links

https://nysed-prod.engageny.org/resource/grade-8-mathematics-module-6-topic-a-lesson-1

https://curriculum.illustrativemathematics.org/MS/teachers/1/6/17/index.html

HS.A.1.c

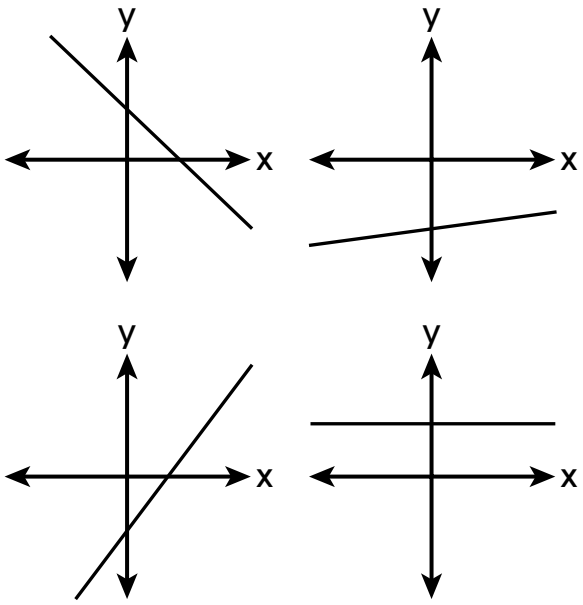
Classify a function given its mapping diagram, function notation, table, or graph as a linear, quadratic, absolute value, exponential, or other function.

Extended: Identify a linear function from a graph.

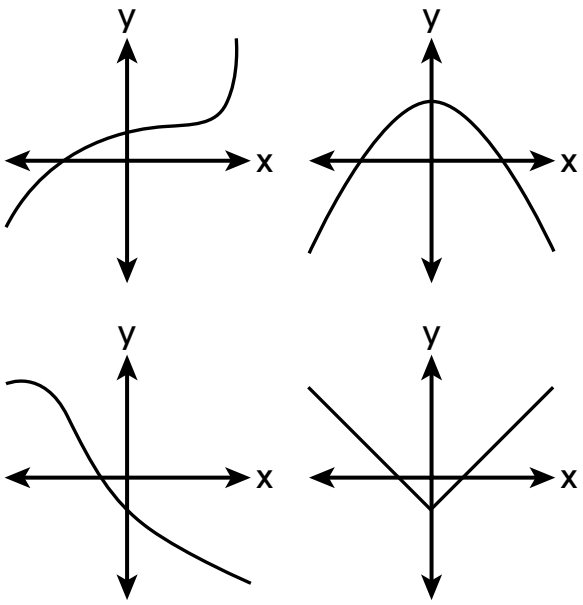
Scaffolding Activities for the Extended Indicator

❑ **Identify a linear function from a graph.**

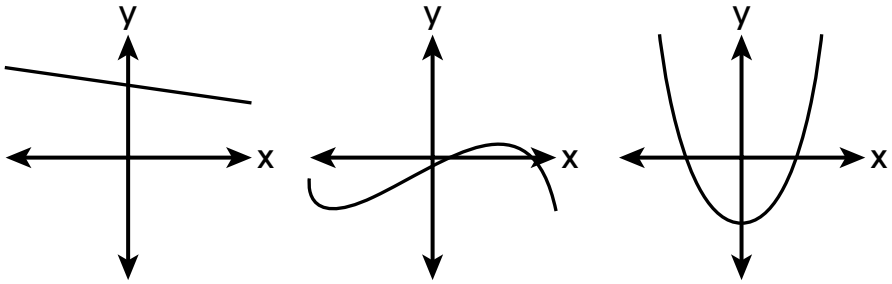
- Use sketches of graphs to show linear functions. Emphasize that each graph is a line that is perfectly straight. This can be verified by using a straight object, like a pencil or ruler, to follow the line.



Also show graphs of functions that are NOT linear. Indicate that when a graph of a function has curves or corners, it is not a linear function. Linear functions can also be called linear relationships since they show the relationship between the *x*- and *y*-values on a graph.



- Ask students to identify a linear function from a graph. For example, give students the following graphs.



Ask students to choose the graph that shows the linear relationship. Students should choose the graph on the left.

Prerequisite Extended Indicators

MAE HS.A.1.b—Given a linear function represented with a table, identify the graph of the linear function.

MAE 5.G.2.b—Identify the x- or y-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

curve, function, graph, line, linear, straight

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1740

https://nysed-prod.engageny.org/resource/grade-8-mathematics-module-5-topic-a-lesson-5

HS.A.1.d

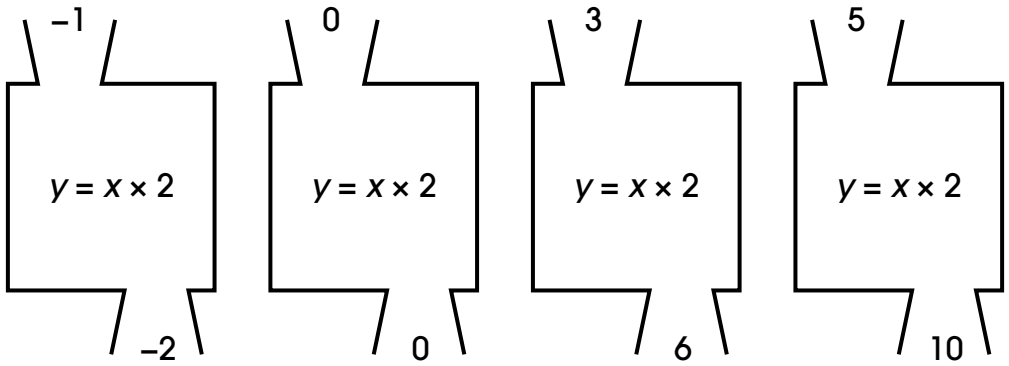
Analyze a function's domain and range to determine if it is one-to-one and has an inverse function both algebraically and graphically.

Extended: Identify the domain and range of a function when given an input-output table.

Scaffolding Activities for the Extended Indicator

❑ **Recognize that the set of input values for a function is called the domain and the set of output values is called the range.**

- Explain to students that functions are relations/relationships or rules that show an association between members of one set with members of a second set. Also, explain that functions can be explained or represented using different views or methods. One way a function can be represented is by using a function machine as shown.



In this example, the function is $y = x \times 2$. This means that when each value for x is selected, the resulting value for y is determined by substituting the value for x into the function or equation.

The input values used at the top of the function machine {−1, 0, 3, 5} are called the **domain** of the function and the output values at the bottom of the function machine {−2, 0, 6, 10} are called the **range** of the function.

- Explain to students that these values can also be written as ordered pairs (x, y) as shown.

  $$(-1, -2), (0, 0), (3, 6), (5, 10)$$

  Show students that all the x-values in the ordered pairs for this function represent the **domain** of the function and that all the y-values in the ordered pairs for this function represent the **range** of the function.

  Domain: −1, 0, 3, 5

  Range: −2, 0, 6, 10

- Ask students to identify the domain and the range for different functions represented in one or more different ways. For example, in this list of ordered pairs that represent a function, which values represent the domain of the function and which values represent the range of the function?

  $$(3, 0), (4, 1), (6, 3), (8, 5), (10, 7)$$

  Students can also be given this information by using a function machine, a mapping diagram, an equation, a word description, or a table. In the example provided, the student should indicate that the domain of the function is {3, 4, 6, 8, 10} and the range of the function is {0, 1, 3, 5, 7}.

❑ **Identify the domain and the range of a function when given an input-output table.**

- Explain to students that another way a function can be represented is by using a table as shown.

Input x	**Output** y
0	2
1	3
3	5
6	8

In this example, the function can be written as $y = x + 2$. This means that when each value for x is selected, the resulting value for y is determined by substituting the value for x into the function or equation.

The input values in the left column of the table {0, 1, 3, 6} are called the **domain** of the function and the output values in the right column of the table {2, 3, 5, 8} are called the **range** of the function.

- Ask students to match the given function with the correct sets of the **domain** and the **range**.

Input *x*	**Output** *y*
1	0
3	2
5	4
7	6

Option 1: Domain: {0, 2, 4, 6}; Range: {1, 3, 5, 7}

Option 2: Domain: {1, 3, 5, 7}; Range: {0, 2, 4, 6}

Option 3: Domain: {2, 4, 6, 8}; Range: {3, 5, 7, 9}

Explain to students why Option 2 correctly names the values for the **domain** and the **range** for the given function.

- Ask students to identify the **domain** and the **range** of a function given as an input-output table.

Prerequisite Extended Indicator

MAE 6.A.2.a—Match a simple word phrase with an input-output box.

Key Terms

domain, equation, function, function machine, input-output table, mapping diagram, ordered pair, range, table

Additional Resources or Links

https://www.insidemathematics.org/modeling-polynomials-post-lesson-debrief-part-1

https://illuminations.nctm.org/Search.aspx?view=search&kw=domain%20&gr=6-8

HS.A.1.e

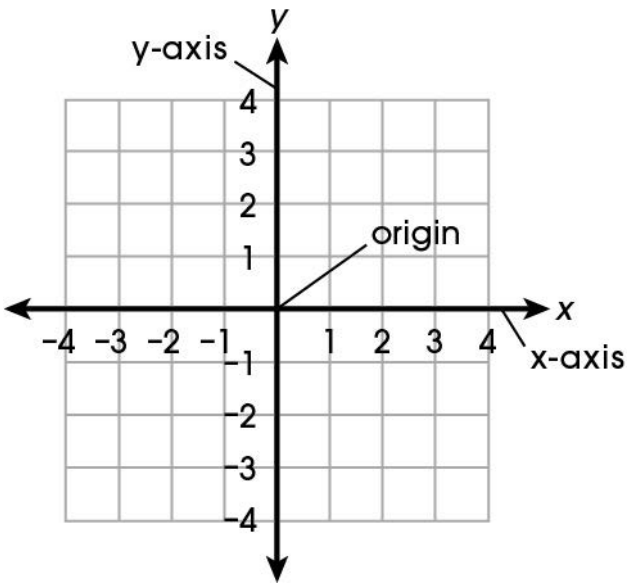
Define, interpret, and analyze linear, quadratic, absolute value, and exponential functions using the points of interest of the functions and graphing technology.

Extended: Given a graph of a linear function, determine the coordinate pair where x = 0.

Scaffolding Activities for the Extended Indicator

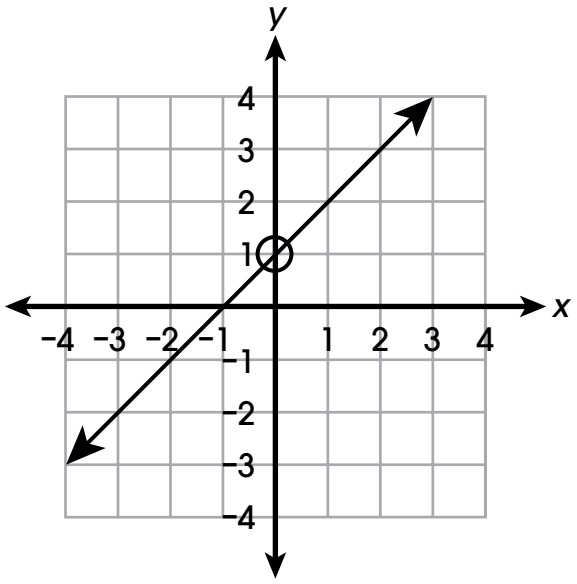
❑ **Identify the location where a graph of a linear function intersects the y-axis.**

- Use a coordinate graph to demonstrate where the *x*-axis, *y*-axis, and origin are located.



Explain that the ordered pair (0, 0) indicates that the origin is where the *x*- and *y*-coordinates are both 0 because ordered pairs are always written as (*x*, *y*).

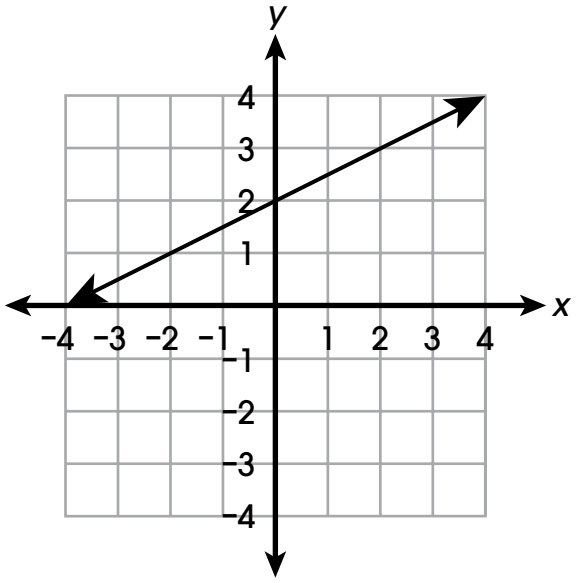
- Use the graph of a linear function to demonstrate how to locate where the function intersects the *y*-axis. Start with locating the origin, and then follow the *y*-axis either up or down to where it crosses the linear function. Indicate that intersection by circling the point or highlighting it in some way. Follow this same process with a variety of graphs of linear functions, each time finding where the line intersects the *y*-axis. Use graphs that cross above the origin as well as below the origin.



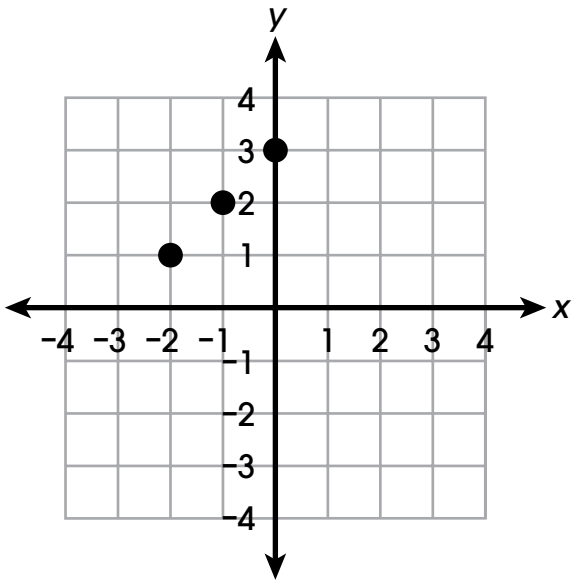
- Ask students to identify the location where a graph of a linear function intersects the *y*-axis.

❑ **Given a graph of a linear function, determine the coordinate pair where x = 0.**

- Use the graph of a linear function to demonstrate how to locate the ordinate pair or coordinate pair where *x* = 0, which is where the graph intersects the *y*-axis.



Since the *x*-coordinate at the point where the graph crosses the *y*-axis is 0, the coordinate pair of that location can be written as (0, *y*), where *y* represents the *y*-coordinate. In the graph shown, the ordered pair is (0, 2). Follow this same process with a variety of linear functions, including those that are graphed with only points, and no line, as shown.



Explain that the location where the function intersects the *y*-axis is still shown by the coordinate pair (0, *y*) whether it is a line or points in a line. For this graph, it is at the point (0, 3).

- Ask students to use the graph of a linear function to locate the coordinate pair where $x = 0$.

Prerequisite Extended Indicators

MAE HS.A.1.b—Given a linear function represented with a table, identify the graph of the linear function.

MAE HS.A.1.c—Identify a linear function from a graph.

MAE 8.A.1.a—Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

MAE 8.A.2.c—Given a graph of a line through the origin and a point on the line, determine another point on the line.

MAE 5.G.2.a—Identify the origin, x-axis, and y-axis of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

function, linear, ordered pair, origin, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://www.insidemathematics.org/performance-assessment-tasks

https://illuminations.nctm.org/Search.aspx?view=search&kw=y-intercept&st=a&g=6-8

(Walking to Class: Modeling Students' Class Schedules with Time-Distance Graphs) (Walking to Class: Modeling Students' Class Schedules with Time-Distance Graphs)

HS.A.2 Algebraic Processes

HS.A.2.a

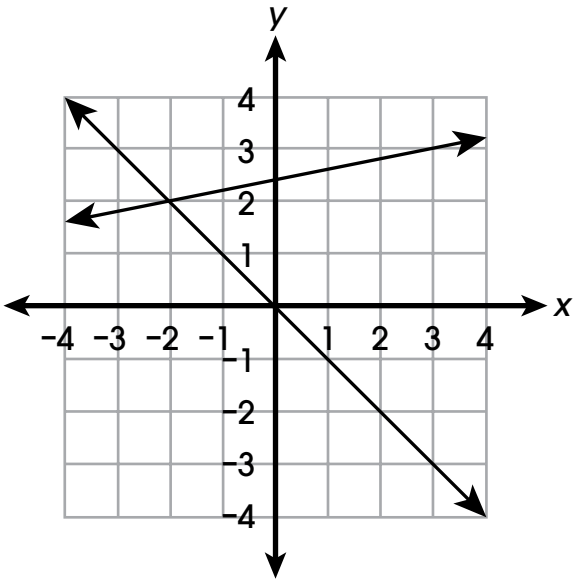
Analyze and explain the properties used in solving equations, inequalities, systems of linear equations, systems of linear inequalities, and literal equations.

Extended: Identify the ordered pair of the graphical solution to a system of two linear equations.

Scaffolding Activities for the Extended Indicator

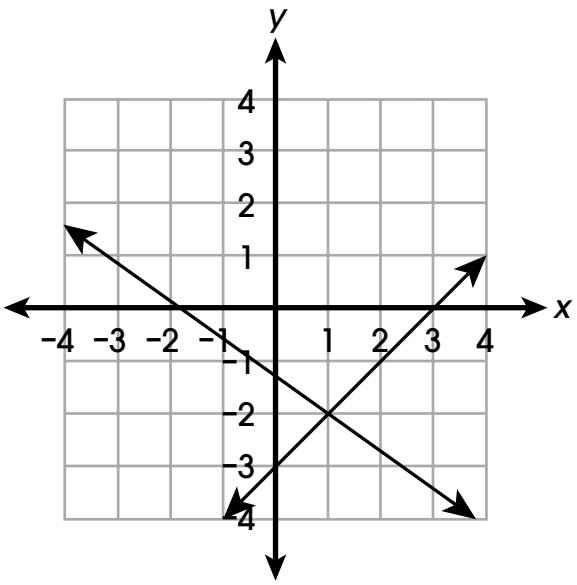
❑ **Identify the ordered pair of the graphical solution to a system of two linear equations.**

- Use a graph with two intersecting lines to show the point of intersection.



The point of intersection is the solution to the system of equations represented by the lines. The location of the point can be shown with the ordered pair (–2, 2). It may be helpful to draw a point at the intersection on the graph or to highlight the point of intersection in some way. Indicate that the ordered pair of a point is always in the form (*x*, *y*), so the ordered pair (–2, 2) shows that the point aligns with the –2 on the *x*-axis and the 2 on the *y*-axis. Show a variety of graphs with intersecting lines. The point of intersection can be in any of the four quadrants on the coordinate plane.

- Ask students to locate the point of intersection on a graph and identify the ordered pair for that location. For example, give students the following graph.



Have students choose from the following possible ordered pairs: (0, 1), (3, 0), or (1, –2). Students should determine that the point of intersection is (1, –2).

Prerequisite Extended Indicators

MAE HS.A.1.c—Identify a linear function from a graph.

MAE 8.A.1.a—Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

MAE 5.G.2.b—Identify the x- or y-coordinate of a point in the first quadrant of a coordinate plane.

Key Terms

coordinate plane, graph, intersection, line, ordered pair, origin, solution, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813

https://www.engageny.org/resource/grade-8-mathematics-module-4-topic-d-lesson-25

HS.A.2.b

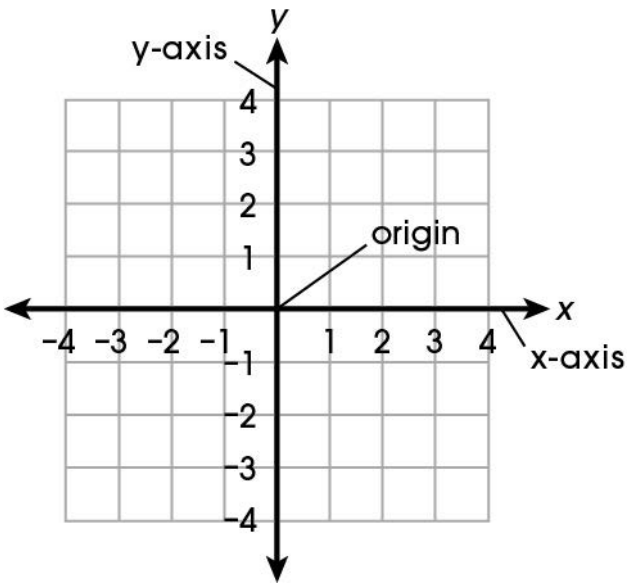
Generate expressions in equivalent forms by using algebraic properties to make different characteristics or features visible.

Extended: Given a graph of a linear function, determine the coordinate pair where $y = 0$.

Scaffolding Activities for the Extended Indicator

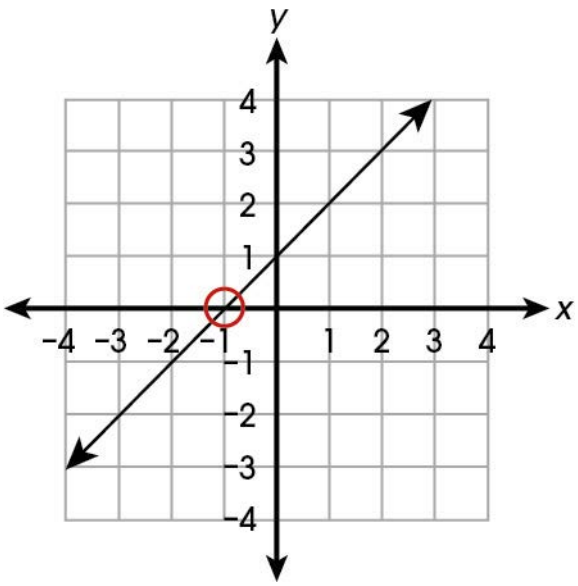
❑ **Identify the location where a graph of a linear function intersects the *x*-axis.**

- Use a coordinate graph to demonstrate where the *x*-axis, *y*-axis, and origin are located.



Explain that the ordered pair (0, 0) indicates that the origin is where the *x*- and *y*-coordinates are both 0 because ordered pairs are always written as (*x*, *y*).

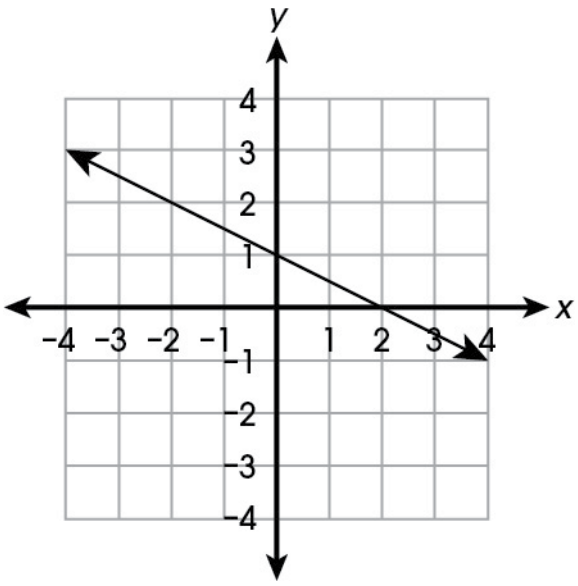
- Use the graph of a linear function to demonstrate how to locate where the function intersects the *x*-axis. Start with locating the origin, and then follow the *x*-axis either left or right to where it crosses the linear function. Indicate that intersection by circling the point or highlighting it in some way. Follow this same process with a variety of graphs of linear functions, each time finding where the line intersects the *x*-axis. Use graphs that cross to the left of the origin as well as to the right.



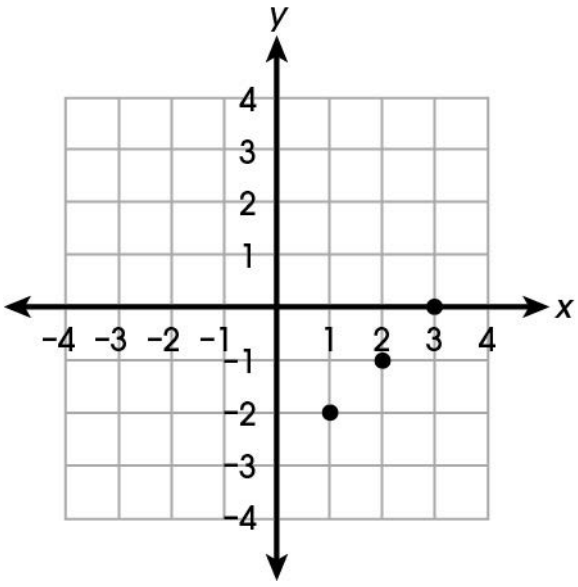
- Ask students to identify the location where a graph of a linear function intersects the *x*-axis.

❑ **Use the graph of a linear function to locate the ordered pair where $y = 0$.**

- Use the graph of a linear function to demonstrate how to locate the ordered pair where $y = 0$, which is where the graph intersects the x-axis.



Since the y-coordinate at the point where the graph crosses the x-axis is 0, the ordered pair of that location can be written as $(x, 0)$, where x represents the x-coordinate. In the graph shown, the ordered pair is (2, 0). Follow this same process with a variety of linear functions, including those that are graphed with only points, and no line, as shown.



Explain that the location where the function intersects the x-axis is still shown by the ordered pair $(x, 0)$ whether it is a line or points in a line. For this graph, it is at the point (3, 0).

- Ask students to use the graph of a linear function to locate the ordered pair where $y = 0$.

Prerequisite Extended Indicators

MAE HS.A.1.e—Given a graph of a linear function, determine the coordinate pair where x = 0.

MAE 8.A.1.a—Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

MAE 8.A.2.c—Given a graph of a line through the origin and a point on the line, determine another point on the line.

MAE 5.G.2.a—Identify the origin, x-axis, and y-axis of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

function, linear, ordered pair, origin, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-2/lesson.html

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-c-lesson-14

https://www.engageny.org/resource/grade-7-mathematics-module-1-topic-lesson-5

HS.A.2.d

Calculate the slope (rate of change) of a line given coordinate points, a graph, or a table of values.

Extended: Given an x-y table of values, determine if the graph of the values forms a horizontal line or a vertical line.

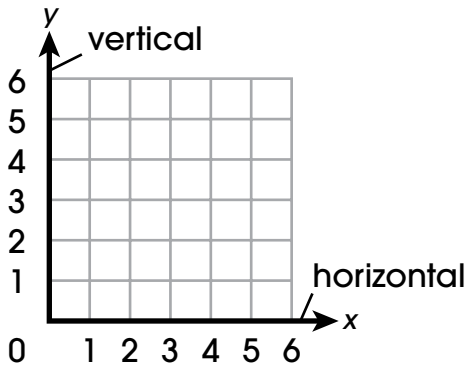
Scaffolding Activities for the Extended Indicator

❑ **Identify a horizontal line and a vertical line.**

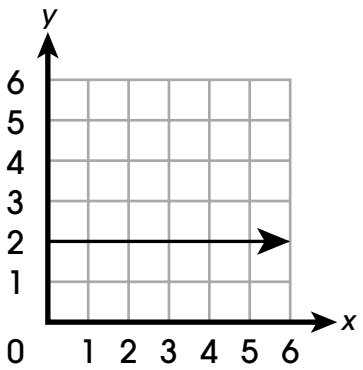
- Use real-world objects and images to teach the meaning of horizontal and vertical. For example, the top of a table is horizontal, and the legs of a table are vertical.
- Ask students to identify horizontal and vertical lines using real-world objects and images.

❑ **Determine whether values graphed in a table form a horizontal line or a vertical line.**

- Use a coordinate plane to show horizontal and vertical lines. For example, the *x*-axis is a horizontal line and the *y*-axis is a vertical line.



In addition to the axes being examples of a horizontal line and a vertical line, a graph of a line can also be a horizontal line or a vertical line. An example of a horizontal line graph is shown.

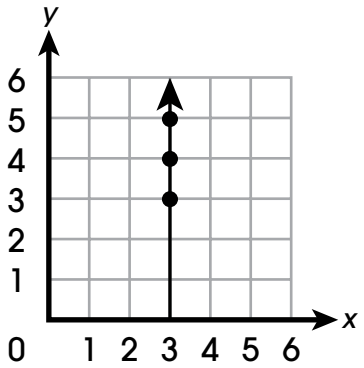


Show a variety of graphs of lines and identify which are horizontal lines and which are vertical lines.

- Use a table to demonstrate that ordered pairs can be formed to represent the relationship in the table and then graphed.

x	***y***
3	3
3	4
3	5

The ordered pairs for this table are (3, 3), (3, 4), and (3, 5) and may be graphed as shown.



Note that since all the *x*-coordinates from the table are 3, a straight line can be drawn through the three points given, so the relationship in the table is represented by a vertical line graph. Show a variety of tables, some with horizontal line graphs and some with vertical line graphs. Make sure to note that a table with all the same values for each *x*-coordinate will make a vertical line and a table with all the same values for each *y*-coordinate will make a horizontal line.

- Ask students to determine whether a given table will have a graph that is a horizontal line or a vertical line.

Prerequisite Extended Indicators

MAE HS.A.1.b—Given a linear function represented with a table, identify the graph of the linear function.

MAE HS.A.1.c—Identify a linear function from a graph.

MAE 5.G.2.a—Identify the origin, x-axis, and y-axis of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

coordinate plane, graph, horizontal, ordered pair, table, vertical, *x*-coordinate, *y*-coordinate

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-5/file/69636

http://tasks.illustrativemathematics.org/content-standards/5/G/A/1/tasks/489

HS.A.2.f

Given a line, write the equation of a line that is parallel or perpendicular to it.

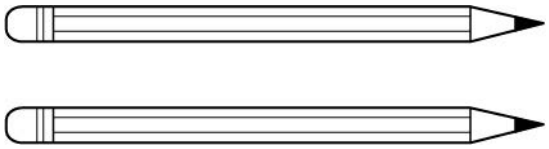
Extended: Distinguish between parallel, perpendicular, and intersecting lines on a coordinate grid.

Scaffolding Activities for the Extended Indicator

❑ **Identify parallel, intersecting, and perpendicular lines.**

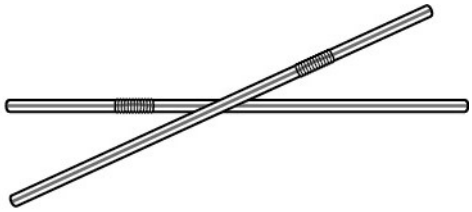
- Show images and reference examples of parallel lines in an authentic context. Indicate that parallel lines never touch and remain the same distance apart.

  Use manipulatives (e.g., pencils, craft sticks, uncooked spaghetti noodles) to represent parallel lines.



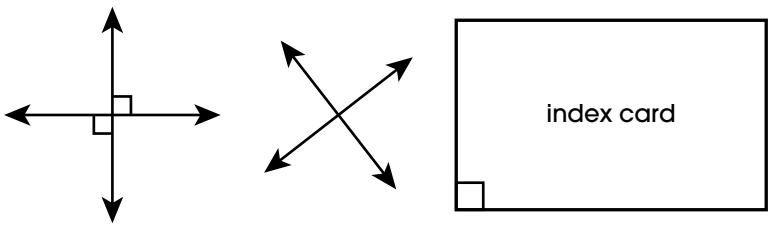
- Show images and reference examples of intersecting lines in an authentic context. Indicate that intersecting lines are lines that cross each other.

  Use manipulatives (e.g., straws, craft sticks, uncooked spaghetti noodles) to represent intersecting lines.

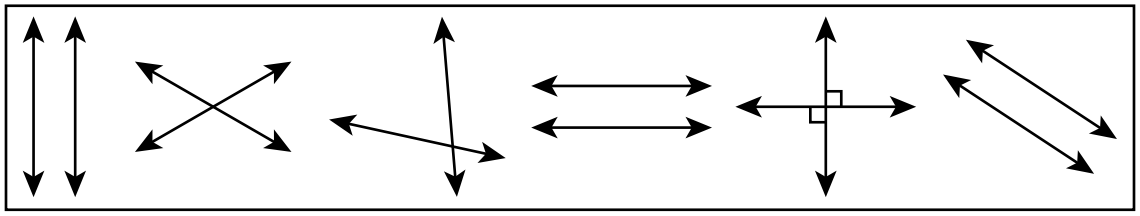


- Show images and reference examples of perpendicular lines in an authentic context. Explain that perpendicular lines are special intersecting lines that make a square corner. The square corner is a 90-degree angle that is called a right angle. Indicate that perpendicular lines sometimes have a small box indicating that the two lines are perpendicular.

  Demonstrate identifying the right angles by placing an index card directly on the angles formed by the perpendicular lines.



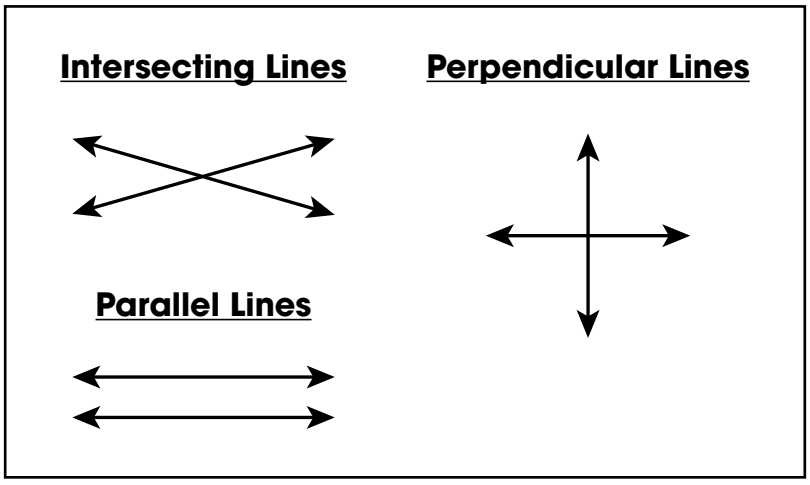
- Place the manipulatives directly on drawings as shown and in each of the orientations shown. In the case of intersecting lines, indicate the point of intersection. Demonstrate identifying parallel, intersecting, and perpendicular lines.



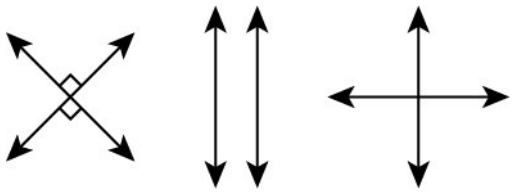
- Ask students to identify parallel lines, intersecting lines, and perpendicular lines.

❑ **Distinguish between intersecting, perpendicular, and parallel lines.**

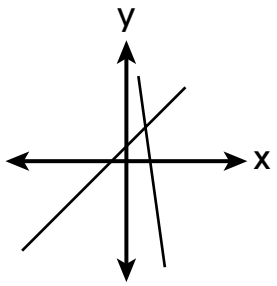
- Reference a visual to review the characteristics of intersecting, perpendicular, and parallel lines.



- Ask students to identify intersecting, perpendicular, and parallel lines. For example, present the following three sets of lines and ask students to identify which set of lines is parallel.

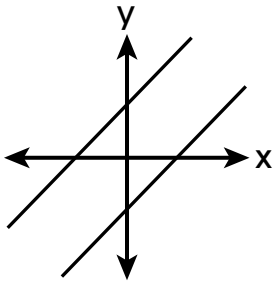


- ❑ **Distinguish between intersecting, perpendicular, and parallel lines on a coordinate grid.**
  - Use the sketch of a coordinate plane to demonstrate lines that are intersecting.



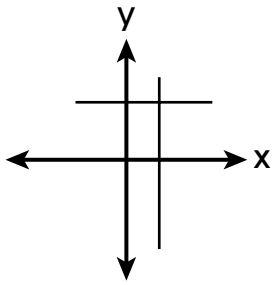
Explain that intersecting lines on a coordinate grid intersect at one point. Emphasize that they intersect in a way that does not always create a right angle. Explain that the intersecting lines cross at one point and will not intersect anywhere else.

  - Use the sketch of a coordinate plane to demonstrate lines that are parallel.



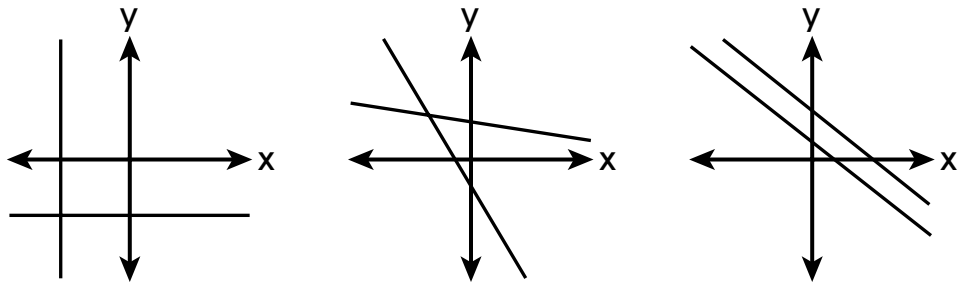
Explain that parallel lines are also lines that have the same slope. Slope is another way to describe the slant of a line. If two lines have the same slope, the lines will never intersect. Show several parallel line graphs with different slopes and with different distances between the lines.

  - Use the sketch of a coordinate plane to demonstrate lines that are perpendicular.



Explain that these lines on the coordinate grid intersect to form a 90-degree angle. Explain that the perpendicular lines intersect at one point on the coordinate grid and will not intersect anywhere else.

- Present various coordinate grids that have intersecting, parallel, and perpendicular lines. Model distinguishing between the different lines.
- Ask students to distinguish between intersecting, perpendicular, and parallel lines on a coordinate grid.



Students should determine that the graph on the left has perpendicular lines, the graph in the middle has intersecting lines, and the graph on the right has parallel lines.

Prerequisite Extended Indicators

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

MAE 8.A.1.a—Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

Key Terms

coordinate grid, coordinate plane, graph, intersecting lines, line, parallel lines, perpendicular lines, right angle, slope

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-4

https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-3

https://www.engageny.org/resource/grade-8-mathematics-module-4-topic-d-lesson-26

https://apps.mathlearningcenter.org/geoboard/

Mathematics—Grade HS
Geometry

HS.G.1 Attributes

HS.G.1.b

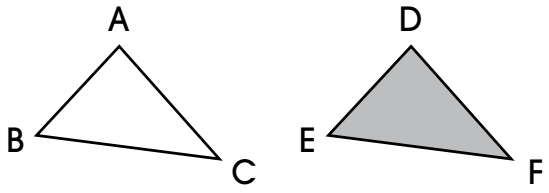
Describe symmetries of a figure in terms of rigid motions that map a figure onto itself and make inferences about symmetric figures (e.g., unknown side lengths or angle measures) in problems both with and without coordinates.

Extended: Identify corresponding angles by position when congruent triangles and similar triangles have the same orientation.

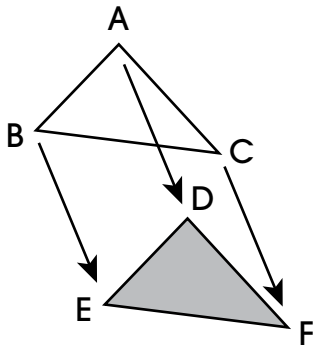
Scaffolding Activities for the Extended Indicator

- ❑ **Identify corresponding angles by position when congruent triangles and similar triangles have the same orientation.**
  - Use cut-out triangles to show corresponding angles as angles in the same position on a triangle. For example, present a pair of congruent triangles with angles labeled differently as shown. It might be helpful to use two different colors for the two triangles. Demonstrate that the triangles are congruent by stacking one triangle on top of the other. Identify that the side lengths are the same and the angles are the same.

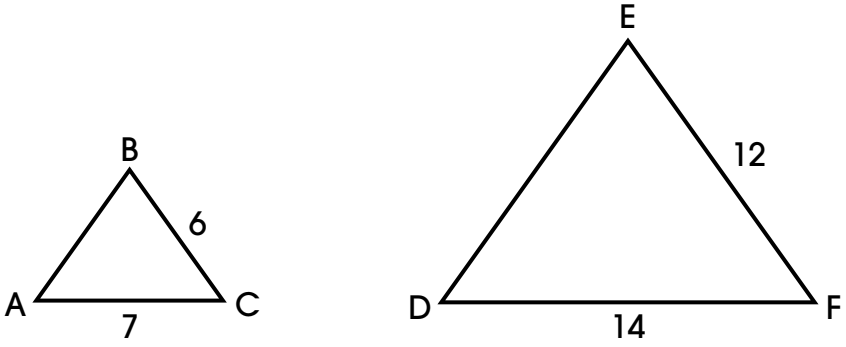
    Reinforce that the triangles are the same size and the same shape because the triangles overlap exactly when one triangle is stacked on top of the other, so all the angles are the same size and all the sides are the same length.



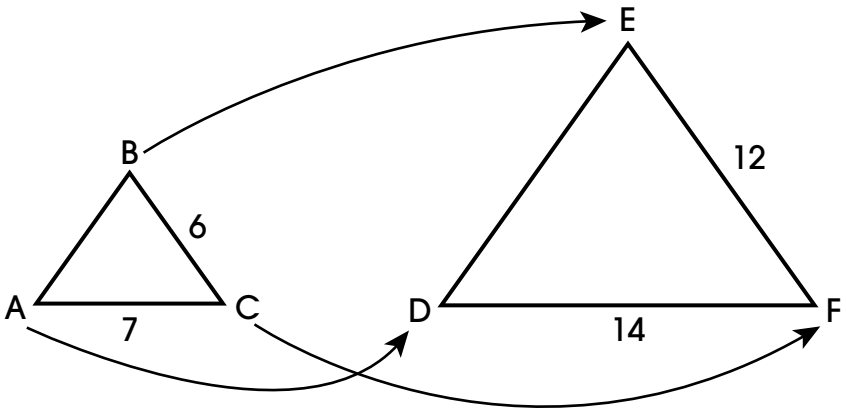
Separate the stacked triangles. Explain that the matching angles are called corresponding angles and that they are in the same position on both triangles. Point to angle A and indicate that it corresponds to angle D. Repeat this for angle B and angle E and then for angle C and angle F.



- Continue to demonstrate how to locate corresponding angles on congruent triangles with manipulatives and then progress to drawings of congruent triangles. Be sure to use a variety of triangle types, including right, obtuse, acute, scalene, isosceles, and equilateral triangles.
- Ask students to locate corresponding angles on two congruent triangles given two triangles with the same orientation.
- Explain that these triangles are similar because they have corresponding sides that are equal in proportion and their corresponding angles are congruent. Explain that even though they are not the same size, they still have corresponding angles just like the angles on congruent triangles. Present a pair of similar triangles that have side lengths and angles labeled differently as shown. Explain that the triangles are positioned in the same orientation. Where side BC on the smaller triangle is 6, the corresponding side EF on the larger triangle is 12. Continue with the remaining side lengths, proving they are similar.



- Explain that the matching angles are called corresponding angles and that they are in the same position on both triangles. Point to angle A and indicate that it corresponds to angle D. Repeat this for angle B and angle E and then for angle C and angle F.



- Continue to demonstrate how to locate corresponding angles on similar triangles with manipulatives and then progress to drawings of similar triangles. Be sure to use a variety of triangle types, including right, obtuse, acute, scalene, isosceles, and equilateral triangles.
- Ask students to locate corresponding angles on two similar triangles given two triangles with the same orientation.

Prerequisite Extended Indicators

MAE 8.G.2.b—Determine if a pair of two-dimensional figures is congruent, non-congruent, similar, or non-similar.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

angle, congruent angles, congruent triangles, corresponding angles, non-congruent triangles, non-similar triangles, similar angles, similar triangles, triangle

Additional Resources or Links

https://www.engageny.org/resource/geometry-module-1-topic-c-lesson-20

https://apps.mathlearningcenter.org/geoboard/

HS.G.1.d

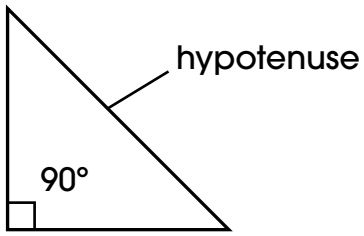
Identify and apply right triangle relationships including converse of the Pythagorean Theorem.

Identify the legs and the hypotenuse of a right triangle.

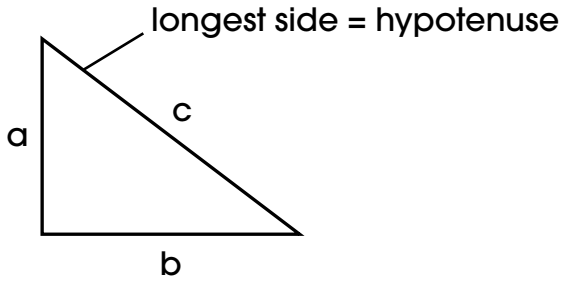
Scaffolding Activities for the Extended Indicator

❑ **Identify the hypotenuse of a right triangle.**

- Explain that the side opposite of a right angle has a special name. This side is called the hypotenuse. Just like a right triangle has only one right angle, it also has only one hypotenuse. Demonstrate first finding the right angle and then finding the hypotenuse on a variety of right triangles rotated in different positions.



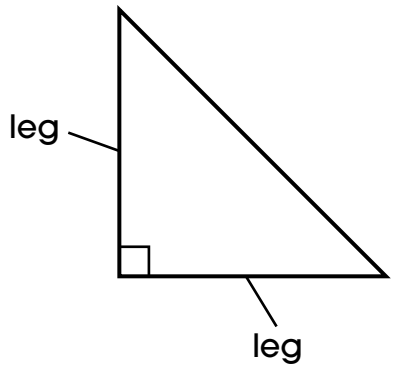
- Explain that the longest side in a right triangle is the hypotenuse. Present a drawing of a right triangle with the sides labeled a, b, and c. Demonstrate that side c is the longest side by using a piece of string, a straightedge, or a ruler to compare the length of each side.



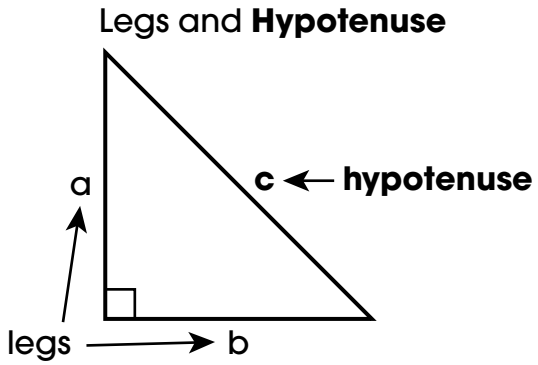
- Ask students to identify the hypotenuse on a variety of right triangles placed in the standard position. Be sure to provide examples of both right triangles that have the right angle labeled and right triangles without the label.
- Ask students to identify the hypotenuse on a variety of right triangles rotated in different positions. Be sure to provide examples of both right triangles that have the right angle labeled and right triangles without the label.

❑ Identify the legs of a right triangle.

- Explain that the two short sides of a right triangle are called legs. Demonstrate that the two legs of a right triangle intersect at the right angle. Demonstrate locating the right angle of the triangle and then locating the two sides adjacent to the right angle.

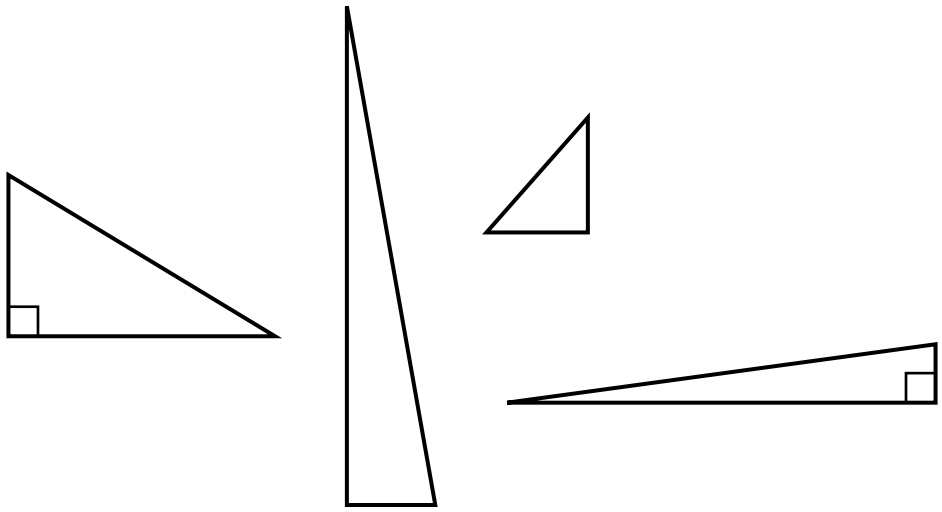


- Explain that the two short sides of a right triangle are called legs. Present a drawing of a right triangle with sides labeled a, b, and c. Using string, a straightedge, or a ruler, compare the lengths of the three sides. Demonstrate that sides a and b are both shorter than side c (the hypotenuse).



- Ask students to identify the two legs on a variety of right triangles placed in the standard position. Be sure to provide examples of both right triangles that have the right angle labeled and right triangles without the label.

- Ask students to identify the two legs on a variety of right triangles rotated in different positions. Be sure to provide examples of both right triangles that have the right angle labeled and right triangles without the label, as well as examples of both isosceles right triangles and right triangles with legs of varying lengths. Some examples are shown below.



Prerequisite Extended Indicators

MAE 8.G.2.b—Determine if a pair of two-dimensional figures is congruent, non-congruent, similar, or non-similar.

MAE 4.G.1.b—Classify angles as acute, obtuse, or right.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

adjacent, hypotenuse, legs, length, opposite, sides

Additional Resources or Links

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Triangle-Classification/

https://apps.mathlearningcenter.org/geoboard/

HS.G.1.h

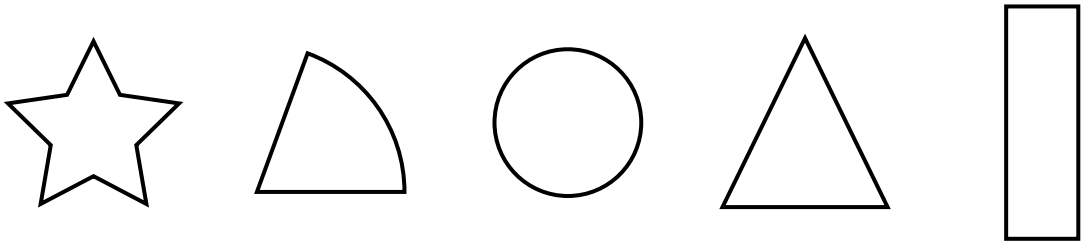
Compare/contrast the properties of quadrilaterals: parallelograms, rectangles, rhombi, squares, kites, trapezoids, and isosceles trapezoids.

Extended: Identify all four-sided polygons as quadrilaterals, and distinguish between parallelograms, rectangles, squares, and kites.

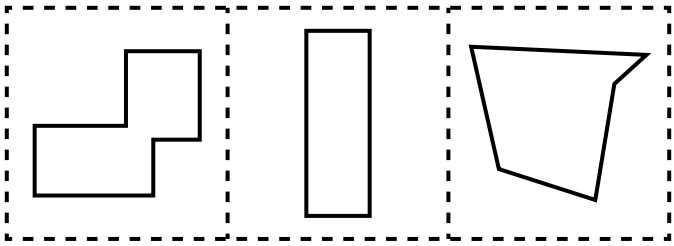
Scaffolding Activities for the Extended Indicator

❑ **Identify a polygon.**

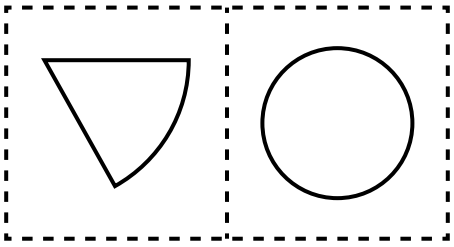
- Demonstrate, using pictures or by drawing, two-dimensional figures that are closed. Include circles and polygons (regular and not regular).



- Explain that two-dimensional closed figures that have straight-line segments for sides are polygons. Present a two-dimensional polygon, point to the straight lines, and model tracing the figure to show that it is closed. Contrast with non-polygons. Model sorting figures into groups of polygons and non-polygons.

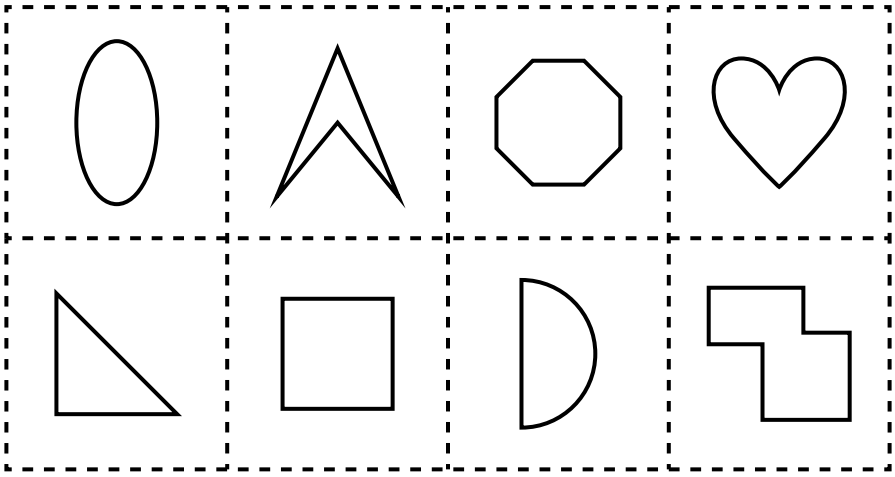


polygons



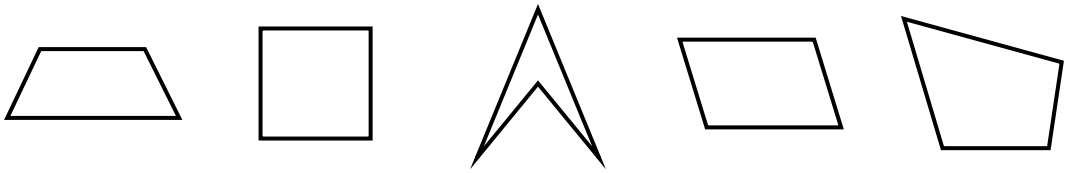
non-polygons

- Ask students to identify the polygons when provided with two-dimensional figures of polygons and non-polygons.

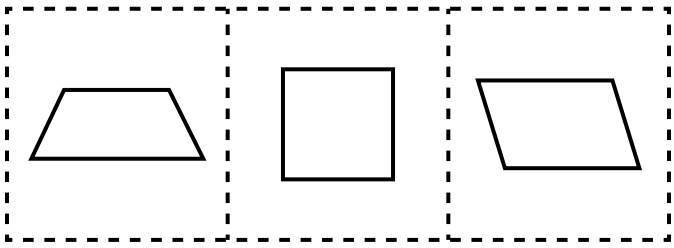


❑ **Identify four-sided polygons as quadrilaterals.**

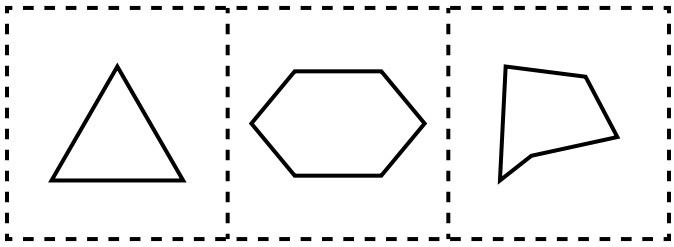
- Demonstrate, using pictures or by drawing, two-dimensional figures that have four sides. Use a variety of four-sided figures as examples.



- Explain that two-dimensional figures with four sides are classified as quadrilaterals. Present a quadrilateral to students and demonstrate pointing to the straight sides and showing that it is a closed figure and also a polygon. Then model counting the four sides and explain that the four sides make the figure a quadrilateral. Demonstrate that not all polygons are quadrilaterals by using non-quadrilateral figures. Continue modeling and sorting.

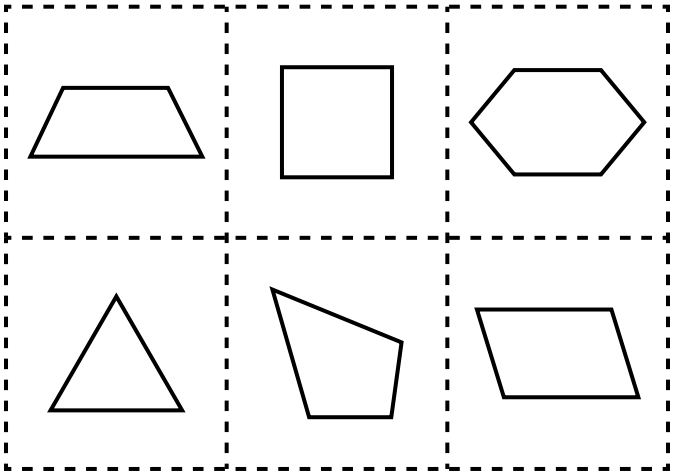


quadrilaterals



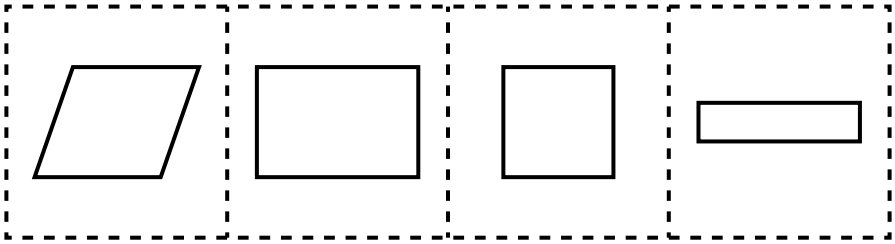
non-quadrilaterals

- Ask students to identify quadrilaterals from a group of at least four polygons.

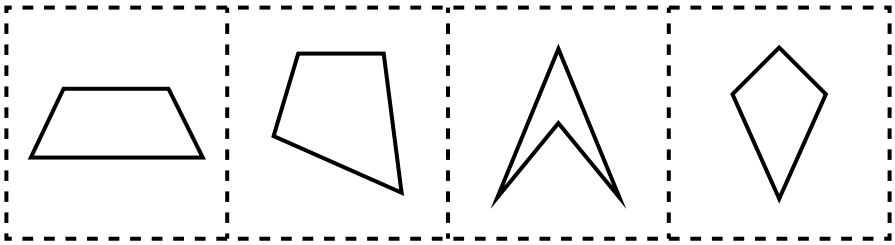


❑ **Distinguish between parallelograms, rectangles, squares, and kites.**

- Demonstrate, using pictures or by drawing, two-dimensional figures that have four sides with the opposite sides parallel.
- Explain that the two-dimensional figures with two sets of parallel sides are called parallelograms. Emphasize that parallel lines are equal distance from each other and never intersect. Present a parallelogram and indicate the parallel sides. Present a non-parallelogram and explain why it is a non-parallelogram. Repeat with various examples and non-examples.

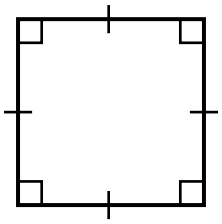


parallelograms

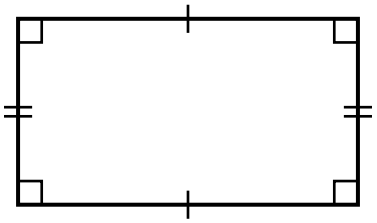


non-parallelograms

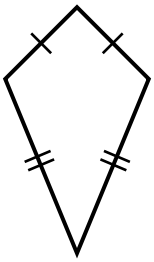
- Demonstrate and explain that if all four sides of a parallelogram are the same length and it has four right angles, it is a special parallelogram called a square.



- Demonstrate and explain that if the parallelogram has two pairs of equal sides and four right angles, it is a special parallelogram called a rectangle. Note that all squares are rectangles, but not all rectangles are squares.

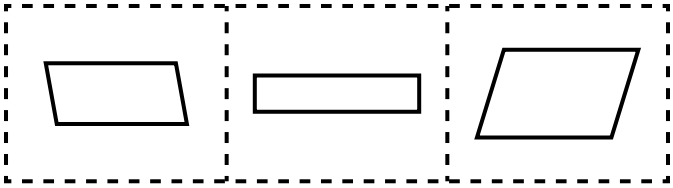


- Explain that there are other special quadrilaterals that are not parallelograms. Demonstrate that a kite is a closed figure that has four sides (which makes it a quadrilateral) and two pairs of sides of equal length. However, none of the sides are parallel.

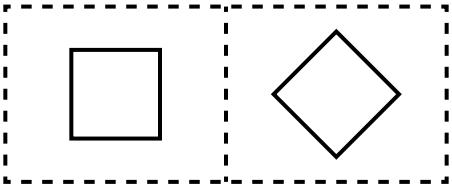


- Ask students to distinguish between parallelograms, squares, rectangles, and kites. For example, ask students to sort or label the figures. Remind students that some figures may belong in two groups (i.e., parallelogram and square).

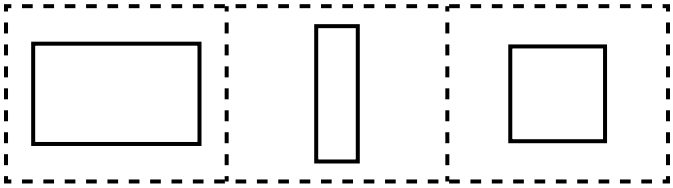
Parallelograms



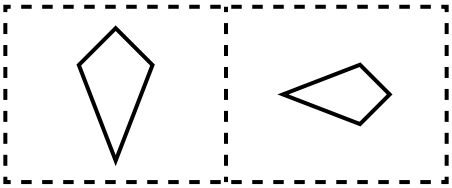
Squares



Rectangles



Kites



Prerequisite Extended Indicators

MAE 8.G.2.b—Determine if a pair of two-dimensional figures is congruent, non-congruent, similar, or non-similar.

MAE 4.G.1.c—Classify quadrilaterals based on the presence or absence of parallel and perpendicular lines and the presence or absence of right angles.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

kite, pair, parallelogram, polygon, rectangle, right angle, sides, square

Additional Resources or Links

https://tasks.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1274

https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-d-lesson-15

HS.G.1.i

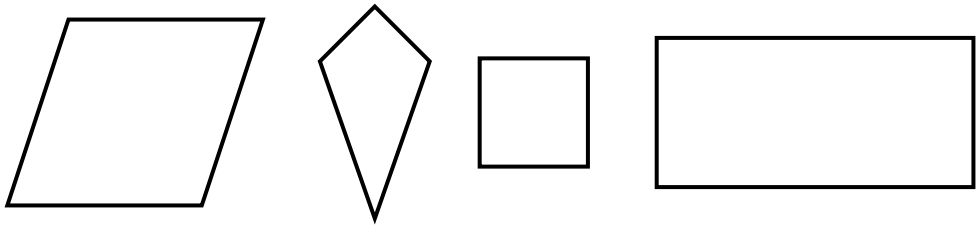
Use slope and the distance formula to determine the type of quadrilateral.

Extended: Identify the quadrilateral on the coordinate grid as a parallelogram, rectangle, square, or kite.

Scaffolding Activities for the Extended Indicator

❑ **Identify parallel lines and right angles on a parallelogram, rectangle, square, or kite.**

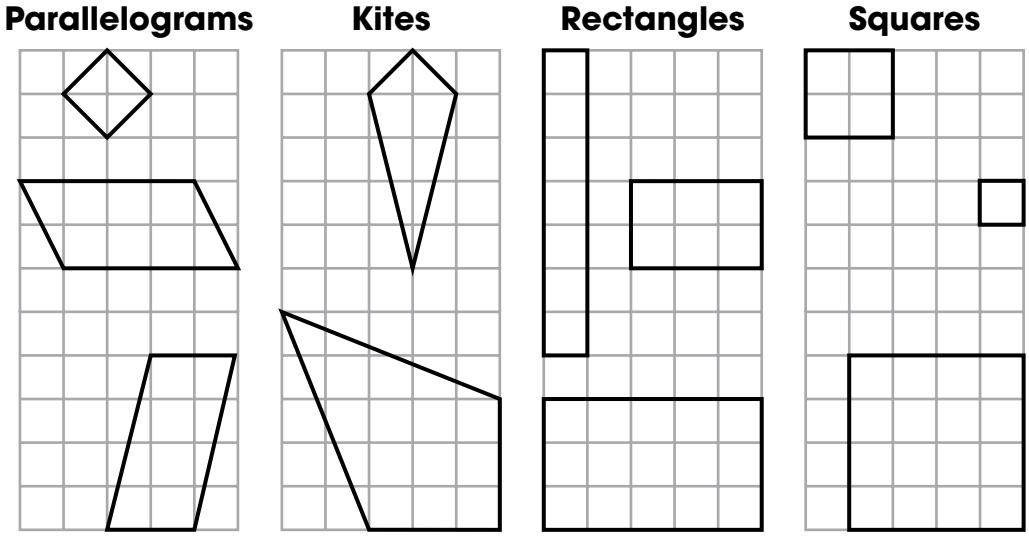
- Use examples of parallelograms, rectangles, squares, and kites in various sizes to describe the attributes of each shape. Explain that parallelograms are quadrilaterals with opposite sides parallel to each other. Explain that rectangles and squares have two pairs of parallel sides with corners that are right angles. Emphasize that rectangles and squares are both also parallelograms. Explain that a kite has two pairs of consecutive congruent sides (sides next to each other that are the same length) but that the sides are not parallel.



- Use examples of parallelograms, rectangles, and squares to identify the right angles in the shapes. Describe rectangles and squares as having four right angles, which are angles that look like the corner of a box, or an “L.” Explain that parallelograms and kites sometimes have right angles, but emphasize that parallelograms may not have 4 of them and kites cannot have 4.
- Use examples of parallelograms, rectangles, squares, and kites to show examples and non-examples of parallel lines that each of these shapes might or might not have.
- Ask students to sort pattern blocks or cutout shapes of parallelograms, rectangles, squares, and kites into categories regarding right angles and parallel lines.
- Ask students to identify a shape that matches a description. For example, present a parallelogram, a rectangle, and a kite and ask the following questions: Which shape has two sets of parallel sides? Which shape has one set of parallel sides? Which shape has four right angles? Which shape has no right angles? Which shape has continuous congruent sides (all sides are the same length or two sides next to each other are the same length)?

- ❑ **Identify the quadrilateral on a coordinate grid as a parallelogram, a rectangle, a square, or a kite.**

  - Present parallelograms, rectangles, squares, and kites drawn on grid paper and describe the attributes of each shape as they relate to the coordinate grid. When appropriate, indicate that the pairs of parallel sides are aligned with pairs of horizontal or vertical lines on the grid. Indicate that right angles are at the intersections of horizontal and vertical lines.



  - Ask students to identify a parallelogram, a rectangle, a square, and a kite drawn on a grid when given an example of the same shape not drawn on a grid for comparison.
  - Ask students to identify a parallelogram, a rectangle, a square, and a kite drawn on a grid.

Prerequisite Extended Indicators

MAE HS.G.1.h—Identify all four-sided polygons as quadrilaterals, and distinguish between parallelograms, rectangles, squares, and kites.

MAE 8.G.2.b—Determine if a pair of two-dimensional figures is congruent, non-congruent, similar, or non-similar.

MAE 4.G.1.b—Classify angles as acute, obtuse, or right.

MAE 4.G.1.c—Classify quadrilaterals based on the presence or absence of parallel and perpendicular lines and the presence or absence of right angles.

MAE 4.G.2.d—Identify benchmark angles of 90° and 180°, and relate those angle measurements to right angles, straight lines, and perpendicular lines.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

congruent, intersecting, kite, parallel, parallelogram, rectangle, right angle, square, unit square

Additional Resources or Links

https://www.engageny.org/resource/grade-1-mathematics-module-5-topic-lesson-2

https://www.engageny.org/resource/grade-2-mathematics-module-8-topic-lesson-1

HS.G.1.j

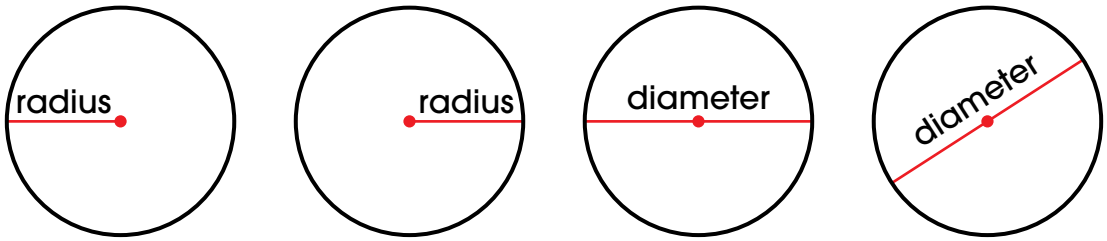
Identify, describe, apply, and reason through properties of central angles, inscribed angles, angles formed by intersecting chords, secants, and/or tangents to find the measures of angles related to the circle, arc lengths, and areas of sectors.

Extended: Differentiate between a chord, radius, diameter, and arc of a circle, and identify the arc length as one-fourth, one-half, or three-fourths of the circle.

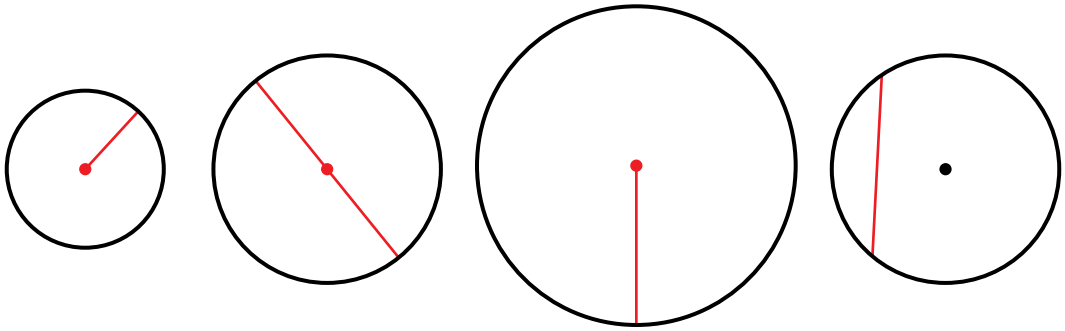
Scaffolding Activities for the Extended Indicator

❑ **Identify and show a chord, a radius, a diameter, and an arc on a circle.**

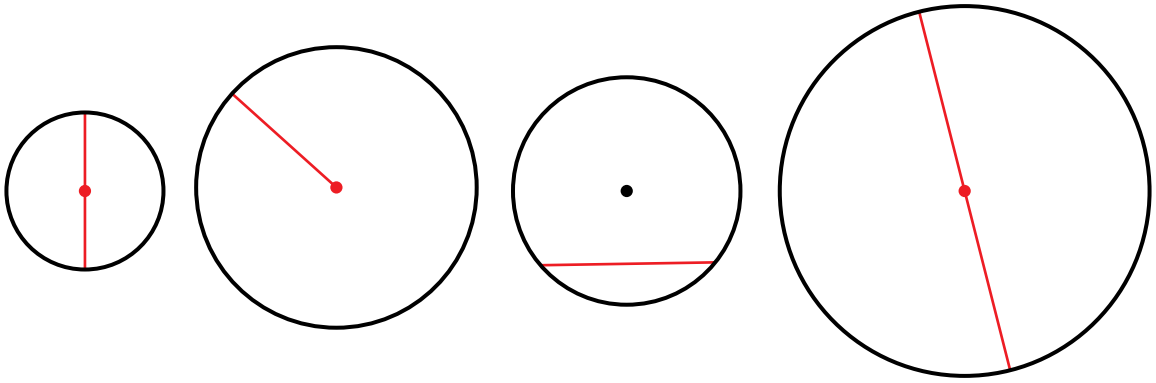
- Describe a circle as a collection of all the points that are the same distance from a point called the center. Show examples of real-life objects such as a Hula-Hoop or a metal ring. Explain that there are many different parts of circles. Two parts of a circle are the radius and the diameter. Explain that the radius of a circle is a line segment equal to the distance from the center of the circle to the edge. Explain that the diameter of a circle is a line segment that connects two points on the circle and passes through the center of the circle. Explain to students that the radius is half the length of the diameter, which makes the diameter twice the length of the radius. Present different examples of the radius and the diameter of circles as shown.



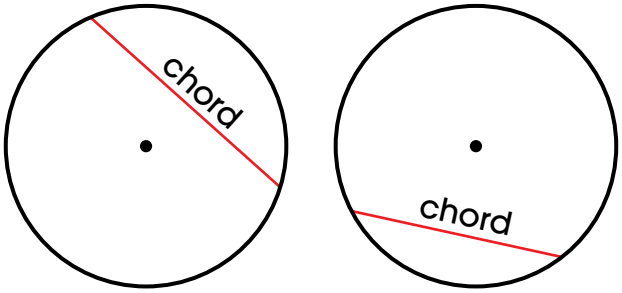
- Ask students to identify a radius on a circle.
- Ask students to identify the circles that show a radius.



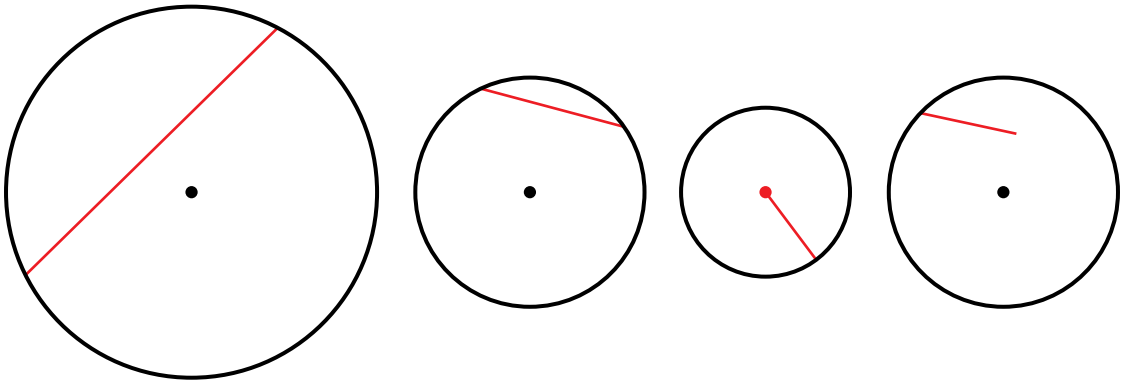
- Ask students to identify a diameter on a circle.
- Ask students to identify the circles that show a diameter.



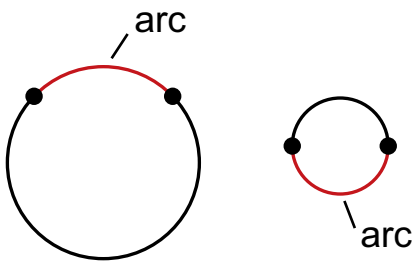
- Describe another part of a circle called a chord. A chord of a circle is a line segment whose endpoints are two points on the circle. Explain that every diameter is also a chord since it is a line segment whose endpoints are two points on the circle. Present different examples of chords as shown.



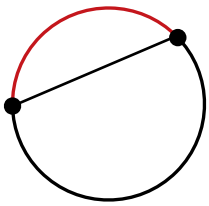
- Ask students to identify a chord on a circle.
- Ask students to identify all the circles that show a chord.



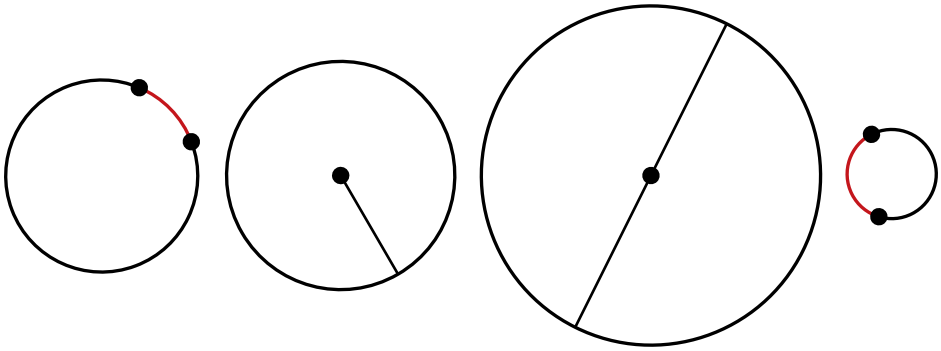
- Describe another part of a circle called an arc. An arc is a fraction or part of the edge of a circle. An arc can be a small fraction or a large fraction of the edge of a circle. Present different examples of arcs as shown.



Emphasize that an arc is a section of the edge of a circle. Arcs are curved, which makes them different from other lines that connect two points inside a circle. Present a picture as shown to demonstrate the difference between a curved line (arc) that connects two points on a circle and a straight-line segment (or chord) inside the circle that connects two points.

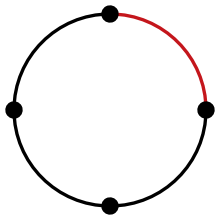


- Ask students to identify an arc drawn on a circle.
- Ask students to identify all the circles with an arc when presented examples and non-examples as shown.

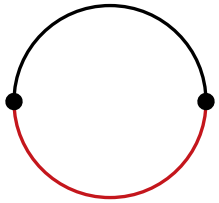


❑ **Identify the arc length of a circle as one-fourth, one-half, or three-fourths of the outside edge of a circle.**

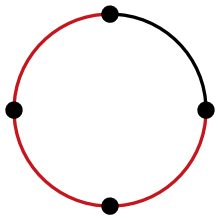
- Explain that the length of the arc of a circle can be one-fourth of the outside edge of a circle. If the edge of a circle is divided into four equal sections as noted by the points on the edge of this circle, the arc length of one of the sections is one-fourth of the circle as shown.



- Explain that if the edge of a circle is divided into two equal sections, the arc length of one of the sections is one-half of the circle as shown.

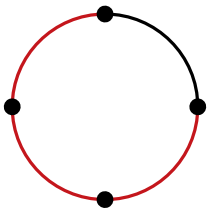


- Explain that if the edge of a circle is divided into four equal sections, the arc length of three of the sections is three-fourths of the circle as shown.

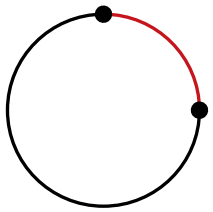


It might be helpful to demonstrate fractional parts of the edge of a circle by cutting a piece of string that is the length of the outer edge of a circle. Then, cut the string into fractional parts to represent an arc length of one-fourth, one-half, or three-fourths of the circle.

- Ask students to identify the given arc length as one-fourth, one-half, or three-fourths of this circle when given a circle with visual support indicating the fractional sections of the edge of the circle as shown.



- Ask students to identify the given arc length as one-fourth, one-half, or three-fourths of this circle when given a circle without additional visual support indicating the fractional sections of the edge of the circle as shown.



Prerequisite Extended Indicators

MAE 7.G.3.c—Identify the center, radius, and diameter of a circle, and distinguish between the area of a circle and the circumference of a circle.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 3.N.2.a—Partition two-dimensional figures (circles, triangles, rectangles, and squares) into two, three, or four equal shares, and express the area of each part as a unit fraction of the whole using 1/2, 1/3, or 1/4.

Key Terms

arc, center, chord, circle, diameter, length, line segment, radius

Additional Resources or Links

https://curriculum.illustrativemathematics.org/MS/students/2/3/2/index.html

https://www.engageny.org/resource/grade-7-mathematics-module-3-topic-c-lesson-16

https://im.kendallhunt.com/HS/students/2/7/10/index.html

HS.G.2 Attributes

HS.G.2.c

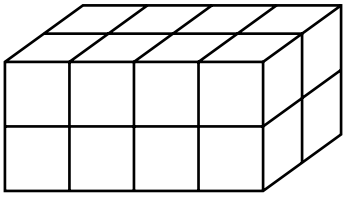
Determine surface area and volume of pyramids, as well as solids that are composites of pyramids, prisms, spheres, cylinders, and cones, using formulas and appropriate units.

Extended: Find the area of one face of a rectangular prism.

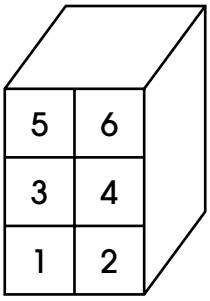
Scaffolding Activities for the Extended Indicator

❑ **Find the area of one face of a rectangular prism by counting unit squares.**

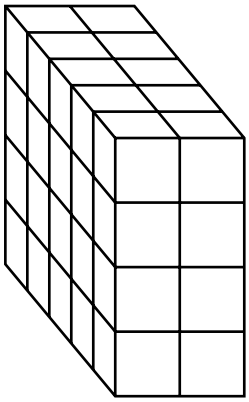
- Explain that a rectangular prism is a shape that looks like a box. It has six faces, which are the flat sides around the shape. Show real-life examples and images of rectangular prisms (e.g., tissue box, cereal box, fish tank).
- Explain that the area of a rectangle is the number of square units that cover the surface. One way to find the area of a face of a rectangular prism is to count each of the unit squares. In the rectangular prism below, the unit squares on the front may be counted to find the area of that face. Count the unit squares on the front. The area of the front face is 8 square units.



- Continue to model finding the area of different faces of rectangular prisms by counting unit squares.
- Ask students to find the area of a face of a rectangular prism with the unit squares numbered on the face as shown. It might be helpful to cover the unit squares of the other faces with paper or tape.



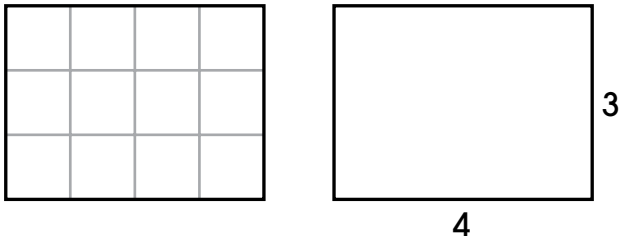
- Ask students to find the area of one face of a rectangular prism by counting unit squares.



Area of the top = ___ square units

❑ **Find the area of one face of a rectangular prism when given the side lengths.**

- Explain that the area of a rectangle is how many square units cover the surface. However, sometimes the unit squares are not shown. Present the following rectangles.



In the first rectangle, the unit squares can be counted to find the area. In the second rectangle, the unit squares are not given, but the length and width of the rectangle are given. The rectangle has a length of 4 units and a width of 3 units. To find the area of the rectangle, multiply 4 × 3. Another way to solve 4 × 3 is to make an array with four rows of three in each row. Model making an array by drawing x's or using manipulatives. Then count the x's (or objects) in the array to determine the area is 12 square units.

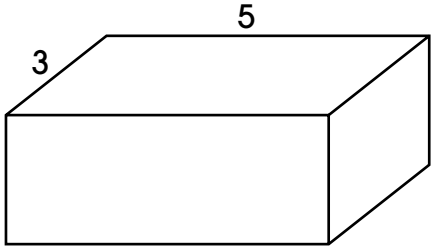
X X X X

X X X X

X X X X

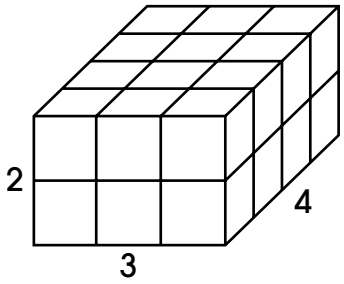
- Continue to model finding the area of other rectangles using the dimensions given by multiplying length times width or making an array and using other computation strategies, including counting, skip counting, or repeated addition.
- Ask students to find the area of rectangles using the given dimensions.

- Explain that finding the area of one of the faces of a rectangular prism is the same as finding the area of a rectangle. In the rectangular prism below, the top of the rectangular prism has a length of 5 units and a width of 3 units. To find the area of the top face of the rectangular prism, multiply 5 × 3 or make an array. The area of the top of the rectangular prism is 15 square units.

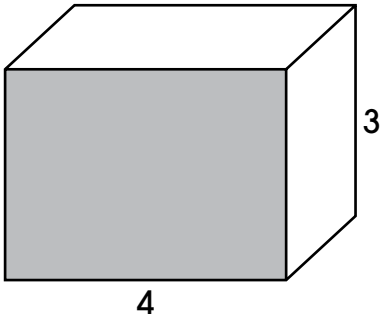


X X X X X
X X X X X
X X X X X

- Continue to model finding the area of one face of other rectangular prisms using the dimensions given by multiplying length times width or making an array and using other computation strategies including counting, skip counting, or repeated addition.
- Ask students to determine which of the 3 faces of the rectangular prism shown has an area of 8 square units.



- Ask students to find the area of one face of a rectangular prism with the given dimensions.



Surface area of the front face = ___ square units

Prerequisite Extended Indicators

MAE 6.G.3.a—Find the area of a rectangle using its whole-number side lengths.

MAE 6.G.3.b—Find the surface area of a rectangular prism by counting unit squares in a net of the figure.

MAE 5.G.1.a—Identify the faces, edges, and vertices of cubes and other rectangular prisms.

Key Terms

area, face, rectangle, rectangular prism, square unit, surface area

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-d-lesson-18

http://nlvm.usu.edu/en/nav/frames_asid_129_g_3_t_3.html?open=activities&from=category_g_3_t_3.html

(Note: Java required for website. Most recent version recommended, but not needed.)

HS.G.4 Logic & Proof

HS.G.4.d

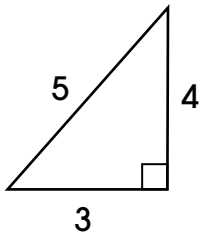
Use coordinate geometry to prove triangles are right, acute, obtuse, isosceles, equilateral, or scalene.

Extended: Identify isosceles, equilateral, or scalene triangles on and off a coordinate grid.

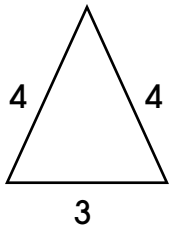
Scaffolding Activities for the Extended Indicator

❑ **Identify the number of sides in a triangle that have the same length.**

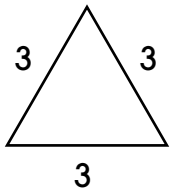
- Demonstrate identifying the number of sides in a triangle with the same length. For example, present a drawing of a right, scalene triangle with side lengths of 3, 4, and 5 labeled. Explain that there are no sides of the same length in scalene triangles.



  Present a drawing of an isosceles triangle with side lengths of 4, 4, and 3 labeled. Explain that there are two sides of the same length in isosceles triangles.



  Present a drawing of an equilateral triangle with side lengths of 3. Explain that all three sides are the same length in equilateral triangles.



- Ask students to identify the number of sides in a given triangle that have the same length.

❑ **Identify isosceles, equilateral, and scalene triangles on and off a coordinate grid.**

- Using cards with pictures of triangles, have students identify the names of triangles based on their relative side lengths. Create a chart with descriptions of an isosceles, an equilateral, and a scalene triangle. Create cards with pictures of isosceles, equilateral, and scalene triangles. Be sure to include examples of triangles with and without the side lengths labeled.

Triangles

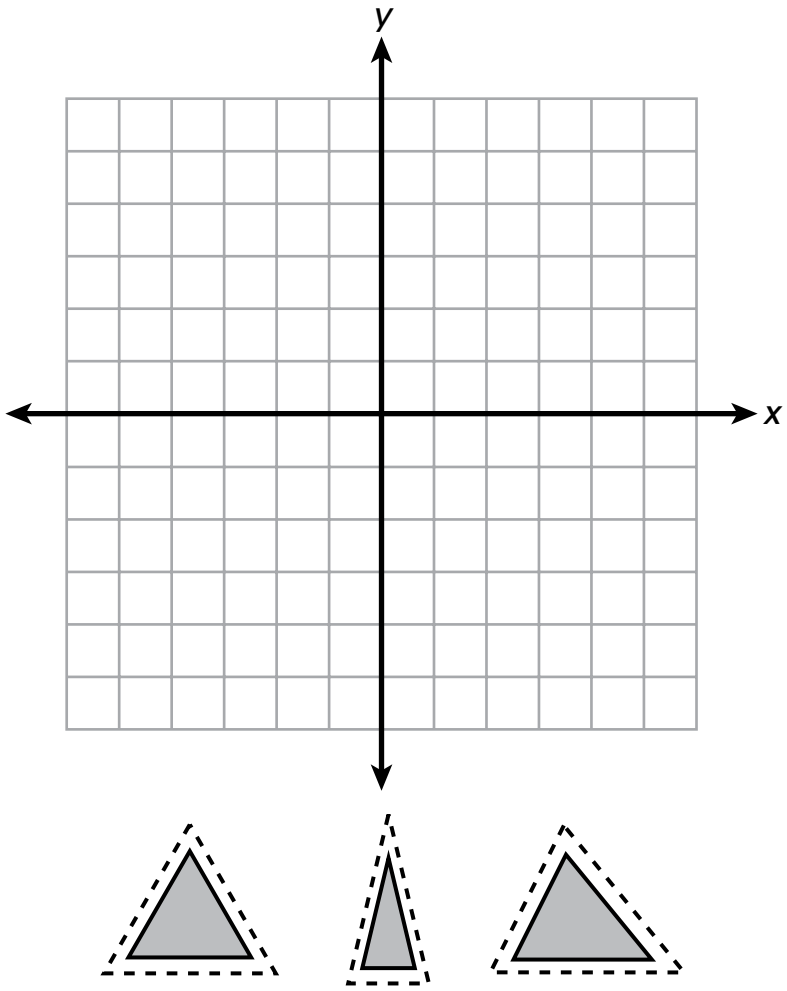
Number of Sides the Same Length		
0 sides Scalene	2 sides Isosceles	3 sides Equilateral

Explain that triangles have different names based on the number of sides that have the same length. Draw a triangle card and demonstrate identifying the number of sides that have the same length and then placing that card in the appropriate column.

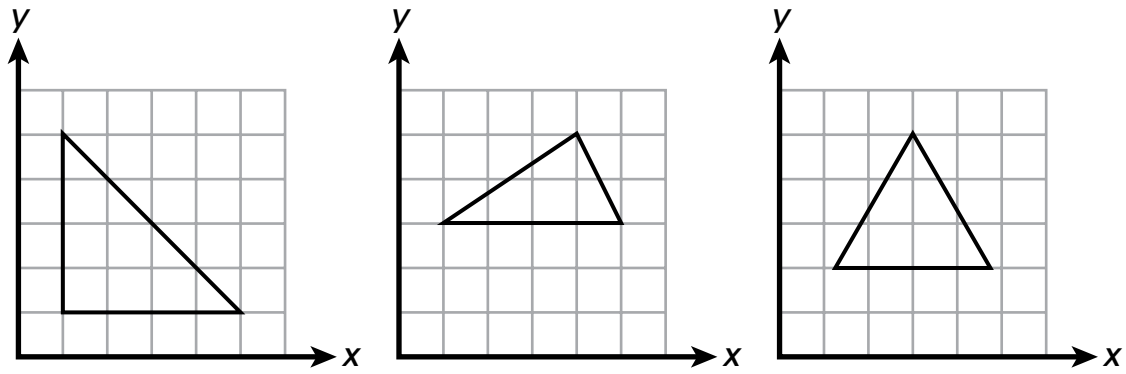
When the chart is complete, explain that each column represents a category of triangles and that a triangle can be labeled based on the number of its sides that have the same length.

- Ask students to identify a triangle as scalene, isosceles, or equilateral.

- Present students with a blank coordinate grid. Explain to students that scalene, isosceles, and equilateral triangles can be shown on coordinate grids. Provide students with cutouts of a scalene, an isosceles, and an equilateral triangle. Position them on the coordinate grid and model identifying each type of triangle and explain your reasoning. You could, for example, show students how to count the side lengths by using the lines on the coordinate grid.



- Ask students to identify which coordinate grid has an isosceles, an equilateral, or a scalene triangle.



Prerequisite Extended Indicators

MAE HS.G.1.d—Identify the legs and the hypotenuse of a right triangle.

MAE 5.G.1.c—Classify triangles as acute, right, or obtuse.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

coordinate grid, equilateral triangle, isosceles triangle, length, scalene triangle, side, triangle

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-d-lesson-13

https://apps.mathlearningcenter.org/geoboard/

Mathematics—Grade HS
Data

HS.D.1 Data Collection and Statistical Methods

HS.D.1.a

Formulate multi-variable statistical investigative questions and determine how data can be collected and analyzed to provide an answer.

Extended: Determine more than one method to collect data to answer an investigative question.

Scaffolding Activities for the Extended Indicator

❑ **Determine whether a specific method of data collection is appropriate to answer an investigative question.**

- Identify for students two types of questions associated with data: investigative questions, which are answered using data, and survey questions, which are used to collect data. An example of an investigative question is shown.

  Investigative Question: “What is the most popular type of pet owned by families on the street where you live?”

  To answer this type of question, the appropriate data must be collected. This can be done by asking each family on that street the type of survey question shown.

  Survey Question: “What types of pets do you own?”

  After the results of this survey question, or a different survey question, are organized, they could be displayed in a way that could be useful in answering the investigative question. A tally chart is shown as an example of how students might display the data.

Pets Living on My Street

Type of Pet	Number of Families
dog	𝍸 𝍸 |
cat	𝍸 ||
fish	||
hamster	|
rabbit	|||

Explain to students that we now have data to answer the investigative question “What is the most popular type of pet owned by families on the street where you live?” Indicate by counting that dogs have been chosen more often than the other animals, which identifies dogs as the most popular type of pet in this survey. Emphasize that collecting data with a survey question and organizing the results in a data table were appropriate methods for answering the investigative question.

- Repeat this process with different types of data collection methods (e.g., consulting books and online sources) and different data-presentation modes (e.g., charts, tables, timelines).
- Ask students to determine whether a specific method of data collection is appropriate to answer the specific investigative question when provided with various examples.

❑ **Determine one method to collect data to answer an investigative question.**

- Explain to students that one common method to collect data that can be used to answer an investigative question is to complete a survey using a targeted survey question.

  For example, if you wanted to answer the question, “What percentage of students in your class walk to school?” you would first need some data to answer this question. You could ask each student in your class, “Do you walk to school?” and use that information along with the number of students in the class to answer the investigative question.

- Present various investigative questions to students. Work together to come up with survey questions that could help collect the data needed to answer the investigative question.

- Ask students to choose a survey question that would be the best method to collect data to answer the corresponding investigative question.

Investigative Question	Survey Question
What was the average number of minutes you spent on your cellphone per day last week?	
What percentage of the United States have you visited?	

Question 1: What is your favorite state in the country?

Question 2: How many minutes did you spend on your cellphone each day last week?

Question 3: Who did you call on your cellphone each day last week?

Question 4: Which states have you visited?

❑ **Determine more than one method to collect data to answer an investigative question.**

- Explain to students that multiple methods can be used to collect data that can be used to answer an investigative question. Some examples of data collection methods are listed below.
    - using questionnaires and surveys
    - conducting interviews
    - looking up historical records
    - searching online sources
    - making firsthand observations
- Provide a situation for students to model how to determine a data-collection method that would be appropriate for a given investigative question. For example, present the investigative question “What percentage of students in this class are eating hot lunch today?” Ask students if the data can be collected by looking at historical records. The answer is no, since the event hasn't occurred yet, so historical records can’t provide the correct answer. Ask students if questionnaires and surveys could collect the data to answer the question. The answer is yes; each student could be asked if they are eating a hot or cold lunch today, and that would provide data that could answer the question. Explain that another way to accomplish this would be through observation. A student could watch other students eating lunch in the cafeteria, the data could be collected, and then the investigative question could be answered. Emphasize that more than one method of collecting data is possible for many investigative questions.

- Ask students to determine more than one method of data collection to answer an investigative question. For example, the investigative question “How many inches of snow accumulated today?” can be answered with data collected using more than one method. The data could be collected online by finding the amounts of snowfall for a specific location. Data could also be collected through observation by going outside with a ruler and measuring the number of inches of snow.

Prerequisite Extended Indicator

MAE 7.D.1.a—Collect data to answer an investigative question.

Key Terms

data, data collection, investigative question, observation, online, search, survey, survey question, questionnaire

Additional Resources or Links

https://illuminations.nctm.org/Search.aspx?view=search&st=d&gr=6-8

https://www.map.mathshell.org/lessons.php?unit=9400&collection=8

HS.D.1.b

Apply an appropriate data collection plan when collecting primary data for the statistical investigative question of interest.

Extended: Follow a plan to collect data for an investigative question of interest.

Scaffolding Activities for the Extended Indicator

❑ **Determine the first step when given a plan to collect data for a statistical investigative question of interest.**

- Explain to students that investigative questions are answered using data and that this process requires a plan or method to collect the data. Explain that the first step to collect data for answering a statistical investigative question of interest is to determine the best method to collect the information desired. Some of the options that could be used by students are listed.
    - Compose survey questions for gathering data.
    - Consult newspaper stories.
    - Create a questionnaire.
    - Collect statistics from books of sports records.
    - Research government publications.
    - Perform in-person interviews.
    - Search internet sites.

  The next steps after the data have been collected would be to organize the results and determine how they can best be displayed. The data should be displayed in a way that is useful for answering the investigative question. This could be as simple as creating a tally chart or as detailed as creating a data table containing several rows and columns or even a graph.
- Ask students to share how they might collect, organize, and display data that could answer a given investigative question. An example of a question that could be provided to students is “What is the average number of hours of sleep students in your class had over the last 5 days?”
- Students might provide a variety of survey questions and ideas such as graphs or data tables to organize and display the data in a way that can be used to answer the question.

- ❑ **Follow a plan to collect data for an investigative question of interest.**
  - Explain to students that they should have a plan to collect the correct data to answer an investigative question and they should follow steps to make their plan successful. Depending on the question, those steps might include the following:

    Step 1: Determine what data are needed.

    Step 2: Decide how to collect the data.

    Step 3: Collect the data.

    Step 4: Organize the data.

    Step 5: Display the data clearly to answer the question.

    Step 6: Answer the question.
  - Provide an investigative question for students such as “What is the average number of letters in the first names of all the students in your classroom?”

    The method for collecting the data might look like this:

    Step 1: Identify that the data required will be the first names of each student in the class.

    Step 2: Determine that you will collect the data by asking the teacher for a list of every student in the class.

    Step 3: Obtain the list of students from the teacher and ensure all first names are spelled correctly.

    Step 4: Organize the data that were gathered. You might just list the first names of each student with the number of letters after the name as shown.

    Melanie (7); John (4); Ashly (5); Raheem (6); Antonio (7); Christine (9); Omar (4)

    Step 5: Display the data in a way that is useful in answering the original question. This could be done in a table that has names in one column and the number of letters in the other column as shown.

First Name	Number of Letters
Melanie	7
John	4
Ashly	5
Raheem	6
Antonio	7
Christine	9
Omar	4
TOTAL:	42

Step 6: Answer the question. At this point, since the data are organized and displayed in a useful manner, all that needs to be done to answer the question is performing a calculation using the data. Since there are 7 students in this class and there are 42 total letters used for all their names, the average number of letters can be determined by dividing 42 by 7. This means that the average number of letters in the first names of the students in this class is 6 letters.

- Provide various examples of investigative questions and the plan for data collection following the same steps.
- Ask students to choose an investigative question that interests them and follow a plan to collect the data and answer the question.

Prerequisite Extended Indicators

MAE HS.D.1.a—Determine more than one method to collect data to answer an investigative question.

MAE 7.D.1.a—Collect data to answer an investigative question.

Key Terms

average, data, data collection, investigative question, observation, plan, survey question,

Additional Resources or Links

https://www.map.mathshell.org/lessons.php?unit=9400&collection=8

https://www.insidemathematics.org/inside-problem-solving/through-the-grapevine

HS.D.1.c

Use appropriate technology, including spreadsheet-based logic, to organize data for analysis.

Extended: Use appropriate technology to organize data collected for an investigative question of interest.

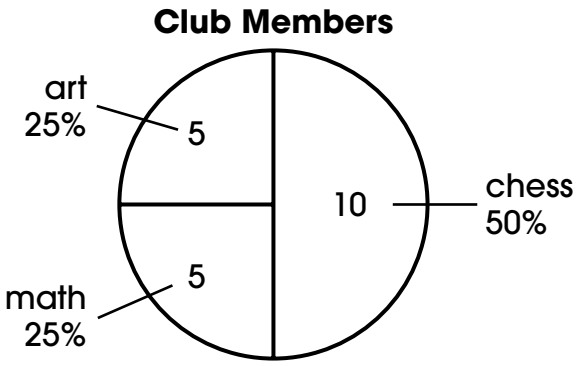
Scaffolding Activities for the Extended Indicator

❑ **Determine the type of technology needed to organize collected data.**

- Remind students that investigative questions are answered using data that has been collected. Once data have been collected, they need to be organized so that they can be displayed in a useful manner. There are many ways to organize data and the one that is best depends on the type of data and the question that needs to be answered.

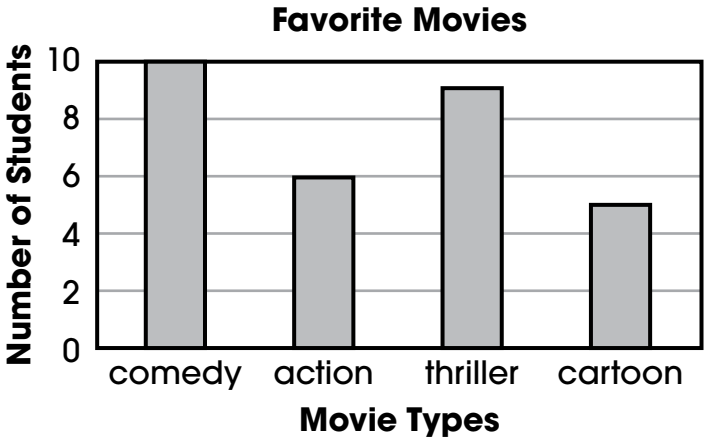
  For example, if the question was "What percentage of the students in your math class are members of the chess club, the math club, or the art club?" In this example, the results of the data collection show there are 20 students in the class, with 10 students in the chess club, 5 students in the math club, and 5 students in the art club. There are multiple ways the data could be displayed, and technology could be used to organize and display the data.

  One type of technology that could be used to organize and display the data would be a pie-chart or circle-graph maker on your computer. An example of a pie chart made by a pie-chart maker is shown.



- Present a different example. The investigative question is "Which type of movie is the most popular among the students at a lunch table?" In this example, the results of the data collection show there are 30 students at the lunch table, with 10 students choosing comedy, 6 students choosing action, 9 students choosing thriller, and 5 students choosing cartoon.

- Ask students to provide some ideas for technology that could be used to organize and collect the data. One example is shown by using a bar graph created on the computer.



- Ask students to come up with an investigative question and create a sample set of data that could have been collected so that they can share ideas of technology that can be used to organize and display the data. Some possible technology options might include different graph generators that are online, a graphing calculator, a software program such as Excel, apps that can be downloaded on computers or tablet, or an automated survey program.

❑ **Use appropriate technology to organize data collected for an investigative question of interest.**

- Explain to students that there are multiple options for organizing and displaying data and there are multiple choices for technology that can be used for this purpose. Present an investigative question to students. Then show an appropriate choice of technology and the corresponding display of the data collected.

  Investigative Question: What were the most popular lunch option this week at the school cafeteria?

  Explain that using a data table on the computer is a way to organize the lunches that were available this week; it's also important to leave enough space to present the data next to each lunch option. Creating a data table on the computer and then printing it out to write in the tally marks is an appropriate way to use technology to collect and organize data.

Lunch	Number of Students
hamburger	𝍸 𝍸 𝍸 𝍸
soup & salad	𝍸
sandwich	𝍸 𝍸
taco	𝍸 𝍸 𝍸

  Some students may be able to recognize that the hamburger column contains the most tally marks, which would be enough to answer the investigative question. Others may find it helpful to input this information into bar-graph-generator technology to visually see the differences. Both ways are appropriate uses of technology to organize collected data.

- Ask students to identify an investigative question of interest and then use appropriate technology to organize the collected data. For example, provide a variety of investigative questions and their accompanying data that aren't yet organized. Ask students to use technology to organize and interpret the data.

Prerequisite Extended Indicators

MAE HS.D.1.a—Determine more than one method to collect data to answer an investigative question.

MAE HS.D.1.b—Follow a plan to collect data for an investigative question of interest.

MAE 7.D.1.a—Collect data to answer an investigative question.

Key Terms

bar graph, circle graph, data, data table, investigative question, pie chart, survey, tally chart, technology

Additional Resources or Links

https://www.map.mathshell.org/lessons.php?unit=9405&collection=8

https://im.kendallhunt.com/MS/students/1/8/16/index.html

HS.D.1.d

Distinguish between surveys, observational studies, and experiments.

Extended: Identify the difference between a survey, an experiment, and/or an observational study.

Scaffolding Activities for the Extended Indicator

❑ **Identify the difference between a survey, an experiment, and/or an observational study.**

- Explain to students that there are different types of statistical studies that are used to collect data. The three major types are a survey, an experiment, and an observational study.

  Survey – A study that gathers data by asking people any number of questions. A survey may include questions that are based on opinions or facts, depending on the purpose of the study. Surveys can be administered in many forms, such as over the phone, on paper, face-to-face, online, and more. Because of the ease with which these surveys can be conducted, they can be used with large numbers of people. **Key features:** Survey participants are randomly selected and then asked questions about a desired topic.

  Observational Study – A study where the sample population being studied is surveyed without being directed or manipulated. The person collecting the data is able to study the variables but does not intervene in any way, meaning they do not influence the results in any way. With an observational study, the data are collected and then information is drawn from the results. **Key features:** The behavior of randomly selected people is observed and studied to collect data about a desired topic.

  Experiment – A study that is more of an ordered procedure, which is performed with the objective of verifying and determining the validity of a hypothesis. While experiments are being conducted, they can be intervened with to collect cause-and-effect data. **Key features:** Randomly selected people are split into groups, one of which serves as a control group. The person collecting data can intervene during data collection to witness cause-and-effect situations.

- Ask students to draw a line from the type of study to the key feature that best fits its description.

Survey	An ordered procedure obtains data by using a control group.
Observational Study	Data are collected through the observation of behaviors.
Experiment	Data are obtained by asking questions.

- Provide students with an example and how each type of study would look using the example.

  Example: You want to know how much students in the lunchroom like pizza.

Survey: A survey could be that you give every student a piece of paper with questions that ask if they like pizza. After every student answers the question, the data can be looked at to determine how many of the students like pizza.

Observational Study: An observational study could be that you give every student in the lunchroom a piece of pizza and observe. You could note which students talked about loving the pizza, which students ate the whole slice, which students didn't take any bites, which students asked for more, etc. This will let you know a little more detail about how much students like the pizza than a regular survey.

Experiment: An experiment involves a control group, so first you could split the students in the lunchroom into two groups. Provide both groups with a slice of pizza and record observations. Then give one of the groups ranch dressing to eat with the pizza. The group without the ranch dressing is the control group since there were no changes made to that group. Now you can see the control group eating the pizza, determine if they like the pizza, and then compare it to the students who eat pizza with ranch dressing. Does the group with ranch like the pizza more than without? Are the students who got ranch more pleased than the control group that didn't get ranch? This experiment provides even more data than the survey and observational study would.

- Ask students to identify the difference between a survey, an observational study, and an experiment. For example, provide a topic and an example of a survey, an observational study, and an experiment that corresponds to the topic. Ask students to match the label with the correct study, as shown.

You want to know how many students in class enjoy listening to country music.

Ask each student, "Do you like country music?" and record answers.	
Separate the class into two groups. Play country music at a low volume for both groups and record data. Then, for one of the groups, play the country music louder and record data.	
Play country music in the classroom and observe the students' behavior while recording data.	

Survey

Observational Study

Experiment

Prerequisite Extended Indicators

MAE HS.D.1.a—Determine more than one method to collect data to answer an investigative question.

MAE HS.D.1.b—Follow a plan to collect data for an investigative question of interest.

MAE HS.D.1.c—Use appropriate technology to organize data collected for an investigative question of interest.

MAE 7.D.1.a—Collect data to answer an investigative question.

Key Terms

data, experiment, observational study, study, survey, questionnaire

Additional Resources or Links

https://www.map.mathshell.org/lessons.php?unit=8405&collection=8

https://www.nctm.org/Classroom-Resources/Illuminations/Lessons/Conduct-an-Experiment/

HS.D.2 Analyze Data and Interpret Results

HS.D.2.e

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data and recognize possible associations and trends in the data.

Extended: Interpret categorical data for two categories in a two-way frequency table.

Scaffolding Activities for the Extended Indicator

❑ **Identify characteristics (e.g., title, categories, quantities) in a two-way frequency table.**

- Describe a two-way frequency table as a way to organize data by separating them into two sets of categorical data. It displays the possible relationships between the two sets of data.

  Present a two-way frequency table about students in sports and music activities. Focus on identifying characteristics of the table including the title, different categories, and quantities of students. Use different colors to indicate the two separate sets of data. Indicate where the title of the table is and that it signifies what data will be represented in the table.

Students in Sports and Music Activities

	Basketball	Baseball	Total
Choir	3	2	5
Band	1	2	3
Total	4	4	8

- Repeat the identification process by identifying the categories (in this case activities) included on the table. Explain that the sports activities of basketball and baseball have been placed across the top of the table. Ask the students to identify where the two music activities are located on the table. Confirm or correct the students' responses.
- Demonstrate the identification process of the quantities of students in the different activities. Identify that there are three students that are in both basketball and choir. Explain that this information can be found by finding the box in the table that is shared by two of the activities.
- Ask students to identify how many students are in basketball and band.
- Ask students to identify how many students are in baseball and band.

- ❑ **Identify total quantities represented in a two-way frequency table.**
  - Present a two-way frequency table about students in sports and music activities. Explain that totals of students in different activities can be found by using the bottom row, which shows the totals for students in basketball and baseball, or the column on the far right, which shows the totals for students in choir and band.

Students in Sports and Music Activities

	Basketball	Baseball	Total
Choir	3	2	5
Band	1	2	3
Total	4	4	8

  - Demonstrate that finding the total number of students in basketball can be found by following the basketball column to the bottom, which shows that a total of four students played basketball. Explain that the total was found by adding together numbers within the basketball column.
  - Ask students to identify how many students sang in the choir.
  - Ask students to identify how many students are represented in this two-way frequency table in total.

- ❑ **Interpret categorical data to make comparisons of quantities represented in a two-way frequency table.**
  - Present a two-way frequency table about students in sports and music activities. Explain that data represented in the table can be used to make comparisons between the different activities.

Students in Sports and Music Activities

	Basketball	Baseball	Total
Choir	3	2	5
Band	1	2	3
Total	4	4	8

  - Indicate that there are four students that play basketball and three students that are in band. Demonstrate that these two numbers can be compared to come to the conclusion that there are more students that play basketball than participate in band.
  - Continue by indicating that there are three students that participate in basketball and choir while there are two students that participate in baseball and choir. Ask the students to identify which sport is more likely to be played for a student who also participates in choir.
  - Ask students to interpret the data from the table and indicate whether there are more students in choir or more students in band.

- Ask students to interpret the data from the table and identify which combination of sport and music activities have the least number of students.

Prerequisite Extended Indicators

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Key Terms

categories, column, data, row, title, total, two-way frequency table, quantities

Additional Resources or Links

https://im.kendallhunt.com/HS/teachers/1/3/2/index.html

https://im.kendallhunt.com/HS/teachers/1/3/1/preparation.html

HS.D.2.f

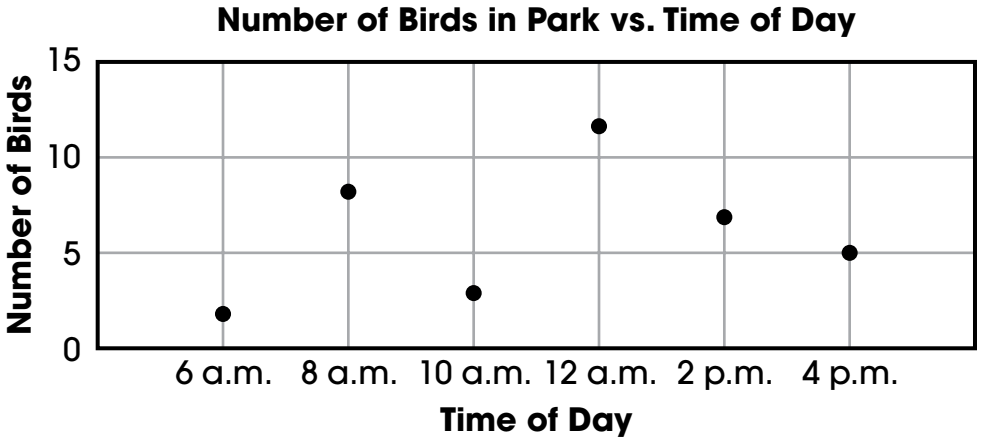
Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

Extended: Given a scatter plot with a clear trend, determine if there is a positive or negative association.

Scaffolding Activities for the Extended Indicator

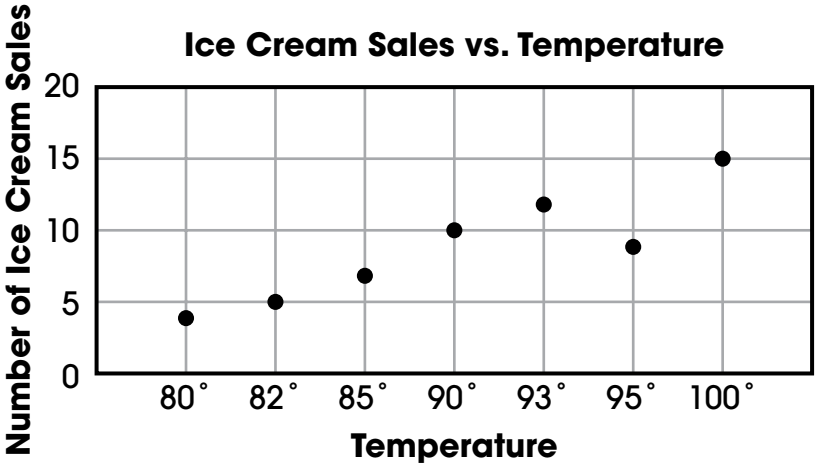
❑ **Recognize a scatter plot.**

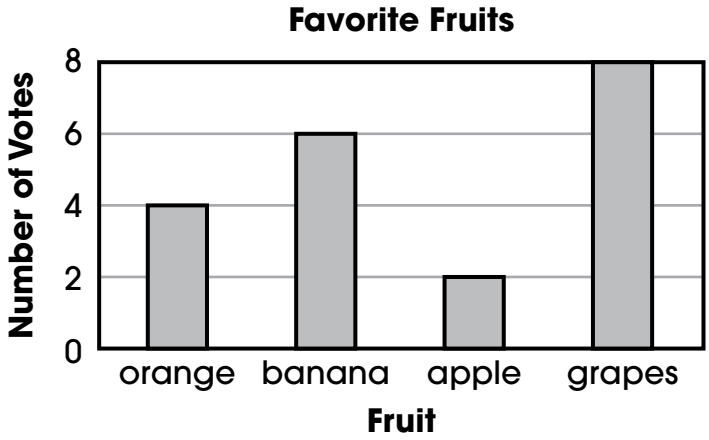
- Describe a scatter plot as a way to display data to show a relationship between two different variables.
- Present a scatter plot with data about the number of birds in the park depending on the time of day. Explain that the two different variables in this scatter plot include the time of day and the number of birds that are in the park. Point out that at 6:00 a.m. there may be only two birds in the park but at noon there could be twelve. Demonstrate that on this scatter plot there are varying numbers of birds in the park depending on the time of day. The relationship between the two variables can change.



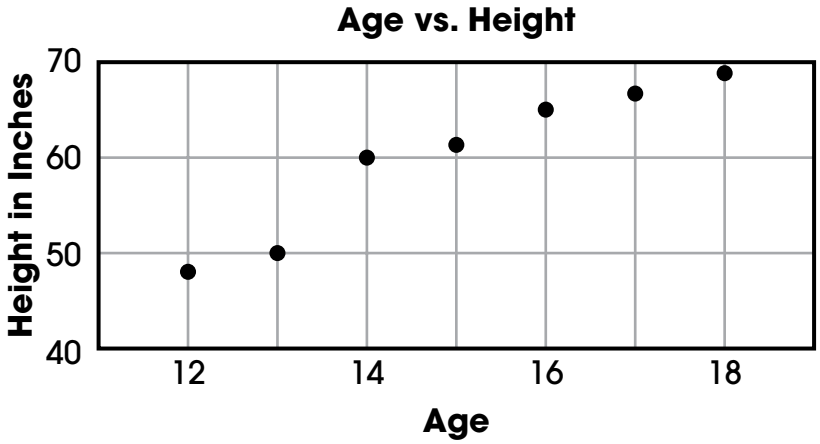
Present a line graph to students. Explain that a line graph also displays data showing the relationship between two variables, but it is not a scatter plot because the data points are connected. Conclude that the line graph is not an example of a scatter plot.

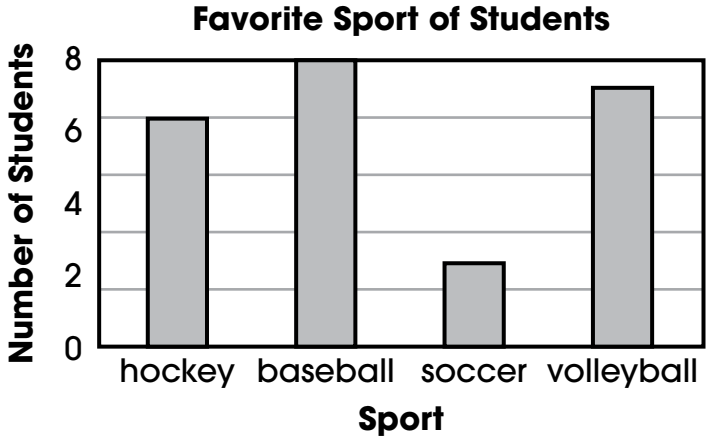
- Present a scatter plot about ice cream sales and temperature and a bar graph about students' favorite fruits. Explain that both graphs are displaying information, but one is a scatter plot, and one is a bar graph. Ask the students to identify the scatter plot. Confirm or correct the students' responses.

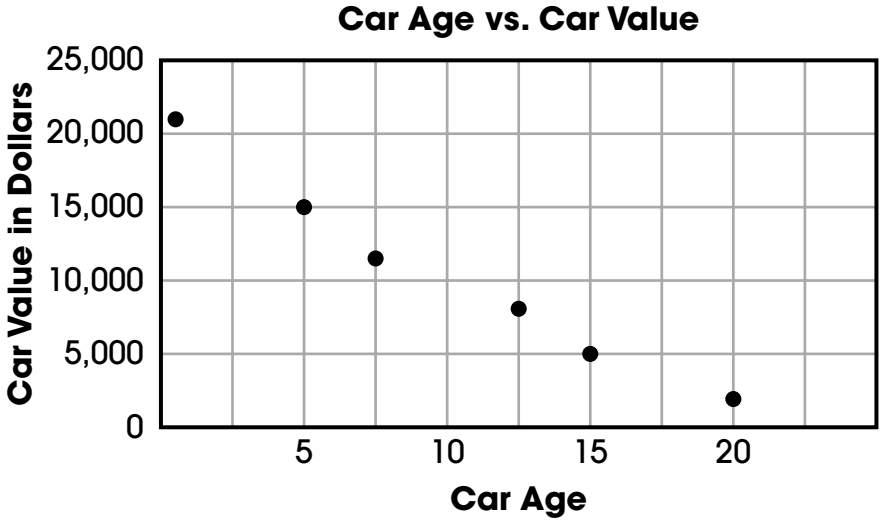


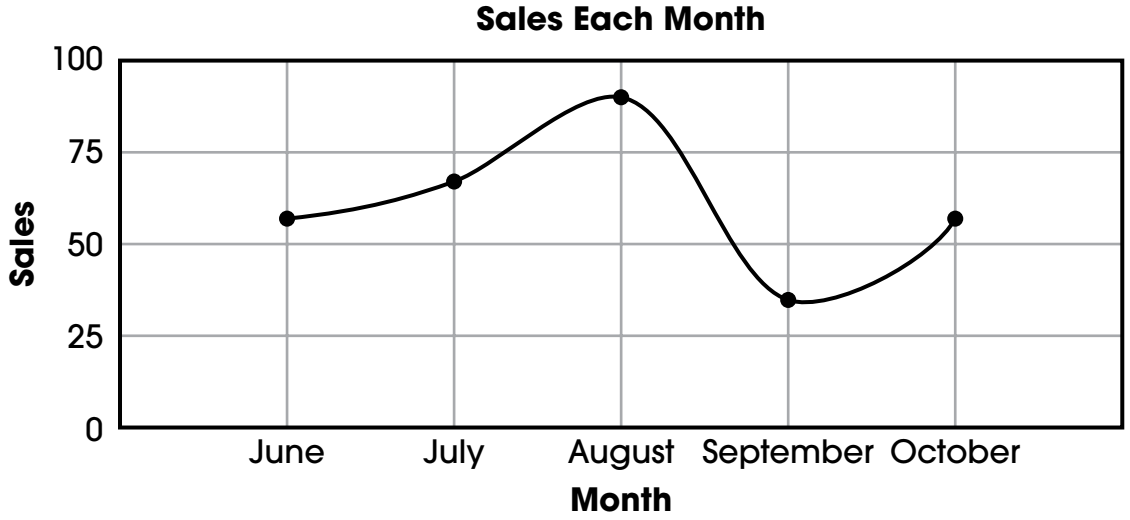


- Ask the students to identify the two scatter plots when provided with four different plots and graphs.



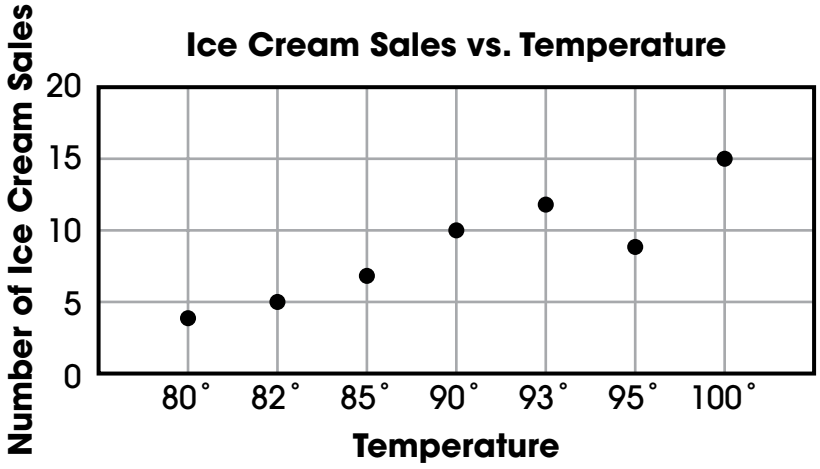




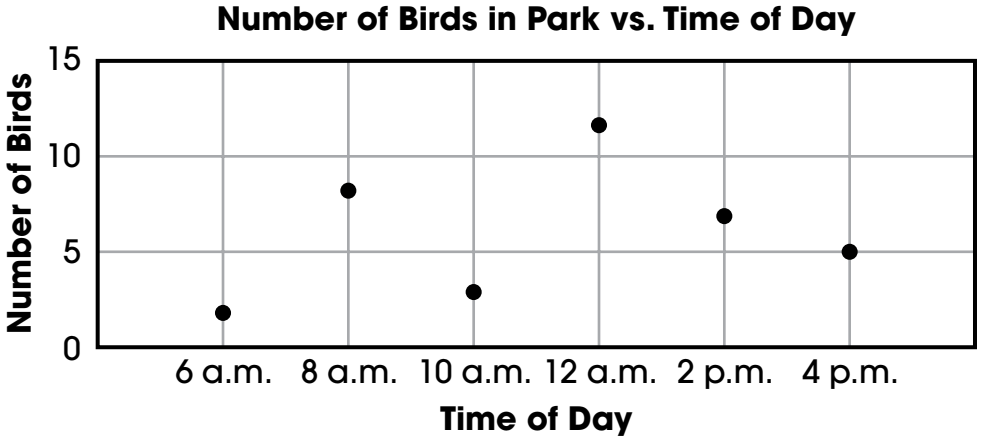


- ❑ **Identify a scatter plot with a clear trend.**

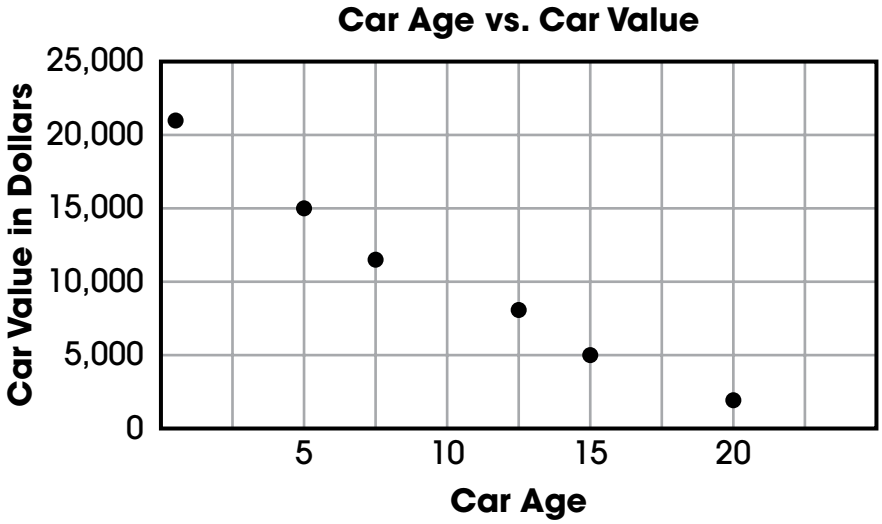
  - Describe a trend as a pattern seen within a set of data. Some patterns that you might see are both variables, or data points, increasing or as one variable increases, the other might decrease. These types of patterns can be described as a trend in the data. Present a scatter plot about the number of ice cream sales and temperature. Indicate on the graph that as the temperature is increasing, the number of ice cream sales is increasing. Explain that the relationship between the two variables shows there is a clear trend.



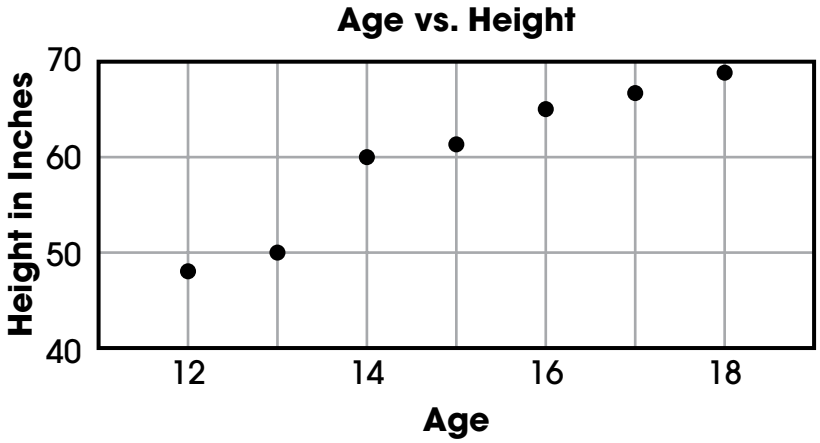
- Present a scatter plot about the number of birds in the park and the time of day. Indicate on the graph that the number of birds in the park can vary depending on the time of day. Explain that the data are showing that the number of birds in the park do not display a clear trend or pattern compared to the time of day. Since the data points are all over the graph, and there is not a clear direction of either increasing or decreasing, there is not a clear trend or pattern.



- Present a scatter plot about the age of a car and the car's value. Ask the students to identify whether or not the data points on the scatter plot indicate a clear trend. Make the connection that as the car becomes older, the value of the car becomes lower, which would indicate that there is a clear trend.



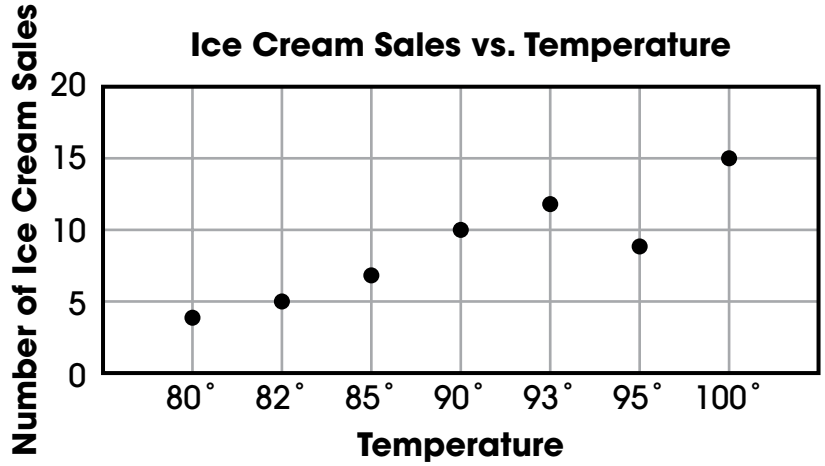
- Present a scatter plot about the age of a person and the person's height. Ask the students to identify whether or not the data points on the scatter plot indicate a clear trend.



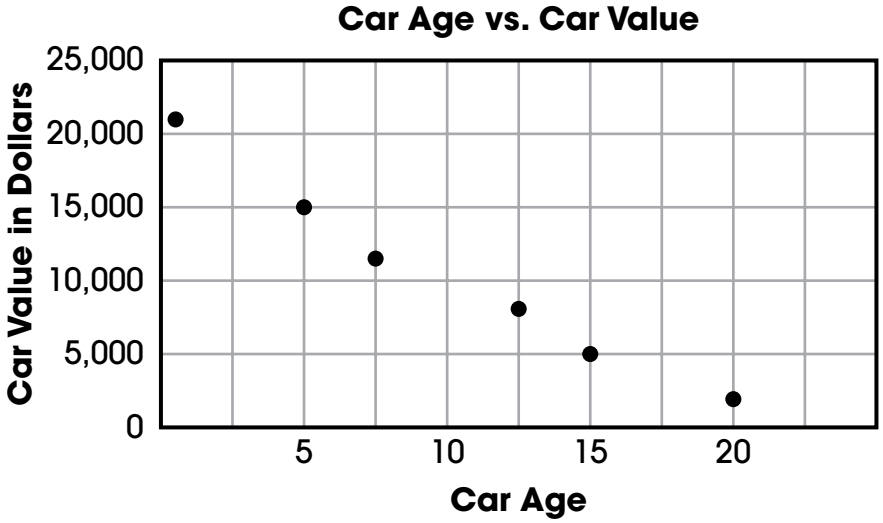
❑ **Determine if a scatter plot with a clear trend is showing a positive or negative association.**

- Describe a trend as a pattern seen within a set of data. Explain that if variables tend to increase together, there is a positive association. Explain that if one variable increases while the other variable decreases, then there is a negative association.

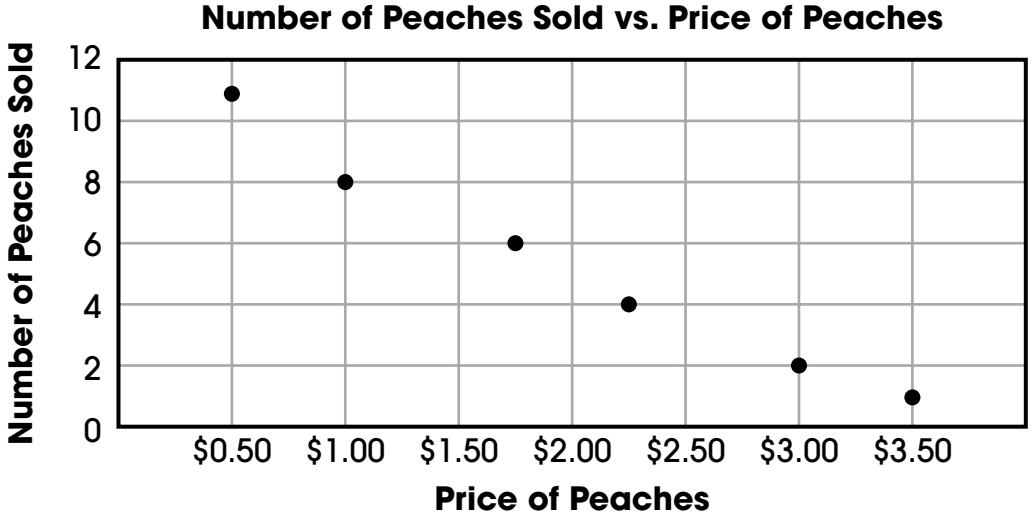
- Present a scatter plot about the number of ice cream sales and temperature. Indicate on the graph that as the temperature is increasing, the number of ice cream sales is increasing. Explain that the relationship between the two variables shows there is a positive association.



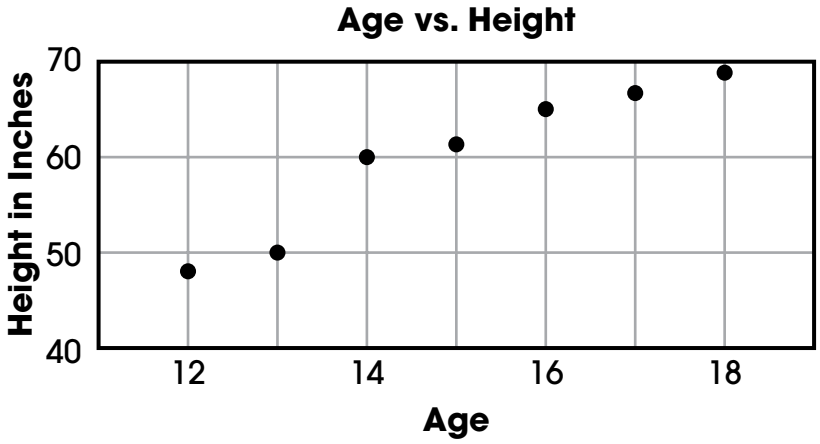
- Present a scatter plot about the age of a car and the car's value. Indicate on the graph that as the car becomes older, the age increases, the value of the car decreases over the years. Explain that the relationship between the two variables shows that there is a negative association.



- Present a scatter plot about the price of peaches and the number of peaches sold. Ask the students to identify whether or not the trend of the scatter plot indicates a positive or negative association. Make the connection that as the price of peaches increases, the fewer peaches that are sold, which would indicate that there is a negative association.



- Ask the students to identify whether or not the trend of the scatter plot indicates a positive or negative association with the trend.



Prerequisite Extended Indicators

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.D.1.a—Identify and compare quantities in line plots, limited to two data points.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Key Terms

data, negative association, positive association, relationship, scatter plot, trend, variables

Additional Resources or Links

https://www.engageny.org/resource/grade-8-mathematics-module-6-topic-b-lesson-6

https://im.kendallhunt.com/MS/teachers/3/6/5/index.html

HS.D.3 Probability

HS.D.3.d

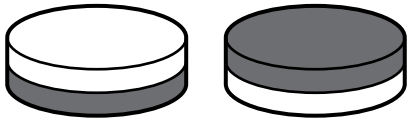
Determine whether or not events are mutually exclusive (disjoint) and calculate their probabilities in either case.

Extended: Identify a pair of mutually exclusive outcomes.

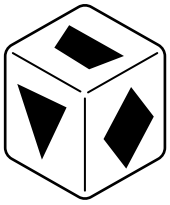
Scaffolding Activities for the Extended Indicator

❑ **Determine possible outcomes to a specific event.**

- Demonstrate how to interpret events and the possible outcomes. Present a counter that is black on one side and white on the other. Explain that because there are two colors, there are two possible outcomes. If tossed in the air, the counter shown could land on white or black but not on both.



Present a cube marked with a different geometric shape on each side. Explain that because the cube has six sides, there are six possible outcomes. When the cube is rolled, the result will be one of the six geometric shapes facing up.



Present the scenario of asking classmates whether they like cats, dogs, both cats and dogs, or neither cats nor dogs. Explain that in this example, there are four possible outcomes. In this case, it is meaningful for the choices to be combined because people can like both cats and dogs at the same time, and people may like neither cats nor dogs.

- Ask students to interpret an event or a scenario in which outcomes cannot occur at the same time or overlap and determine the possible outcomes by, for example, rolling a number cube, flipping a coin, or using a spinner.
- Ask students to interpret a scenario or an event in which two outcomes can occur at the same time or overlap and determine the number of possible outcomes. For example, present options of activities that can occur simultaneously during lunch.

❑ **Identify a pair of mutually exclusive outcomes.**

- Demonstrate when multiple events can and cannot be combined. First, present a scenario in which two events can be combined or occur at the same time. For example, Selena wants to study and listen to music after school. Explain that Selena can combine the activities and do both activities at the same time, or she could study first and then listen to music or vice versa.

  Next, present a scenario in which two events can't be combined to introduce the concept of mutually exclusive. For example, Tyson wants to play soccer and play baseball, but the two teams practice every day at 5:30 p.m. Explain that since the two sports have the same practice time, Tyson will have to choose soccer or baseball. When two events can't occur at the same time, the events are called mutually exclusive.

  Continue to demonstrate examples of events or outcomes that are mutually exclusive. For example, present a cube with sides numbered 1 through 6. Demonstrate that tossing the cube has six possible outcomes, but the toss will result in only one outcome at a time: 1, 2, 3, 4, 5, or 6. The outcomes of 1 and 6, for example, are mutually exclusive.

- Ask students to identify mutually exclusive outcomes for a given scenario or event (e.g., rolling a number cube, flipping a coin, spinning a spinner).
- Ask students to identify which outcomes are mutually exclusive. For example, present four cards with numbers on them in different colors. Present two black cards with the numbers 2 and 3 and two red cards with the numbers 4 and 5. Give students the scenario of picking one card and ask them which outcomes are mutually exclusive: picking a number card and a red card or picking a number 2 card and a number 5 card.

Prerequisite Extended Indicators

MAE 6.D.3.a—Identify a list of possible outcomes for a simple event, limited to four possible outcomes.

MAE 6.D.3.c—Identify the probability of an event as always, sometimes, or never.

Key Terms

event, mutually exclusive, outcome

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1660

https://im.kendallhunt.com/MS/teachers/2/8/index.html

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Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators High School