NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 8

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

NEBRASKA DEPARTMENT OF EDUCATION

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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 8
Number

8.N.1 Numeric Relationships

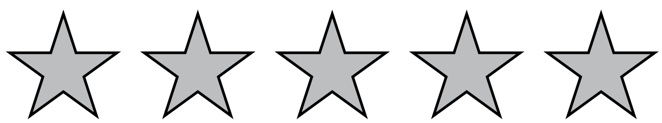
8.N.1.a

Determine subsets of numbers as natural, whole, integer, rational, irrational, or real based on the definitions of these sets of numbers.

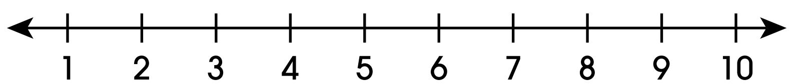
Extended: Distinguish between whole numbers, fractions, and decimals (e.g., $\frac{3}{5}$, 4, 1.7).

Scaffolding Activities for the Extended Indicator

- ❑ **Identify a whole number.**
  - Present a group of whole objects. For example, present five stars. Count the stars. The whole number that indicates how many stars there are is 5. Continue presenting other small groups of objects and the whole number that identifies how many objects there are. Be sure to make the connection that since the objects presented are whole, whole numbers can be used to represent the values. Show non-examples of wholes, or parts, if necessary.

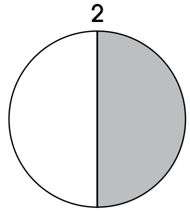


  - Ask students to identify the whole numbers on a number line.



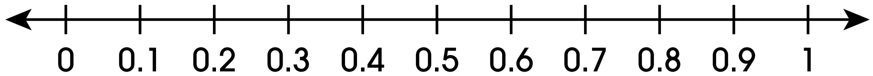
- ❑ **Identify fractions and decimals.**
  - Explain that different types of numbers are used to show the value of a part of a whole. For example, the part of the circle that is shaded can be represented by the fraction $\frac{1}{2}$. Indicate that a fraction has a numerator (top number) and a denominator (bottom number), with a line separating the numerator and the denominator. Continue to show other examples of fractions used to represent the part of a whole or the part of a set.

$\frac{1}{2}$



  - Demonstrate identifying fractions in a variety of scenarios.
  - Ask students to identify the fractions when given examples of both whole numbers and fractions.

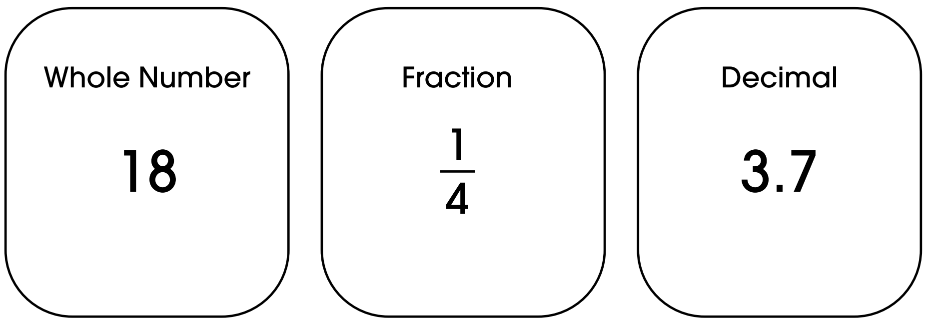
- Explain that another way to show a part of something is by using a decimal. For example, one whole can be divided into ten equal parts, and the value of each part can be represented as a decimal. Show the ten equal parts on a number line from 0 to 1 that has been labeled in tenths. Reference the decimal point in each decimal.



- Demonstrate identifying decimals in a variety of scenarios.
- Ask students to identify the decimals when given examples of both fractions and decimals.

❑ **Distinguish between whole numbers, fractions, and decimals.**

- Use cards showing whole numbers, fractions, and decimals to make comparisons. Present the following number cards. Identify the characteristics of each number form.



Present a number card that is not labeled and demonstrate asking clarifying questions to help students identify whether the number is a whole number, fraction, or decimal. For example, does this number have a top and bottom number? Does this number have a decimal point?

- Demonstrate sorting number cards into groups of whole numbers, fractions, and decimals.
- Ask students to sort number cards into groups of whole numbers, fractions, and decimals.

8.N.1 Numeric Relationships

- Use a chart to show the differences in the written form for whole numbers, fractions, and decimals. For example, create a three-column chart with the columns labeled Whole Number, Fraction, and Decimal with examples of each number form. List defining characteristics for each number form.

<table>
<tr><th colspan="3">Whole Number</th><th colspan="3">Fraction</th><th colspan="3">Decimal</th></tr>
<tr><td>5</td><td>68</td><td>135</td><td>$\frac{1}{2}$</td><td>$\frac{2}{5}$</td><td>$\frac{7}{10}$</td><td>0.5</td><td>6.9</td><td>12.7</td></tr>
<tr><td colspan="3">● counting numbers and 0
● no decimal point</td><td colspan="3">● part of a whole
● top number
● bottom number</td><td colspan="3">● part of a whole
● number/decimal/ number</td></tr>
</table>

- Ask students to identify whether a given value meets the criteria (defining characteristics) of a whole number, fraction, or decimal.

Prerequisite Extended Indicators

MAE 4.N.1.d—Use decimal notation for fractions from 0 to 1 with a denominator of 10 (e.g., $\frac{2}{10}$ = .2), and identify those decimals on a number line from 0 to 1.

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

Key Terms

decimal, decimal point, denominator, fraction, numerator, sort, whole number

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/documents/sample_materials/br3-tg-u4-m3.pdf

https://www.mathlearningcenter.org/sites/default/files/documents/sample_materials/br4-tg-u3-m3.pdf

8.N.1.b

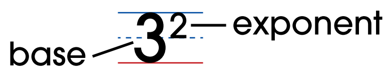
Represent numbers with positive and negative exponents and in scientific notation.

Extended: Represent numbers with the bases of 2, 3, 4, or 5 and positive exponents of 2 and 3 in expanded form (e.g., 4^3 = 4 × 4 × 4).

Scaffolding Activities for the Extended Indicator

- ❑ **Recognize the base and the exponent in an exponential expression.**

  - Use lined or gridded paper to place an exponent in superscript.

    Describe the base as the bigger number on the left and the exponent as the smaller number written above and on the right. Make explicit the difference between 3^2 and 32. Explain that the exponent can be read as “three squared,” “three to the second power,” or “three raised to the power of two.”

  - Ask students to identify parts of an exponential expression. For example, list 3^2, 4^3, and 5^2 and ask students to identify the expression with a base of 4. List 2^3, 5^2, and 4^3 and ask students to identify the expression with an exponent of 2.

  - Ask students to identify an expression that is read to them. For example, present 4^2, 5^2, 4^3, and 5^3. Ask students to identify four to the second power or five squared.

- ❑ **Represent numbers with a base of 2, 3, 4, or 5 and a positive exponent of 2 or 3 in expanded form.**

  - Make explicit connections between repeated multiplication and exponential notation. For example, with the expression 3 × 3, there are two threes being multiplied, or 3 is a factor two times. Another way to write 3 × 3 is 3^2, where the exponent of 2 means there are two threes being multiplied. In the same way, 4^3 means there are three fours being multiplied, or 4 × 4 × 4.

  - Ask students to determine how many times the base is multiplied in these expressions: 4^3, 5^2, 2^3. Ask students to determine the expanded form of values raised to the second power by presenting 2^2, 3^2, 4^2, and 5^2. Then ask students to identify that the correct expanded forms are 2 × 2, 3 × 3, 4 × 4, and 5 × 5, respectively. Repeat the same process for the values raised to the third power.

8.N.1 Numeric Relationships

Prerequisite Extended Indicator

MAE 5.N.1.c—Represent 10, 100, 1,000, or 10,000 as a power of 10.

Key Terms

base, cubed, expanded form, exponent, exponential notation, factor, power, squared

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-b-lesson-5/file/43511

http://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/2225

8.N.1.d

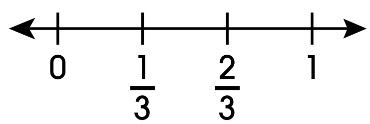
Approximate, compare, and order real numbers, both rational and irrational, and locate them on the number line.

Extended: Compare and order tenths, fifths, fourths, thirds, halves, and whole numbers 1–100 using a number line.

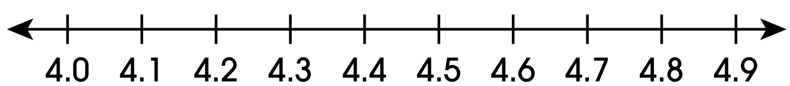
Scaffolding Activities for the Extended Indicator

- ❑ **Identify tenths, fifths, fourths, thirds, halves, and whole numbers on a number line.**

  - Use a number line from 0 to 1 to demonstrate how to identify tenths, fourths, thirds, halves, and whole numbers on a number line. Present the number line as shown to model the location of thirds on a number line. Explain that the distance between 0 and 1 has been divided into three equal sections called thirds. As the number line goes from left to right, the number of thirds increases. Emphasize that $\frac{3}{3}$ is equal to 1 whole.



  - Use sections of a number line from 1–100 to show values greater than 1 and less than 100. For example, use the number line shown to demonstrate how to identify tenths from 4.0 to 4.9.



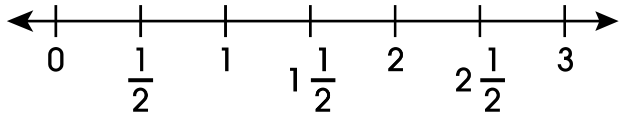
    Show a variety of number lines involving numbers greater than 1 and less than 100 using tenths, fifths, fourths, thirds, and halves. The number lines can be labeled with decimal numbers as shown in the example above or with whole numbers and mixed numbers using increments of fifths, fourths, thirds, and halves.

  - Ask students to identify tenths, fifths, fourths, thirds, and halves on a number line from 0 to 1.
  - Ask students to identify tenths, fifths, fourths, thirds, halves, and whole numbers on sections of a number line from 1 to 100.

8.N.1 Numeric Relationships

- ❑ **Compare tenths, fifths, fourths, thirds, halves, and whole numbers on a number line.**

  - Demonstrate comparing numbers on a number line. Present the number line as shown.

    Explain that the numbers increase in value from left to right. Demonstrate using the symbols greater than (>) and less than (<) to compare whole numbers.

    $$1 < 2$$
    $$3 > 2$$

    Then compare whole numbers and mixed numbers on the number line.

    $$1\frac{1}{2} < 2$$
    $$1\frac{1}{2} > 1$$
    $$2\frac{1}{2} > 1\frac{1}{2}$$
    $$2\frac{1}{2} < 3$$

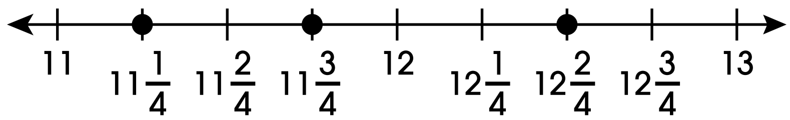
    Show a variety of number lines containing numbers greater than 1 and less than 100 using tenths, fifths, fourths, thirds, and halves. The number lines can be labeled with mixed numbers or decimal numbers.

  - Ask students to use the symbols greater than (>) and less than (<) to compare tenths, fifths, fourths, thirds, halves, and whole numbers on sections of a number line using numbers from 1 to 100.

8.N.1 Numeric Relationships

❑ **Order tenths, fifths, fourths, thirds, halves, and whole numbers on a number line.**

- Use a number line to demonstrate ordering tenths, fourths, thirds, halves, and whole numbers from least to greatest. For example, present the number line as shown.



  Explain that to order the numbers from least to greatest, the points should be read from left to right because the numbers increase in value from left to right. Reading from left to right, the points are plotted at the locations $11\frac{1}{4}$, $11\frac{3}{4}$, and $12\frac{2}{4}$, so that is the order from least to greatest.

  Continue to demonstrate ordering tenths, fourths, thirds, halves, and whole numbers using a variety of number lines containing numbers greater than 1 and less than 100. The number lines can be labeled with mixed numbers or decimal numbers.

- Ask students to order tenths, fifths, fourths, thirds, halves, and wholes using sections of a number line with numbers from 1 to 100.

Prerequisite Extended Indicators

MAE 6.N.1.e—Compare and order halves with halves, quarters with quarters, and tenths with tenths from 0 to 1 on a number line and compare and order integers from -10 to 10 on a number line.

MAE 4.N.2.a—Compare and order mixed numbers with denominators up to 5.

MAE 4.N.1.b—Use symbols <, >, and = to compare whole numbers up to 50.

MAE 4.N.1.d—Use decimal notation for fractions from 0 to 1 with a denominator of 10 (e.g., $\frac{2}{10}$ = .2), and identify those decimals on a number line from 0 to 1.

MAE 3.N.2.c—Represent halves and wholes on a number line.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

decimal number, fifth, fourth, fraction, greater than, half, less than, number line, tenth, third, whole number

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/NBT/A/tasks/1813

https://www.insidemathematics.org/sites/default/files/materials/decimals.pdf

https://apps.mathlearningcenter.org/number-line/

8.N.2 Operations

8.N.2.a

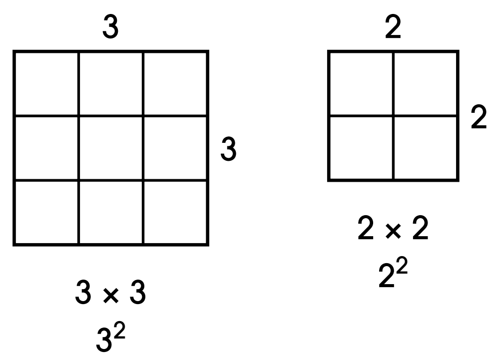
Evaluate the square roots of perfect squares less than or equal to 400 and cube roots of perfect cubes less than or equal to 125.

Extended: Identify the squares of whole numbers up to 10.

Scaffolding Activities for the Extended Indicator

❑ **Identify connections between square shapes and square arrays.**

- Present different-size square arrays by labeling their length and width and writing corresponding expressions. Explain that the number of rows in a square array matches the number of columns. Therefore, the array is considered a square. The area of the square is 9 because 3 × 3 = 9. 3 × 3 can also be written as 3^2 and read as “three squared” or “three to the second power.” The area is 9 because 9 square units cover the square.



Continue making the connections with squares of whole numbers up to 10.

- Ask students to complete a table with the equivalent array.

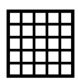
Array	Expression	Exponent
	2×2	2^2
	3×3	3^2
	4×4	4^2
	5×5	5^2
	6×6	6^2
	7×7	7^2
	8×8	8^2
	9×9	9^2
	10×10	10^2

Repeat the process by providing the array and expression and asking students to identify the exponent. Then repeat the process by providing the array and exponent and asking students to identify the expression.

- Ask students to complete a table with the equivalent array, expression, or exponent.

❑ **Identify the squares of whole numbers up to 10.**

- Present a table to make explicit connections between the base of a squared expression, the size of the corresponding array, and the value of the squared expression. For example, 5^2 can be represented as a square array with the base of 5 indicating the number of rows and columns. The area can be determined to be 25 by counting each unit square or by multiplying 5 × 5. Therefore, 5^2 is 25. Continue filling in additional rows of the table to demonstrate the squares of all whole numbers up to 10.

Array	Expression	Exponent	Value
	5 × 5	5^2	25

- Create cards with the numbers 1, 4, 9, 16, 25, 36, 48, 64, 81, and 100 written on them. Then create a second set of cards with the corresponding square arrays and squared expressions (such as 4^2) written on the cards. Demonstrate matching the cards. Use counting strategies and multiplication strategies when appropriate to determine the area. For example, the card with a 4 × 4 square array can be matched to the card with the number 16 by either counting the 16 squares or by multiplying 4 × 4.

- Ask students to match cards to determine squares of whole numbers up to 10.

Prerequisite Extended Indicators

MAE 8.N.1.b—Represent numbers with the bases of 2, 3, 4, or 5 and positive exponents of 2 and 3 in expanded form (e.g., 4^3 = 4 × 4 × 4).

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

Key Terms

array, base, column, exponent, expression, factor, row, square, squared, value

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-b-lesson-5/file/43511

http://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/2225

8.N.2.c

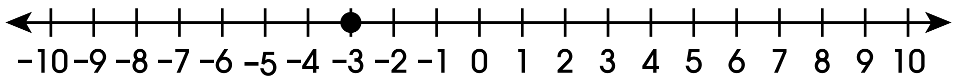
Evaluate numerical expressions involving absolute value.

Extended: Determine absolute value using a model (e.g., temperature below zero).

Scaffolding Activities for the Extended Indicator

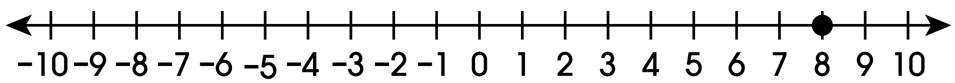
❑ **Determine absolute value using horizontal and vertical number lines.**

- Explain that the absolute value of a number is its distance from 0. Present a horizontal number line, as shown, with a point plotted at –3. Count the units (or distance) from –3 to 0 on the number line. Indicate that the absolute value of –3 is 3, or |–3| = 3, because –3 is 3 units from 0. Explain that two vertical lines with a number between them indicate that the absolute value of a number is being sought.



- Demonstrate how to identify a number plotted on a horizontal number line and determine the absolute value of the number.

What is the number?
What is the absolute value of the number?

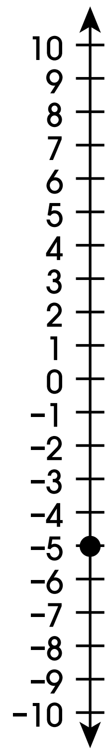


|8| = 8
The absolute value of 8 equals 8.

- Demonstrate how to identify a number plotted on a vertical number line and determine the absolute value of the number.

What is the number?

What is the absolute value of the number?

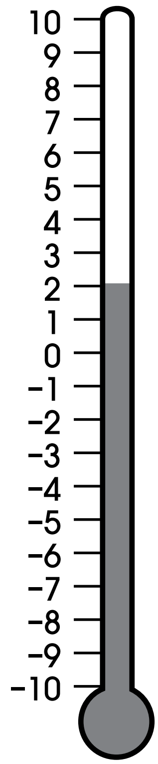


$|-5| = 5$

The absolute value of –5 equals 5.

- Ask students to determine the absolute value of a number on a horizontal number line and on a vertical number line.

- ❑ **Determine absolute value on a thermometer.**
  - Present a picture of a thermometer that illustrates a temperature of 2 degrees. Explain that the temperature is 2 degrees because the top of the bar is at 2. Indicate that the absolute value of 2 degrees is 2 because it is a distance of 2 from 0.

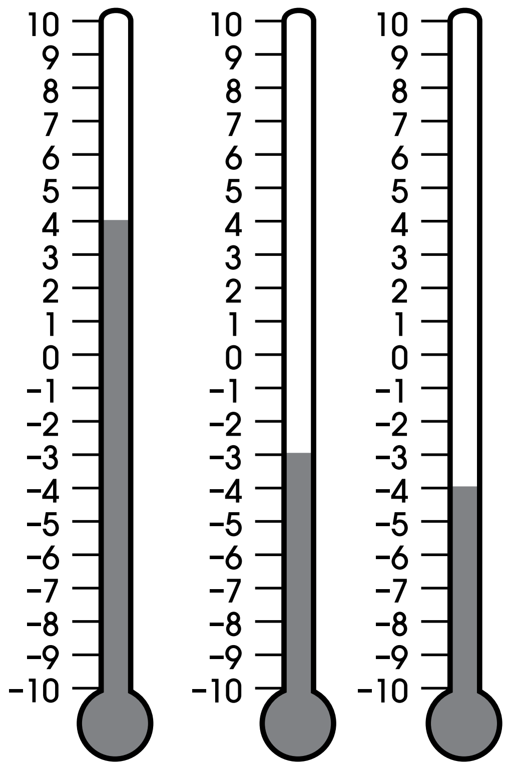


- Demonstrate matching absolute-value sentences to thermometers. Present three cards with absolute-value sentences and pictures of three thermometers. Match each card to a thermometer.

$$\boxed{|-4| = 4}$$

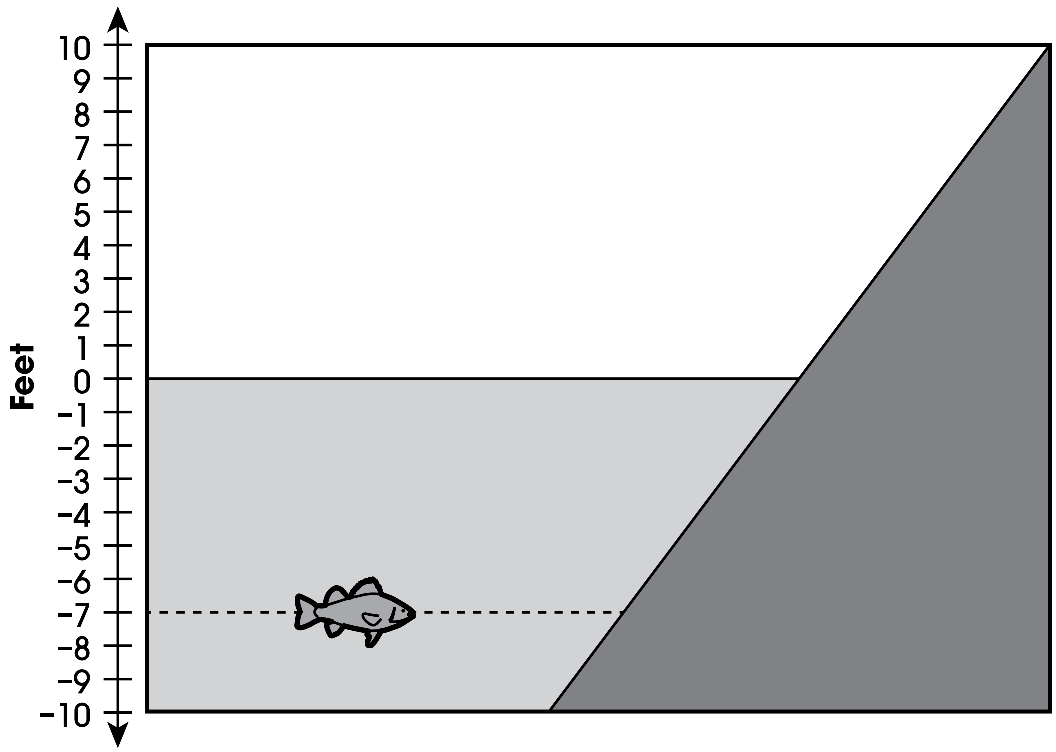
$$\boxed{|4| = 4}$$

$$\boxed{|-3| = 3}$$



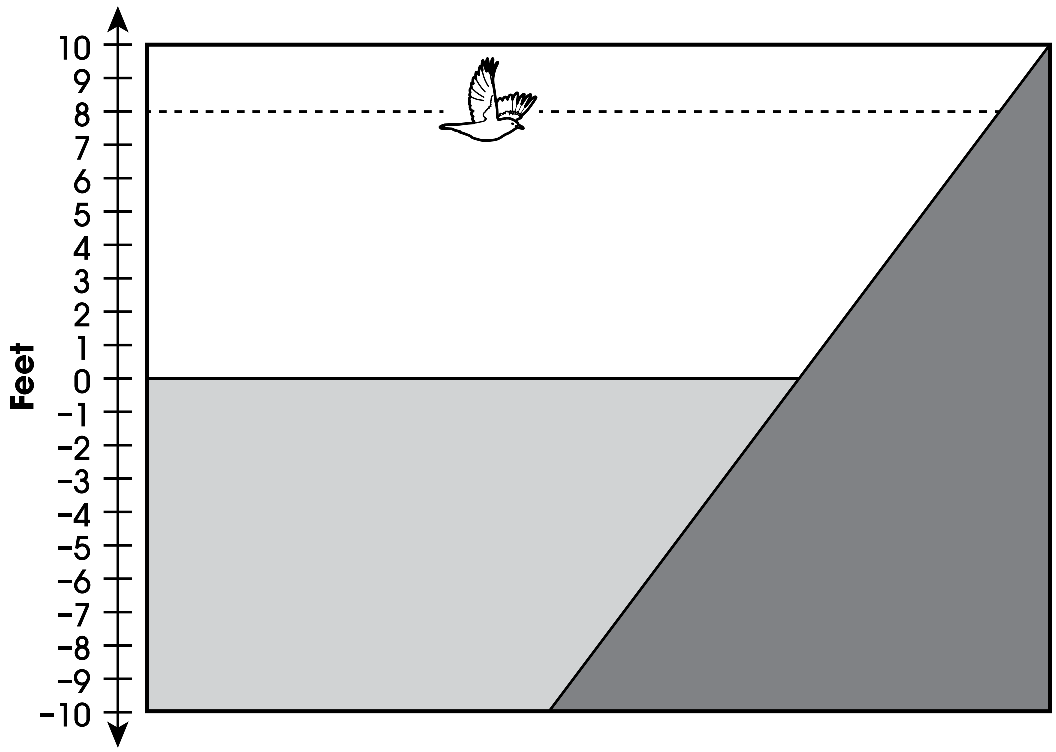
- Ask students to match absolute-value sentences with the temperature shown on a thermometer.

- ❑ **Determine absolute value on a model showing distance above and below sea level.**
  - Present a vertical sea-level number line diagram that illustrates a fish located at a depth of –7 feet. Explain that in this context, sea level has a location of 0 feet. Point to the 0 on the number line and the sea-level line to show how this is represented in the diagram. Indicate that the fish is located at –7 feet because its location aligns with the –7 on the number line. Explain that in this context, the absolute value represents how far an object is located above or below sea level. The fish in the diagram is located at a depth of –7 feet. This means that the fish is located 7 feet below sea level.



- Ask students to identify the locations of other objects, from –10 feet to 10 feet, on a vertical sea-level number-line diagram. Then ask students to connect this to absolute value by stating how far above or below sea level the object is located.

What is the location of the bird?
What is the bird's distance above or below sea level?



The bird is located at a height of 8 feet.
The bird is located 8 feet above sea level.

- Ask students to match absolute-value sentences with descriptions of an object's location and/or pictures of an object's location.

$\lvert -2 \rvert = 2$	$\lvert 2 \rvert = 2$
$\lvert -9 \rvert = 9$	$\lvert 9 \rvert = 9$

The shell is located at a depth of –9 feet. The shell is located 9 feet below sea level.	
The bird is located at a height of 9 feet. The bird is located 9 feet above sea level.	
The shell is located at a depth of –2 feet. The shell is located 2 feet below sea level.	
The rock is located at a height of 2 feet. The rock is located 2 feet above sea level	

Prerequisite Extended Indicators

MAE 6.N.1.c—Identify models of integers from –10 to 10 using drawings, words, manipulatives, number lines, and symbols.

MAE 6.N.1.d—Identify the absolute value of an integer between –10 and 10.

MAE 6.N.1.e—Compare and order halves with halves, quarters with quarters, and tenths with tenths from 0 to 1 on a number line and compare and order integers from –10 to 10 on a number line.

Key Terms

absolute value, altitude, depth, distance, height, location, measurement, temperature, thermometer

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-13

https://curriculum.illustrativemathematics.org/MS/students/1/7/7/index.html

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Mathematics—Grade 8
Algebra

8.A.1 Algebraic Processes

8.A.1.a

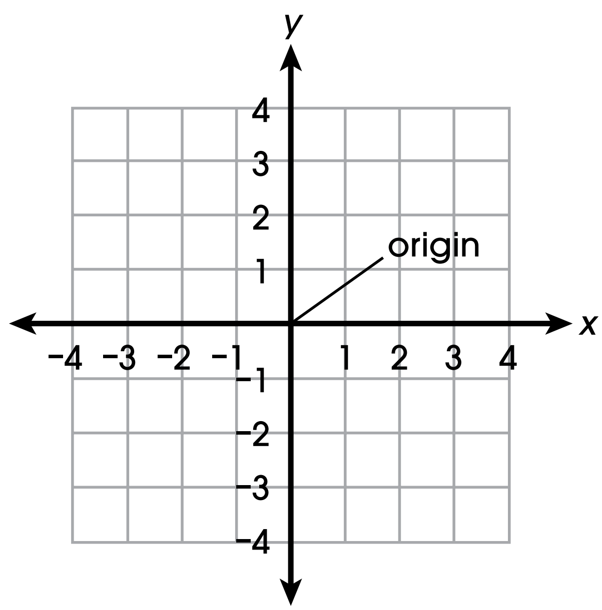
Describe single variable equations as having one solution, no solution, or infinitely many solutions.

Extended: Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

Scaffolding Activities for the Extended Indicator

❑ **Identify the origin on a coordinate graph that shows all four quadrants.**

- Use a coordinate plane to identify the origin. Explain to students that when two lines intersect, they are crossing at a point.

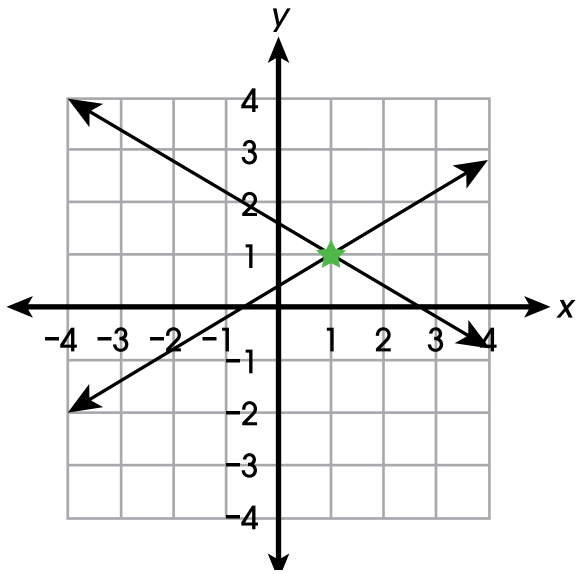


Indicate that the origin is the location where the *x*-axis and *y*-axis intersect. It can be represented with the ordered pair (0, 0). In the graph shown, the 0 is not labeled, but since the numbers 1 and –1 are on either side of the origin, the missing number in the middle is 0 for each axis. This is the same as a number line that shows both positive and negative numbers.

- Ask students to locate the origin on a coordinate plane.

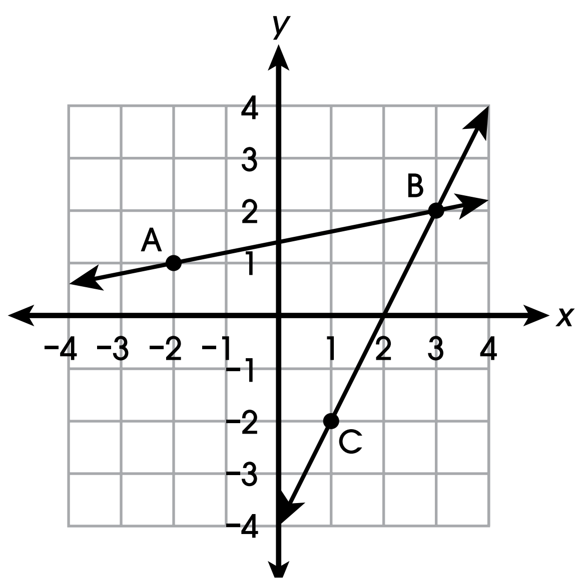
- **Identify the point where two lines intersect on a coordinate plane.**

  - Use a graph with two intersecting lines to show the point of intersection.



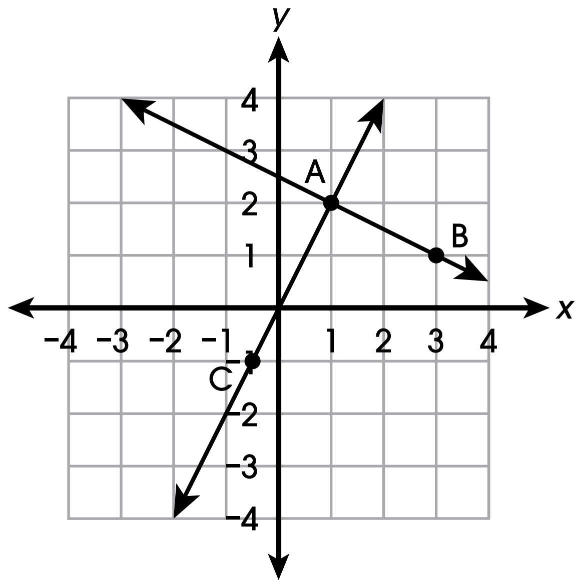
The point of intersection in the figure shown is marked with a star. Show a variety of graphs with intersecting lines. Emphasize that the point of intersection is also called the solution.

  - Indicate that this coordinate plane shows intersecting lines. Identify point A, point B, and point C. Explain to students that they are looking for the point that is the solution, or the point of intersection. Explain that point B is where the intersection of two lines is located, so point B is the solution.



Show students a variety of intersecting lines on coordinate planes and have them find the point of intersection.

- Ask students to locate the point of intersection (solution) on a graph. For example, present the following graph.



Then have students choose between point A, point B, and point C. Students should determine that the point of intersection is point A.

Prerequisite Extended Indicators

MAE 5.G.2.b—Identify the *x*- or *y*-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

Key Terms

coordinate plane, graph, intersection, line, ordered pair, origin, solution, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://nysed-prod.engageny.org/resource/grade-8-mathematics-module-4-topic-d-lesson-25

https://www.map.mathshell.org/lessons.php?unit=8220&collection=8

8.A.1.b

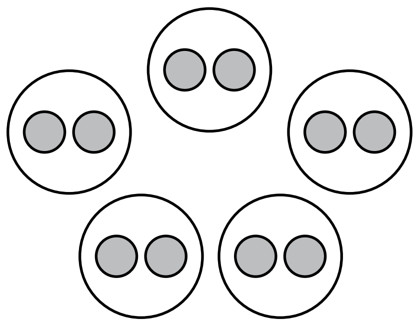
Solve multi-step equations involving rational numbers with the same variable appearing on both sides of the equation.

Extended: Use substitution to determine if a given value for a variable makes a two-step equation true.

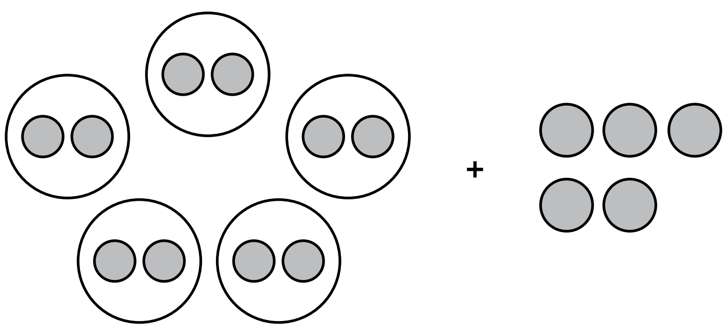
Scaffolding Activities for the Extended Indicator

❑ **Use the appropriate order of operations to evaluate a two-step expression.**

- Use an equation such as 5 × 2 + *n* = 15 to demonstrate solving a two-step equation. Explain that the first step is to multiply 5 × 2 because multiplication comes before addition and subtraction based on the order of operations. Demonstrate 5 × 2 with manipulatives as "5 groups of 2." This results in a product of 10.



Next, explain that the equation can be rewritten as 10 + *n* = 15. Therefore, *n* is equal to the quantity added to 10 that results in a sum of 15. Again, this may be modeled with manipulatives. Begin with 10 and count on until 15 is reached.

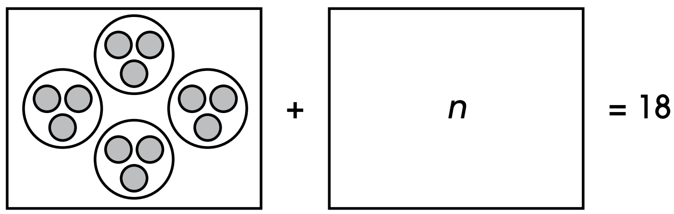


Since 5 more were added to reach 15, *n* = 5. Demonstrate replacing *n* with 5 in the original equation, so 5 × 2 + 5 = 15.

Repeat the process with an equation of the type 8 + *y* – 5 = 7. First, subtract 8 – 5 by combining like terms. Then, count on from 3 to 7 to demonstrate that 4 more must be added to get to 7. Therefore, *y* = 4.

- Ask students to complete a model with manipulatives or a drawing to solve a two-step equation. For example, provide a model that shows the equation $4 \times 3 + n = 18$. Provide a box with 4 groups of 3 along with an empty box to represent *n*. Ask students to determine the quantity that must be in the empty box so that the total of both boxes is 18. Have students use manipulatives or draw objects to count 6 more from 12 to get to 18 and conclude that *n* is 6.

$4 \times 3 + n = 18$

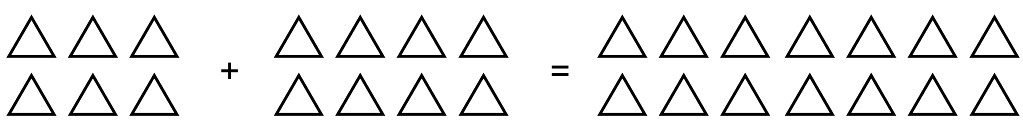


Repeat the process by starting with the equation $2 + c - 1 = 10$. Provide students with a box with 1 to represent $2 - 1$ along with an empty box to represent *c*. Ask students to count on from 1 to 10 to determine that *c* must be 9.

- Ask students to use a drawing or manipulatives to solve a two-step equation such as $a + 2 \times 4 = 20$ or $9 + b + 3 = 15$.

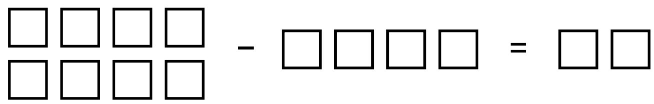
❑ **Use substitution to determine whether a given value for a variable makes a one-step equation true.**

- Demonstrate substituting a given value for a variable in an equation and explain whether that value makes the equation true. Use an equation such as $6 + d = 14$ given that $d = 8$ and substitute for *d*. When d = 8 is used to substitute into the equation becomes $6 + 8 = 14$. Since 6 + 8 is equal to 14, the equation is true. Demonstrate this by using manipulatives as shown.

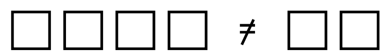


6 + 8 = 14

Repeat the process by using an equation such as $y - 4 = 2$ and use a value for y that makes the equation not true. For example, use $y = 8$. When 8 is substituted for y in the equation, the equation will be $8 - 4 = 2$. Ask students what $8 - 4$ is equal to. Then, ask students if 4 is equal to 2. Therefore, when $y = 8$, the equation $y - 4 = 2$ is not true. Again, this can be demonstrated by using manipulatives as shown.



$$8 \quad - \quad 4 \quad = \quad 2$$



$$4 \quad \neq \quad 2$$

- Demonstrate determining whether a given value for a variable makes an equation true. This table can be used to have students substitute values in different equations to show which values make equations true. Write a “T” or the word “True” in the correct row and column for the value of z that makes the given equation true.

	$z = 2$	$z = 5$	$z = 7$
$z + 4 = 9$			
$7 \times z = 14$			
$11 - z = 4$			

It should be determined that $z = 2$ is true for the second row, $z = 5$ is true for the first row, and $z = 7$ is true for the third row.

- Ask students to determine whether given values for variables in several different equations result in making the equations true. Some examples are:

$$6 \times t = 18 \text{ when } t = 3$$

$$15 - w = 10 \text{ when } w = 5$$

$$12 + p = 18 \text{ when } p = 2$$

- **Use substitution to determine whether a given value for a variable makes a two-step equation true.**

  - Use a two-step equation with a variable to show how to determine whether a given value for the variable makes the equation true. Use the equation shown to determine whether $t = 2$ or $t = 3$ makes the equation true.

    $$2 \times 3 - t = 3$$

    Substitute the value 2 for t to determine whether this value for t makes the equation true. Model putting parentheses around the substituted value.

    $$2 \times 3 - (2) = 3$$

    Multiply 2 × 3 which is 6. Now we can replace the first part of the equation with 6. The next step is to subtract 2 from 6. This results in a statement 4 = 3, which is not true. So, when the value 2 is substituted for the variable t, the equation is not true.

    $$\begin{aligned} 6 - 2 &= 3 \\ 4 &= 3 \quad \text{is not true} \end{aligned}$$

    Repeat the process using $t = 3$, as shown. This value for t makes the equation true.

    $$\begin{aligned} 2 \times 3 - t &= 3 \\ 2 \times 3 - (3) &= 3 \\ 6 - 3 &= 3 \\ 3 &= 3 \quad \text{is true} \end{aligned}$$

    Demonstrate determining whether a given value for a variable makes an equation true. Students can substitute the different given values for s for the variables in the different given equations to determine which value makes each equation true. Each of the values makes one of the given equations true.

$20 - 2 \times s = 6$	$s = 4$
$8 + 1 - s = 3$	$s = 5$
$4 + s + 2 = 10$	$s = 6$
$2 \times s + 1 = 11$	$s = 7$

    Ask students to use substitution to determine which two-step equation is true when $x = 2$.

    $$\boldsymbol{x = 2}$$

    $$4 \times x + 3 = 11$$

    $$12 + x - 10 = 2$$

Prerequisite Extended Indicators

MAE 7.A.1.c—Solve a one-step equation using multiplication.

MAE 5.A.1.d—Evaluate two-step numerical expressions involving addition or subtraction and multiplication using order of operations, limited to the digits 1–5 (e.g., 4 × (5 – 2), 4 + 2 × 3).

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

Key Terms

equation, group, multiplication, product, substitution, sum, symbol, true, variable

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1686

https://im.kendallhunt.com/MS/students/2/6/11/index.html

8.A.2 Applications

8.A.2.a

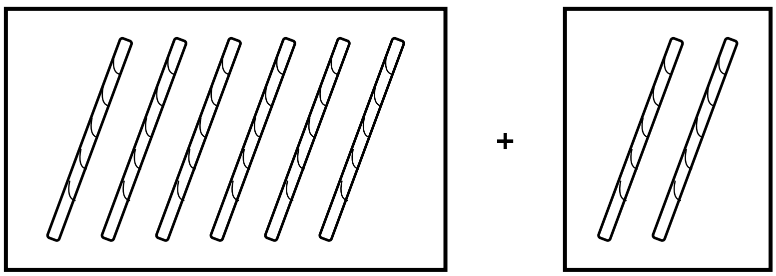
Write multi-step single variable equations from words, tables, and authentic situations.

Extended: Identify a two-step expression that represents an authentic situation, limited to addition, subtraction, and multiplication.

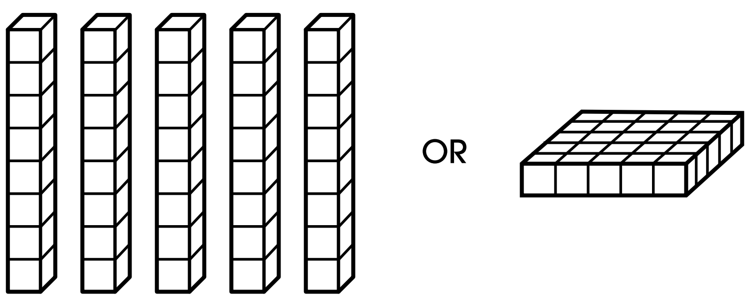
Scaffolding Activities for the Extended Indicator

❑ **Use a model to demonstrate how to represent an authentic situation with an expression.**

- Use a one-step authentic situation to create a model that can be used to represent the situation. For example, in the situation where one student has 6 straws and another student has 2 straws, a model can show the sum or total number of straws the two students have. One possible model of the expression 6 + 2 is shown.

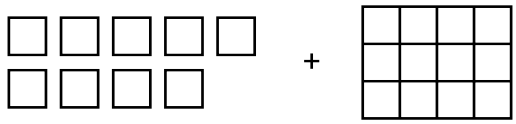


- Present an authentic situation using multiplication. For example, explain that a person works 8 hours each day for 5 days. Emphasize that models can be used to represent the information in the multiplication problem. Demonstrate creating two different models. Ask students which model represents the multiplication expression 8 × 5 from the scenario.



Students should choose the model that shows 5 stacks of 8 cubes. Explain that the other type of model shown would work if it were 5 units by 8 units instead of 5 units by 5 units.

- Use a two-step authentic situation to create a model that can be used to represent a situation. For example, present a situation where a student has 9 golf balls and then buys 4 packages of golf balls that each contain 3 golf balls. This situation can be represented using the following model and expression. Remind students that 4(3) is another way to show 4 × 3.



9 + 4(3)

- Ask students to model a situation using a drawing, manipulatives, or an expression that shows a two-step expression involving addition, subtraction, and/or multiplication. For example, present a situation where someone bought 12 eggs and then used 3 eggs for one recipe and 6 eggs for another recipe. Which expression can be used to represent this situation? Remind students to use models to represent the expression.

  12 + 3 + 6

  12 – 3 × 6

  12 – 3 – 6

  Students should choose 12 – 3 – 6 since it shows three eggs and six eggs being removed from the original twelve eggs that were purchased.

❑ **Identify a one-step expression that represents an authentic situation.**

- Use an authentic situation to demonstrate a one-step expression involving addition, subtraction, or multiplication. For example, a student bought 3 tickets that cost $4 each. The expression that represents this situation is shown.

  4 × 3

  Explain that, since each ticket costs $4 and 3 tickets were purchased, the two values should be multiplied if the total cost is to be determined. If appropriate, make the connection between skip counting and multiplication to help students decide when an authentic situation involves multiplication.

  Show examples of other one-step expressions that use addition, subtraction, or multiplication. Some examples are shown.

  9 + 15

  8 – 3

  2 × 11

  15 – 8

  4 × 7

- Use a variety of relevant authentic situations to connect the expressions to familiar situations. Use visual representations for support when appropriate.
- Ask students to identify a one-step expression that represents an authentic situation. For example, a student brought 3 chairs into a classroom. There were already 12 chairs in the classroom. Students can be given a variety of expressions to choose from, such as 12 – 3, 3 + 12, and 3 × 12. The correct expression in this example is 3 + 12.

❑ **Identify a two-step expression that represents an authentic situation.**

- Use an authentic situation to demonstrate a two-step expression. For example, a student made 24 cupcakes. She gave one cupcake to each of the 13 students in her class. Then she gave one cupcake to 5 different teachers. The expression that represents this situation is shown.

  $$24 - 13 - 5$$

  Show examples of other two-step expressions. Some examples are shown.

  $$33 - 22 + 2$$

  $$18 \times 2 - 5$$

  $$3 \times 5 + 8$$

  $$8 + 9 - 4$$

- Use a variety of relevant authentic situations to connect the expressions to familiar situations. Use picture representations for support when appropriate.
- Ask students to identify a two-step expression that represents an authentic situation. For example, an art teacher currently has 21 paintbrushes. She will buy 12 more paintbrushes for each of her 5 art classes. Ask students to choose the expression that represents this situation.

  $$21 \times 12 + 5$$

  $$21 + 12 \times 5$$

  $$21 - 12 + 5$$

  The correct expression for this situation is 21 + 12 × 5.

Prerequisite Extended Indicators

MAE 7.A.1.d—Identify equivalent expressions using the distributive property, limited to digits 1–9 (e.g., $2(3 + 4) = (2 \times 3) + (2 \times 4)$).

MAE 6.A.2.a—Match a simple word phrase with an input-output box.

MAE 6.A.1.a—Identify equivalent expressions with one variable by combining like terms, limited to digits 1–9 (e.g., $2n + 3n = 5n$).

MAE 5.A.1.d—Evaluate two-step numerical expressions involving addition or subtraction and multiplication using order of operations, limited to the digits 1–5 (e.g., $4 \times (5 - 2)$, $4 + 2 \times 3$).

Key Terms

addition, expression, one-step expression, multiplication, subtraction, two-step expression

Additional Resources or Links

https://im.kendallhunt.com/MS/students/2/6/23/index.html

https://illuminations.nctm.org/Search.aspx?view=search&st=a&gr=6-8&page=3

8.A.2.b

Determine and describe the rate of change for given situations through the use of tables and graphs.

Extended: Given a table, determine the rate of change of a proportional relationship.

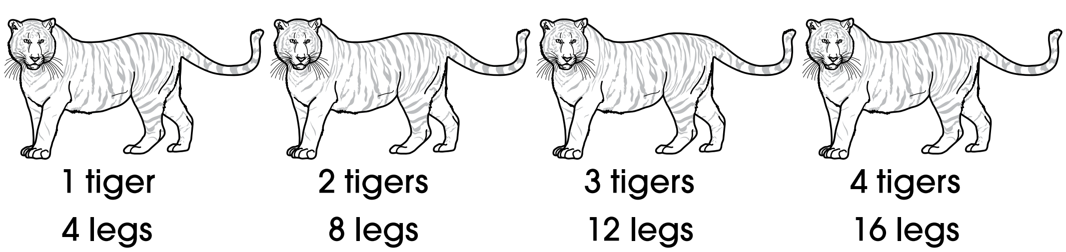
Scaffolding Activities for the Extended Indicator

❑ **Identify the rate of change of a proportional relationship given a table.**

- Use a table to show a proportional relationship. Explain that a proportional relationship always has the same rate of change and that proportional relationships are often represented in real-life scenarios (for example, price per pound of apples is a proportional relationship). Emphasize that the rate of change shown in the table doesn't change, because the relationship between the number of wings and the number of birds is a constant ratio of 2:1. Each bird has 2 wings, so the number of birds can always be multiplied by 2 to find the number of wings. The rate of change for this example is 2 wings per bird.

Number of Birds	Rate of Change (wings per bird)	Number of Wings
1	2	1 × 2 = 2
2	2	2 × 2 = 4
3	2	3 × 2 = 6
4	2	4 × 2 = 8

It may also be helpful to show an example that includes a model of the proportional relationship. Present the figure shown and explain that a tiger has 4 legs, so the ratio of legs to tigers is 4:1. Use multiplication or skip counting to find the rate of change.



The relationship of the tigers and their legs can then be put into a table. The rate of change for this example is 4 legs per tiger.

Number of Tigers	Rate of Change (legs per tiger)	Number of Legs
1	4	1 × 4 = 4
2	4	2 × 4 = 8
3	4	3 × 4 = 12
4	4	4 × 4 = 16

Continue to demonstrate proportional relationships using models and tables that include the rate of change to represent a variety of scenarios.

- Ask students to identify the rate of change of a proportional relationship given a table.

❑ **Determine the rate of change of a proportional relationship given a table.**

- Use a table to demonstrate how to find and describe a rate of change for a proportional relationship. Present the table shown, which represents the costs of packages of snacks.

Number of Packages	1	2	3	4
Total Cost ($)	5	10	15	20

Explain that the rate of change is the multiplication pattern from the number of packages to the cost. To find the cost in this example, each number of packages is multiplied by 5. The rate of change can be described as “5 dollars per package.” Demonstrate using an appropriate computation method to show that multiplying by 5 works for each pair of numbers in the table to determine the cost from the number of packages.

1 × ___ = 5

2 × ___ = 10

3 × ___ = 15

4 × ___ = 20

Continue to demonstrate identifying and describing the rate of change using a variety of tables that represent proportional relationships (e.g., minutes and pages read, hours and miles, cost and number of tickets).

8.A.2 Applications

- Ask students to determine the rate of change of a proportional relationship when given a table. For example, give students the following table, which shows how many bracelets have been made after certain numbers of days.

Making Bracelets

Number of Days	1	2	3	4
Number of Bracelets	10	20	30	40

Students should determine that the rate of change can be described as "10 bracelets per day."

Prerequisite Extended Indicators

MAE 6.R.1.a—Determine ratios from concrete models and drawings.

MAE 6.R.1.e—Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

MAE 7.R.1.a—Determine unit rate when given a table, limited to ratios of 1:2, 1:3, 1:5, and 1:10.

MAE 7.R.1.b—Given a proportional relationship that represents an authentic situation, determine the missing quantity.

Key Terms

proportional relationship, rate of change, ratio, table

Additional Resources or Links

https://www.engageny.org/resource/grade-7-mathematics-module-1-topic-lesson-2

https://www.map.mathshell.org/download.php?fileid=1610

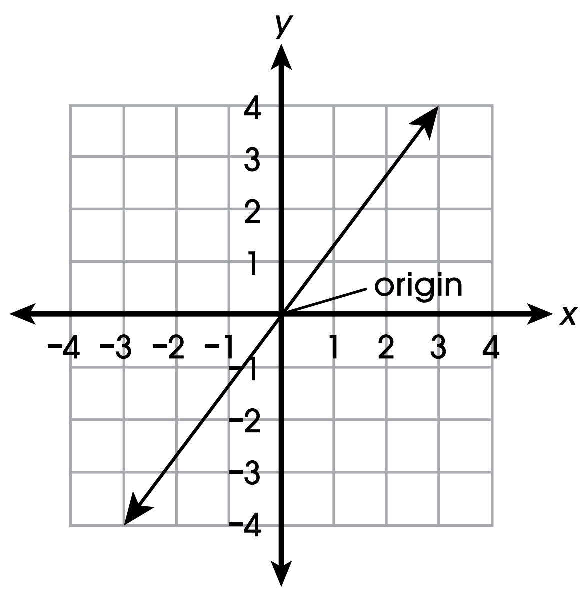
8.A.2.c

Graph proportional relationships and interpret the rate of change.

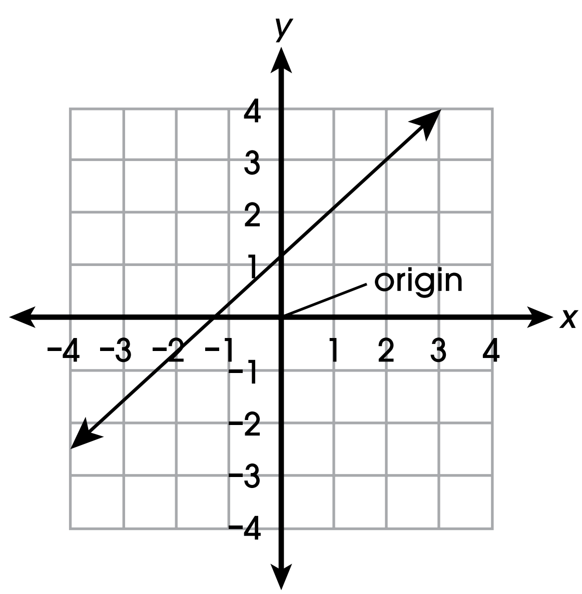
Extended: Given a graph of a line through the origin and a point on the line, determine another point on the line.

Scaffolding Activities for the Extended Indicator

- ❑ **Identify a graph of a line that goes through the origin.**
    - Use a coordinate graph to show that a line can go through the origin. Indicate that the origin is represented as the point (0, 0).



Demonstrate that lines do not always go through the origin by graphing other lines.

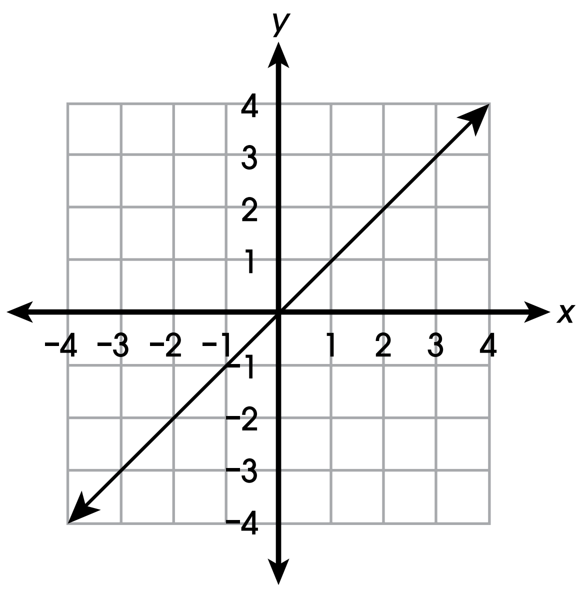


With each graph shown, be sure to emphasize to students when the line DOES and DOES NOT go through the origin. It may be helpful to show the origin in a different color to highlight its location.

- Ask students to identify graphs of lines that go through the origin.
- Ask students to create graphs of lines that go through the origin. This can be done using an online graphing tool, paper and a drawing, or a blank graph and manipulatives (like straws or wooden sticks) to represent the lines.

❑ **Identify points on a graphed line that goes through the origin.**

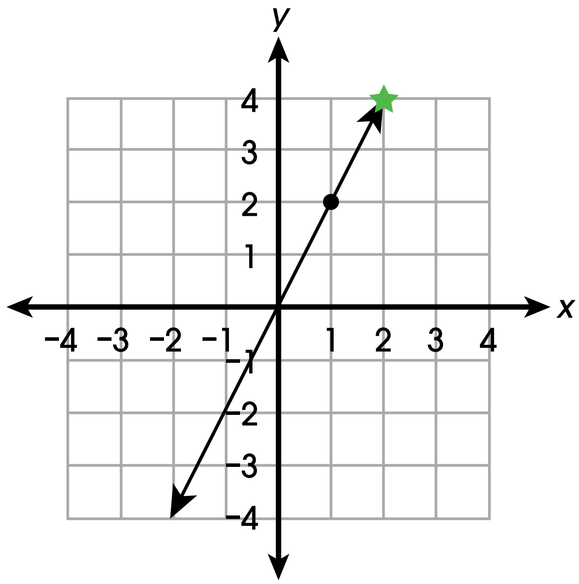
- Use the graph of a line that goes through the origin to show points on that line.



For example, the line shown here could have a point placed at (1, 1), (2, 2), or (3, 3), among other locations. Show a variety of graphs of lines that go through the origin and demonstrate locating points on the lines using ordered pairs, (*x*, *y*). Indicate that the *x*-axis is horizontal and has the label *x* next to it and that the *y*-axis is vertical and has the label *y* above it.

8.A.2 Applications

- Ask students to identify a point on a line that goes through the origin when given another point on that line. For example, present the following graph.



Give students three possible ordered pairs to choose from: (1, 1), (2, 1), and (2, 4). Students should determine that the star is located at (2, 4).

Prerequisite Extended Indicators

MAE 8.A.1.a—Identify the point of intersection (solution) for intersecting lines on a coordinate plane, limited to naming the point without determining the coordinate pair.

MAE 5.G.2.b—Identify the *x*- or *y*-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

Key Terms

coordinate plane, graph, horizontal, line, ordered pair, origin, vertical, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/100

https://www.engageny.org/resource/grade-7-mathematics-module-1-topic-b-lesson-10

Mathematics—Grade 8
Geometry

8.G.1 Attributes

8.G.1.a

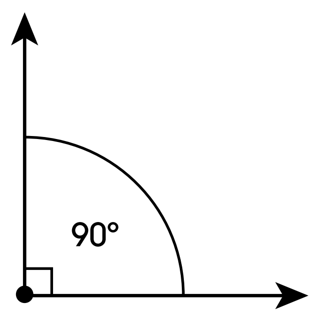
Determine and use the relationships of the interior angles of a triangle to solve for missing measures.

Extended: Identify the missing angle measure in 45-45-90 triangles and 30-60-90 triangles when given two of the angles and a drawing of the triangle.

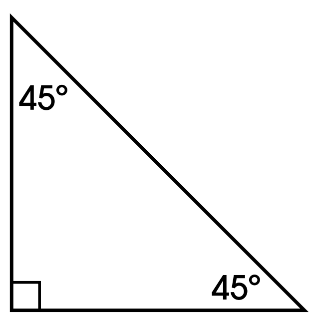
Scaffolding Activities for the Extended Indicator

❑ **Identify the missing angle measure in a 45-45-90 triangle.**

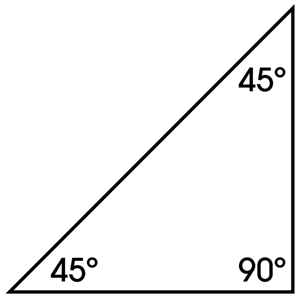
- Explain that all right triangles have one 90° angle and that a symbol is often used to mark that angle. Point to the symbol and reference the square corner of an index card or a piece of paper as another example of a right angle.



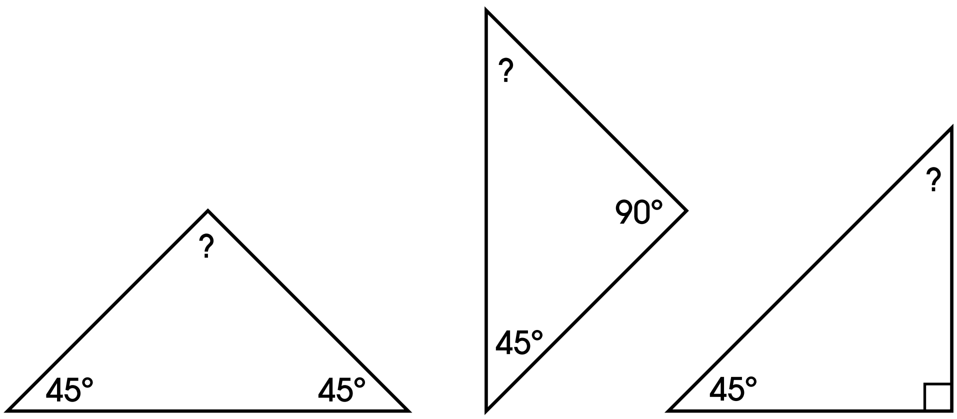
Use 45-45-90 triangles to show that the three angle measurements in a triangle total 180°. Demonstrate adding the three angle measurements together: 45 + 45 + 90 = 180.



Show 45-45-90 triangles in various positions, using both the symbol for the right angle and the label of 90°.



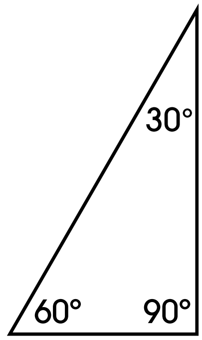
Model identifying a missing angle measure in a 45-45-90 triangle. Be sure to include triangles in a variety of positions. Some possible examples are shown.



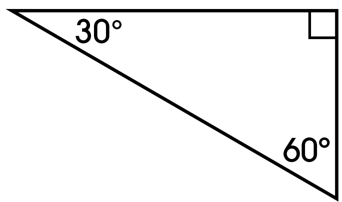
- Ask students to identify the missing angle measure in a 45-45-90 triangle.

- ❑ **Identify the missing angle measure in a 30-60-90 triangle.**

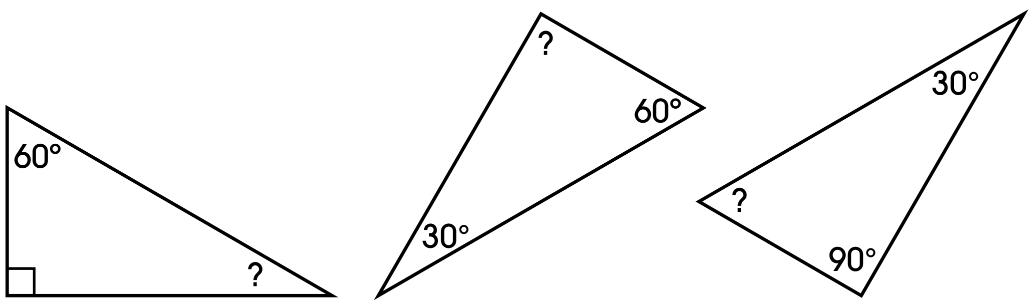
  - Use 30-60-90 triangles to show that the three angle measurements in a triangle total 180°. Demonstrate adding the three angle measurements together: 30 + 60 + 90 = 180.

    Show 30-60-90 triangles in various positions. Present triangles that have the symbol for the right angle and others that have the label of 90°.

    Model identifying a missing angle measure in a 30-60-90 triangle. Be sure to include triangles in a variety of positions. Some possible examples are shown.

  - Ask students to identify the missing angle measure in a 30-60-90 triangle.

Prerequisite Extended Indicators

MAE 7.G.1.a—Identify a pair of angles as complementary (equal to 90°) or supplementary (equal to 180°).

MAE 5.G.1.c—Classify triangles as acute, right, or obtuse.

Key Terms

30-60-90 triangle, 45-45-90 triangle, degree, right angle, triangle

Additional Resources or Links

https://www.engageny.org/resource/grade-8-mathematics-module-2-topic-c-lesson-13

https://im.kendallhunt.com/MS/teachers/3/1/15/preparation.html

8.G.1.b

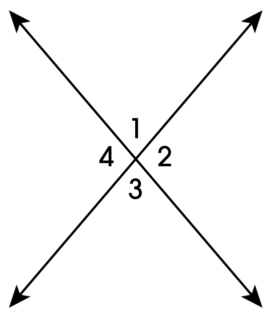
Identify and apply geometric properties of parallel lines cut by a transversal and the resulting corresponding same side interior, alternate interior, and alternate exterior angles to find missing measures.

Extended: Identify any pair of congruent angles in two intersecting lines or in two parallel lines cut by a transversal, limited to locating but not naming as vertical, corresponding, alternate interior, or alternate exterior.

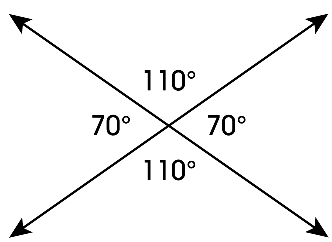
Scaffolding Activities for the Extended Indicator

❑ **Identify pairs of congruent angles created by intersecting lines.**

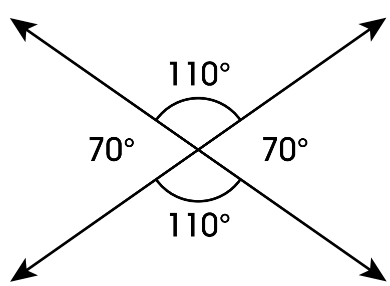
- Use two intersecting lines to show opposite angles. In the figure shown, the opposite angles are 1 and 3 and 2 and 4.



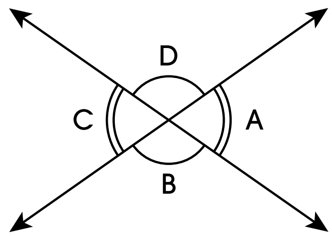
- Ask students to identify the opposite angles in a figure of two intersecting lines.
- Use two intersecting lines with angle measurements labeled to show that opposite angles are equal in size.



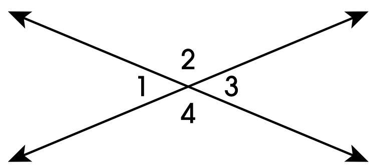
Equal angle measurements can also be called congruent, so the opposite angles in two intersecting lines are congruent. This is true for any two intersecting lines. Mark the congruent angles with symbols, as shown.



Repeat this process with a variety of intersecting lines, always showing the angle measurements and identifying the congruent, opposite angles. Then move on to intersecting lines without the angle measurements given and only a letter or number to label each angle.



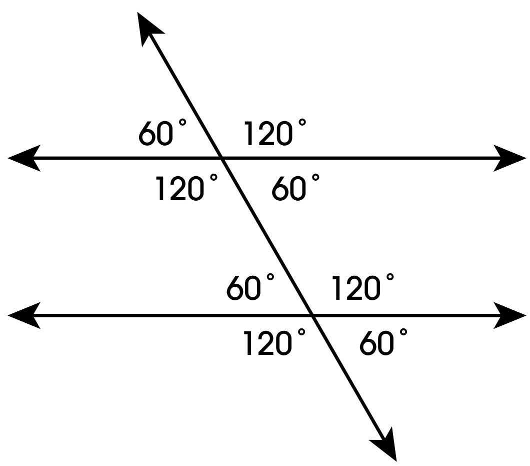
- Ask students to identify a pair of congruent angles in two intersecting lines. For example, ask students to find which angle is congruent to angle 1 in the figure shown.



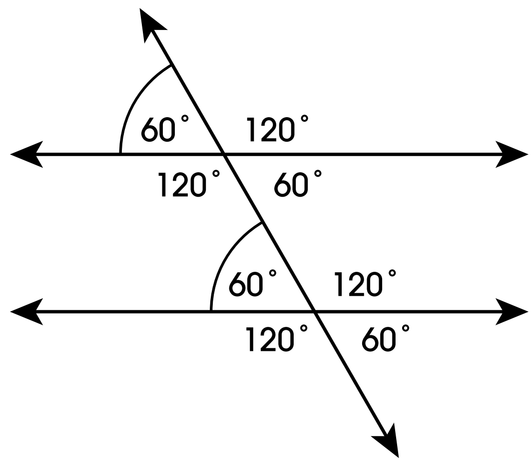
Students should determine that angle 3 is congruent to angle 1.

- [ ] **Identify pairs of congruent angles created by two parallel lines cut by a transversal.**

  - Use two parallel lines with a transversal with angle measurements labeled to show that the different types of angle pairs are equal in measure. Emphasize that the angles with the same measurements are congruent.

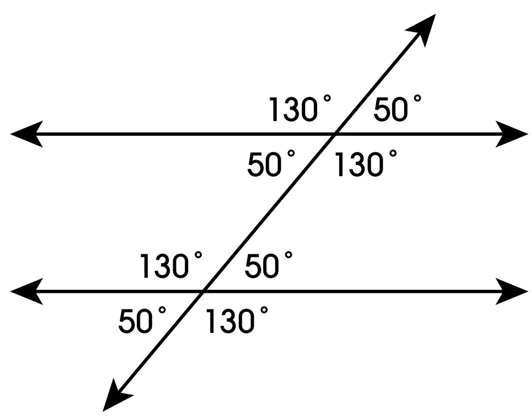


Explain to students that angles are considered corresponding angles if they are on the same corner at each intersection. Point out a pair of corresponding angles and show how their angle measures are equal. An example is shown.

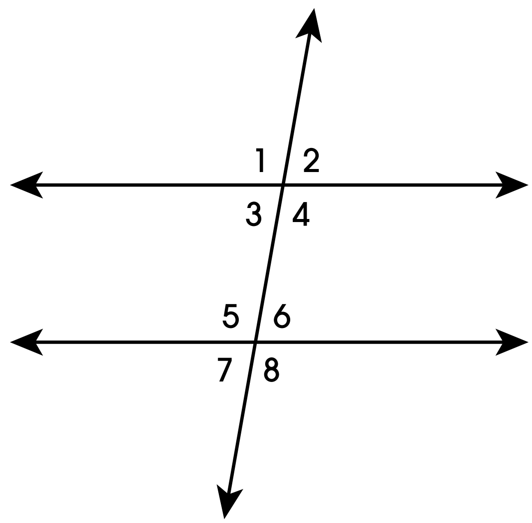


Repeat this process with a variety of parallel lines cut by a transversal, always showing the angle measurements and identifying congruent, corresponding angles. Then repeat this process for alternate interior and alternate exterior angles. It is not necessary to use these names of congruent angles. Once students demonstrate understanding, try this same exercise using figures without the angle measurements and only a letter or number to label each angle.

- Ask students to identify congruent angles created by two parallel lines cut by a transversal when angles are labeled with measurements.



- Use two parallel lines with a third line that intersects both parallel lines. In the figure shown, the pairs of opposite angles are 1 and 4; 2 and 3; 5 and 8; and 6 and 7. The angles in each pair of opposite angles are congruent because they have equal measurements.



Explain to students that there are several other types of pairs of congruent angles when parallel lines are cut (intersected) by a transversal (a line that intersects two or more lines). There are corresponding angles, alternate interior angles, and alternate exterior angles.

Use the three following figures to show the corresponding angle pairs, the alternate interior angle pairs, and the alternate exterior angle pairs.

In figure 1, the pairs of corresponding angles are 1 and 5; 2 and 6; 3 and 7; and 4 and 8. The angles in the congruent pair of 1 and 5 are marked with symbols, as shown.

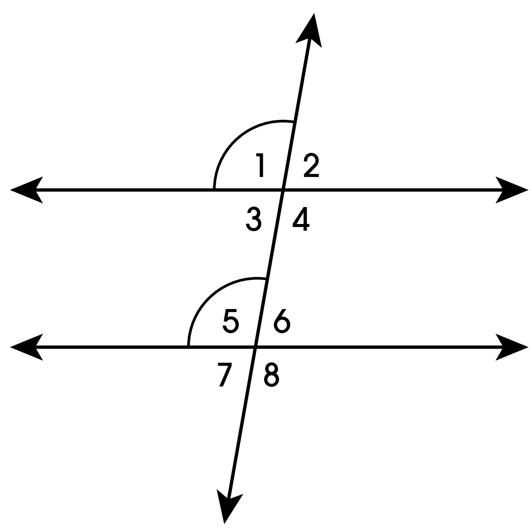


figure 1

Explain to students that alternate interior angles are formed on the opposite sides of the transversal. In figure 2, the pairs of alternate interior angles are 3 and 6 and 4 and 5. The congruent pairs of angles 3 and 6 and angles 4 and 5 are marked with symbols, as shown.

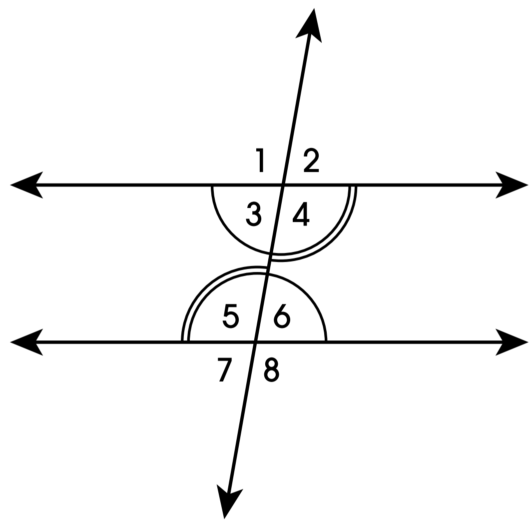


figure 2

Explain to students that alternate exterior angles are formed on the opposite sides of the transversal and are outside of the parallel lines. In figure 3, the pairs of alternate exterior angles are 1 and 8 and 2 and 7. The congruent pairs of angles 1 and 8 and angles 2 and 7 are marked with symbols, as shown.

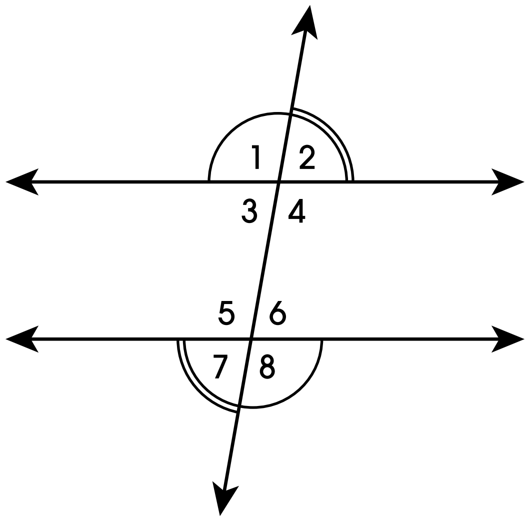
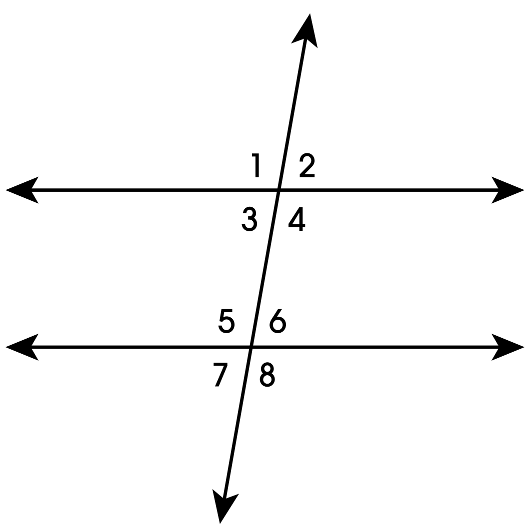


figure 3

- Ask students to identify pairs of congruent angles in two parallel lines cut by a transversal.



Prerequisite Extended Indicators

MAE 7.G.1.a—Identify a pair of angles as complementary (equal to 90°) or supplementary (equal to 180°).

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

MAE 4.G.2.d—Identify benchmark angles of 90° and 180°, and relate those angle measurements to right angles, straight lines, and perpendicular lines.

Key Terms

alternate interior angles, alternate exterior angles, angle, congruent, corresponding angles, intersect, line, opposite, pair, parallel, ray, transversal

Additional Resources or Links

https://illuminations.nctm.org/Search.aspx?view=search&kw=transversal&st=g

https://tasks.illustrativemathematics.org/content-standards/8/G/A/5/tasks/1503

8.G.2 Coordinate Geometry

8.G.2.a

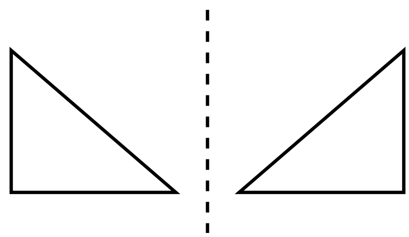
Perform and describe positions and orientations of shapes under single transformations including rotations in multiples of 90 degrees about the origin, translations, reflections, and dilations on and off the coordinate plane.

Extended: Identify the image of a shape or letter following a reflection.

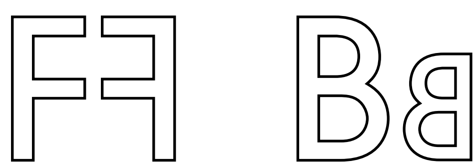
Scaffolding Activities for the Extended Indicator

❑ **Demonstrate that the reflection of a shape or letter is a new shape that is the same size and same shape as the original shape or letter.**

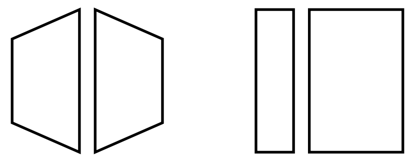
- Explain that a reflection is created when a shape is flipped over an imaginary line. The triangle shows a reflection because a triangle has been flipped over the given line.



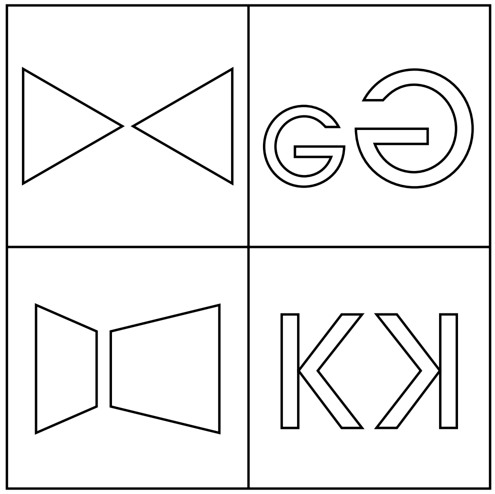
- Use die-cut letters and geometric shapes to demonstrate reflections over a line that is drawn on paper or made with tape or string. Explain that when a shape is reflected it stays the same size. The letter *F* shows a reflection because the letter has been flipped and both letters are the same size. The letter *B* does not show a reflection because the second *B* is smaller.



- Explain that when a shape is reflected it remains the same shape. The trapezoids show a reflection because both trapezoids are the same shape. The rectangles do not show a reflection because the shapes of the rectangles are different.



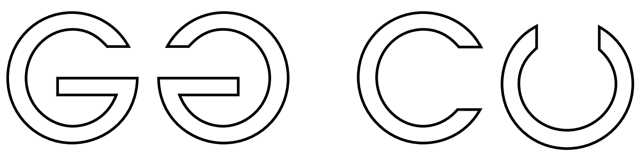
- Model identifying whether a pair of shapes or letters is a reflection or is not a reflection.



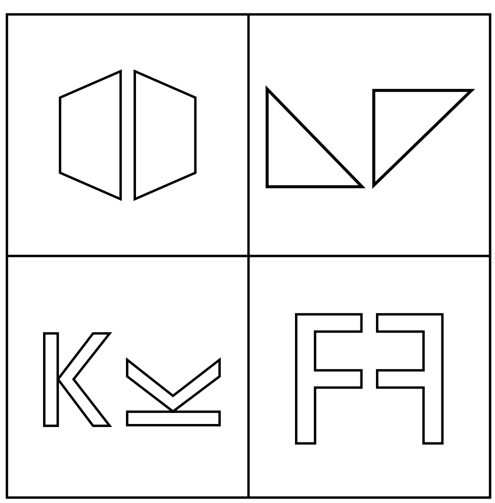
- Ask students to identify whether a pair of shapes or letters is a reflection or is not a reflection.

❑ **Identify the image of a shape or a letter following a reflection.**

- Explain that when a shape or a letter is reflected, the shape and the size stay the same; the only thing that changes is that the shape has been flipped over an imaginary line. The letter *G* shown below is an example of a reflection because the letter has been flipped over and remains the same size and shape. The letter *C* is **not** an example of a reflection because the *C* has been turned instead of flipped. Use die-cut letters and geometric shapes to demonstrate the difference between a flip and a turn.



- Model identifying whether a pair of shapes or letters is a reflection or is not a reflection.



- Ask students to identify whether a picture of a pair of shapes or letters shows a reflection.

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

image, flip, reflection, shape, size, turn

Additional Resources or Links

https://www.engageny.org/resource/grade-8-mathematics-module-2-topic-lesson-4

http://nlvm.usu.edu/en/nav/frames_asid_294_g_2_t_3.html?open=activities&from=category_g_2_t_3.html

(Note: Java required for website. Most recent version recommended, but not needed.)

8.G.2.b

Determine if two-dimensional figures are congruent or similar.

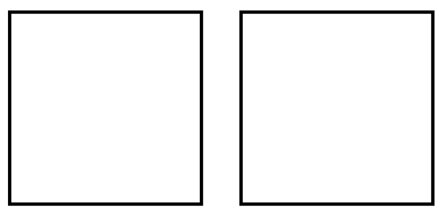
Extended: Determine if a pair of two-dimensional figures is congruent, non-congruent, similar, or non-similar.

Scaffolding Activities for the Extended Indicator

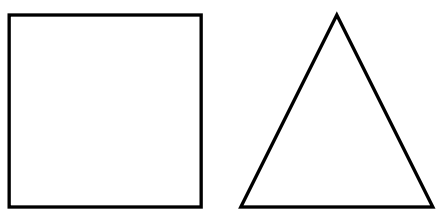
❑ **Identify congruent shapes.**

- Use two different shapes (e.g., circles, triangles, squares, rectangles) of various sizes to describe attributes of the shapes. Since squares and rectangles have similar attributes, intentionally choose to compare a rectangle and a square after comparing other shapes. Explain that a shape is determined by the number of angles, the number of sides, the length of the sides, and whether the sides are straight or curved.

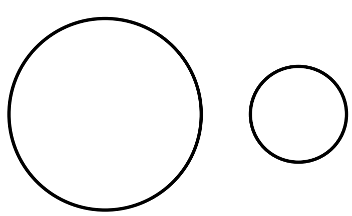
  Define the term "congruent" as meaning the exact same size and the exact same shape. Present two squares as shown. The squares are both the same shape and the same size. Therefore, the squares are congruent.



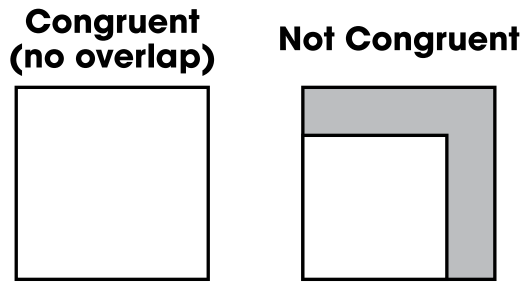
  Present a square and a triangle as shown. Identify the attributes of the two shapes. One shape has four sides, while the other shape has only three sides. While the lengths of the sides appear to be the same, the shapes are not the same. Therefore, the shapes are not congruent, or non-congruent.



  Present two circles as shown. The circles are both the same shape, but the circles are not the same size. Therefore, the circles are not congruent.



Show additional pairs of shapes, explicitly demonstrating how to describe each shape's attributes. Cutouts or other manipulatives may be placed or stacked on top of each other to determine whether the two shapes are the same size and shape.

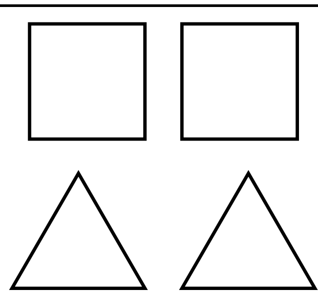
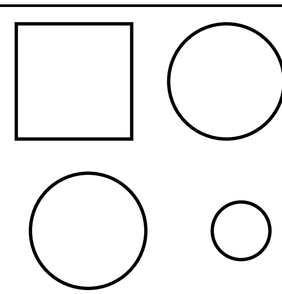


- Ask students to select a shape that is congruent to a given shape. For example, present one object that models a two-dimensional shape (e.g., pattern block, cutout) and a choice of three other objects that are the same shape but different sizes. The students should select the object that models a congruent shape.
- Ask students to select a shape that is congruent to a given shape when presented a choice of several other objects that model two-dimensional shapes (e.g., pattern blocks, cutouts) that are different shapes and different sizes.

❑ **Distinguish between pairs of congruent and non-congruent shapes.**

- Present two congruent shapes (e.g., two circles, two triangles, two squares, two rectangles) and describe the attributes of the two shapes that make them congruent (e.g., same shape, same size, identical). Introduce additional non-congruent examples and point out how some of the shapes are not the same shape, some of the shapes are the same shape but not the same size, and some of the shapes are the same size but not the same shape.

  Demonstrate how to determine whether two shapes are congruent by checking to see if they are the same shape and the same size.

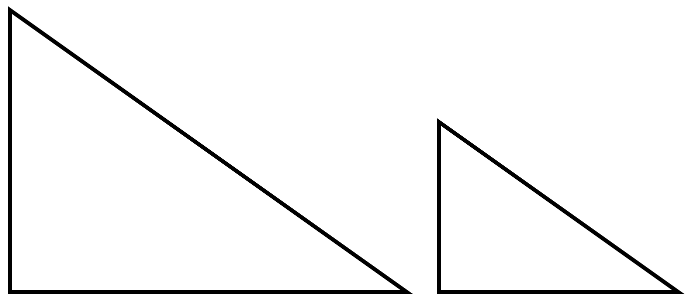
Congruent pairs Same shape and same size	Non-congruent pairs Different shape or different size
	

- Present two objects that model two-dimensional shapes (e.g., pattern blocks, cutouts) and ask students to determine whether the two shapes are congruent or non-congruent.
- Present a collection of many two-dimensional shapes (e.g., pattern blocks, cutouts) and ask students to create pairs of congruent shapes and pairs of non-congruent shapes.

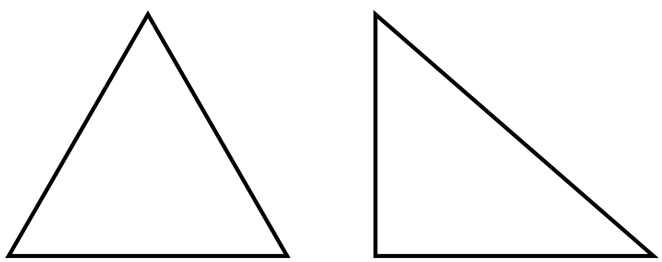
8.G.2 Coordinate Geometry

❑ **Identify similar shapes.**

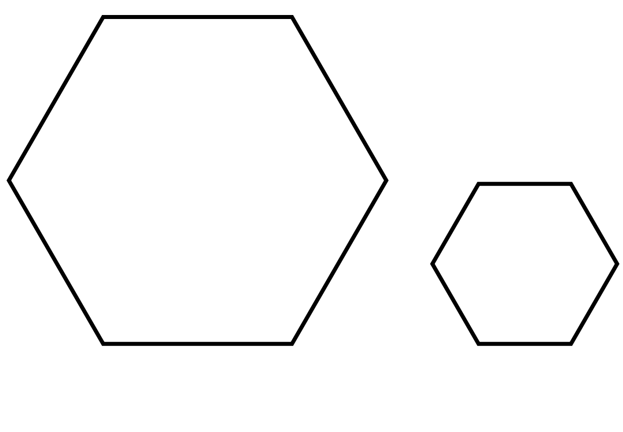
- Use sets of two shapes to show similarity and non-similarity. Define the mathematical term "similar" as shapes that are the same, have corresponding angles that are equal, but may be different sizes. For similar shapes, the corresponding lengths are in the same ratio. Present a pair of triangles as shown. The triangles are different sizes but are the same shape and have corresponding angles that are equal. Therefore, the triangles are similar.

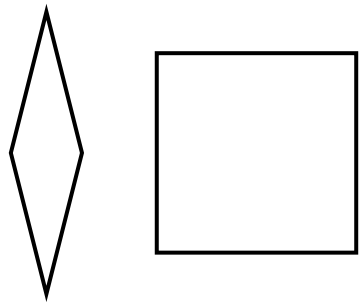
  Next, present non-examples by first showing two shapes from different categories (e.g., a circle and a square) and indicating that the circle and the square are not similar, or non-similar. Progress to presenting the pair of triangles shown. Explain that the two triangles are not identical shapes because one triangle has a right angle and the other triangle does not. Therefore, the triangles are non-similar.

- Continue to demonstrate with a variety of shapes of different sizes and discuss the attributes of each that make two shapes similar or non-similar. For example, present two hexagons as shown. Explain to students that the larger hexagon is an enlargement of the smaller hexagon, defining them as similar shapes.

Present two quadrilaterals as shown.



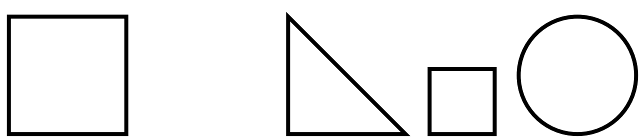
Both shapes have four sides, but the shapes look different because only one shape has right angles. The quadrilaterals shown are not similar. Be sure to include examples of contrasting a square and a rectangle to identify a square and a rectangle as shapes that are not similar.

- Ask students to determine whether two shapes with a different number of sides are similar or non-similar.
- Ask students to determine whether two shapes with the same number of sides are similar or non-similar.

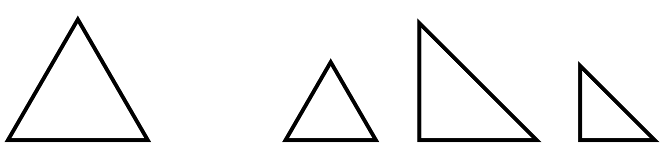
❑ **Distinguish between pairs of similar and non-similar two-dimensional shapes.**

- Use two-dimensional shapes such as circles, squares, similar rectangles, similar right triangles, or equilateral triangles to find similar shapes. Model how the attributes of a shape may be determined based on the number of sides and the number of angles. Then, describe how the shapes may be categorized based on the sizes of the angles and the lengths of the sides. Begin with more obvious examples to demonstrate that shapes that are smaller or larger copies of each other can be similar.

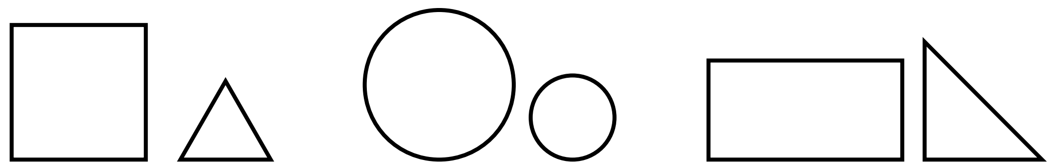
  Present one shape as a model and three other shapes to demonstrate how to find the similar shape in a group. For example, present the four shapes shown. Note that the given model is a square and has four sides, so the similar shape must also be a square that has four sides.



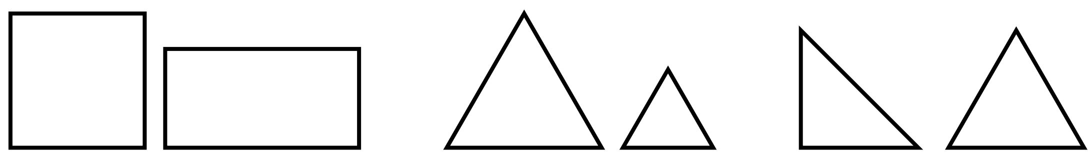
  Present one shape as a model and three other shapes with the same number of sides for comparison as shown. In this case, note that the three other shapes have the same number of sides as the given model. However, the angles in the given model are all smaller than a right angle, so the similar shape must also be a triangle whose three angles are less than 90 degrees (no right angles).



- Present three pairs of shapes, of which only one pair is similar, and determine the similar pair. Describe characteristics of each pair to determine similarity, and ask the question, “Do the shapes have the same number of sides?”



- Present three pairs of shapes in which each shape in the pair has the same number of sides and determine which pair is similar. Ask a series of questions to determine which pair of shapes is similar. “Do the shapes have the same number of sides?” “Are the angles the same size?” “Do the lengths of the sides make the shapes look like a smaller or larger copy of each other?”



- Ask students to select a similar shape when given one shape and a choice of three other shapes, of which only one of the choices is from the same shape category.
- Ask students to select a pair of similar shapes when given three pairs of shapes in which each shape in the pair has the same number of sides.
- Ask students to select the two shapes that make a pair of similar shapes when given three or more shapes.

Prerequisite Extended Indicators

MAE 5.G.1.b—Identify the difference between two-dimensional (flat) and three-dimensional (solid) figures.

MAE 4.G.1.c—Classify quadrilaterals based on the presence or absence of parallel and perpendicular lines and the presence or absence of right angles.

MAE 4.G.1.d—Identify lines of symmetry in two-dimensional shapes.

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

congruent, identical, same, shape, size

Additional Resources or Links

https://tasks.illustrativemathematics.org/content-standards/tasks/1935

https://apps.mathlearningcenter.org/geoboard/

8.G.3 Measurement

8.G.3.c

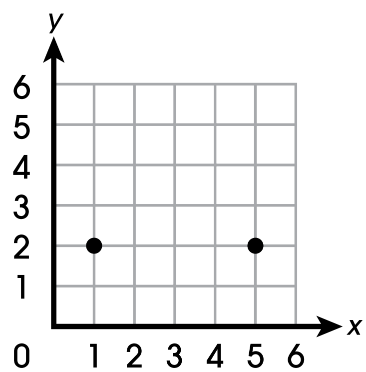
Find the distance between any two points on the coordinate plane using the Pythagorean Theorem.

Extended: Find the distance between two points on horizontal and vertical lines on a coordinate graph, limited to the first quadrant.

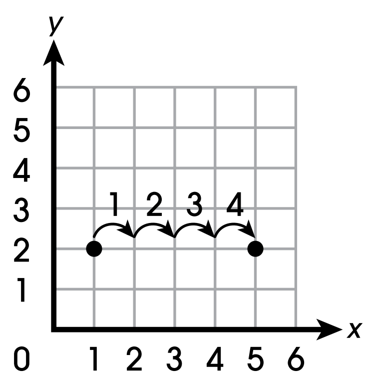
Scaffolding Activities for the Extended Indicator

❑ **Find the distance between two points on a coordinate graph.**

- Use a coordinate graph to demonstrate how to count the distance, in units, between points.

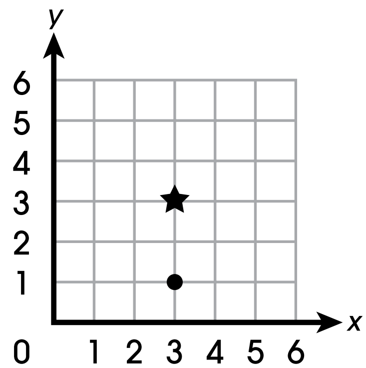


Since the two points shown have a *y*-coordinate of 2, the distance between the points can be found by finding the difference of the numbers on the *x*-axis. The difference of 5 and 1 is 4, so the distance between the two points is 4. Or, demonstrate counting the number of grid lines to get from one point to the other.



The same process can be followed to determine the distance between two points that have the same *x*-coordinate. Show several pairs of points on a coordinate graph and demonstrate finding the distance between the points. Always use points with either an *x*-coordinate or a *y*-coordinate in common.

- Ask students to find the distance between two points on a coordinate graph. For example, present the graph shown.



Ask students to determine the distance from the star to the point. Students should determine the distance is 2 units.

Prerequisite Extended Indicators

MAE 7.G.2.a—Given a triangle in quadrant 1 with one vertex on the origin, identify the location of one of the other vertices.

MAE 5.G.2.a—Identify the origin, x-axis, and y-axis of a coordinate plane.

MAE 5.G.2.b—Identify the x- or y-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.c—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

coordinate graph, coordinate plane, distance, point, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://apps.mathlearningcenter.org/geoboard/

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-c-lesson-19

8.G.3.d

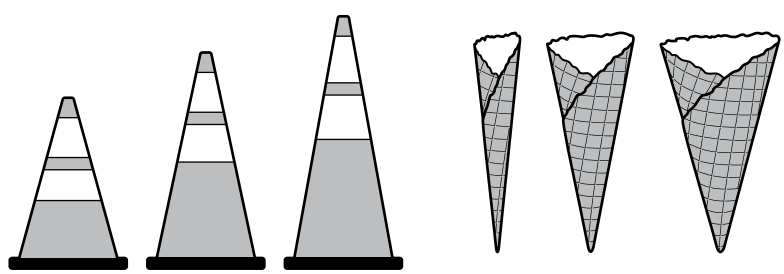
Determine the volume of cones, cylinders, and spheres and solve authentic problems using volumes.

Extended: Identify the cone, cylinder, and sphere with the greatest volume when given three cone-shaped containers with either the same base or the same height, three cylinder-shaped containers with either the same base or the same height, or three spheres.

Scaffolding Activities for the Extended Indicator

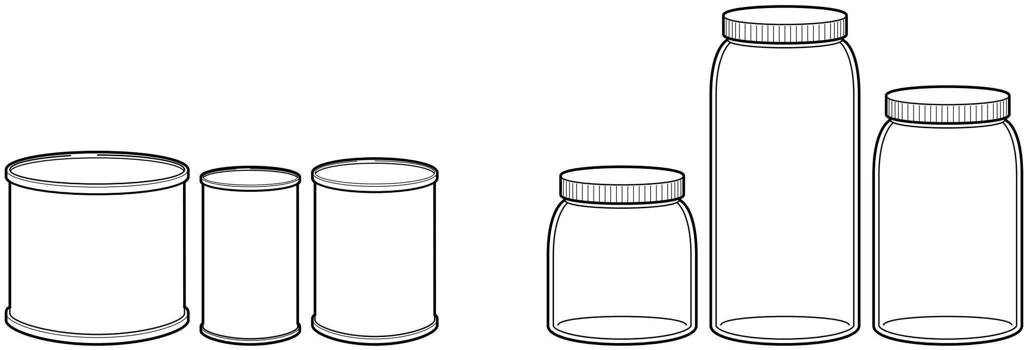
❑ **Identify the cone with the greatest volume when given three cones with either the same base or the same height.**

- Explain that volume is the amount of space inside an object, which can also be represented as the number of unit cubes that fit inside an object. Explain that when one object is larger than another object with the same shape, the larger object holds more unit cubes. Therefore, the larger object has a greater volume. For example, a traffic cone and an ice-cream cone are both cones that can be made in different sizes. The larger the cone, the greater the volume.

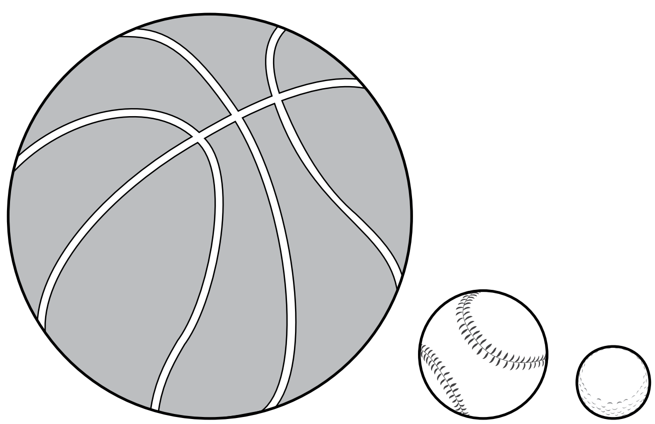


- Ask students to identify the cone with the greater volume when given two cones with one cone having a greater height and a greater base.
- Ask students to identify the cone with the greatest volume when given three cones with the same base but different heights.
- Ask students to identify the cone with the greatest volume when given three cones with the same height but different bases.

- ☐ **Identify the cylinder with the greatest volume when given three cylinders with either the same base or the same height.**
  - Present real-world examples of cylinders in different sizes and shapes (e.g., canned goods, jars). Compare the sizes of the bases and the heights of different-size cans or jars. When cylinders have the same base and different heights, the taller the cylinder, the greater the volume. When cylinders have different bases but the same height, the larger the base of the cylinder, the greater the volume.



  - Ask students to identify the cylinder with the greater volume when given two cylinders, one of which has a greater height and a greater base.
  - Ask students to identify the cylinder with the greatest volume when given three cylinders with the same base but different heights.
  - Ask students to identify the cylinder with the greatest volume when given three cylinders with the same height but different bases.
- ☐ **Identify the sphere with the greatest volume when given three different-size spheres.**
  - Present real-world examples of spheres of different sizes (e.g., basketball, baseball, golf ball). The bigger the sphere, the greater the volume.



  - Ask students to identify the sphere with the greatest volume when given three spheres of different sizes.

Prerequisite Extended Indicator

MAE 6.G.1.a—Use two-dimensional representations (e.g., drawings, nets) and/or three-dimensional models to identify cubes, cylinders, cones, rectangular prisms, pyramids, and spheres.

MAE 5.G.4.d—Find the volume of a cube or another rectangular prism with whole-number side lengths by counting unit cubes and showing that repeated addition is the same as multiplying the side lengths (e.g., 9 + 9 + 9 = 27 unit cubes in a 3 × 3 × 3 cube).

Key Terms

base, cone, cylinder, greater, height, sphere, volume

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html?from=search.html?qt=ffeet

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/kindergarten-mathematics-module-3-topic-d-lesson-13

Mathematics—Grade 8
Data

8.D.2 Analyze Data and Interpret Results

8.D.2.c

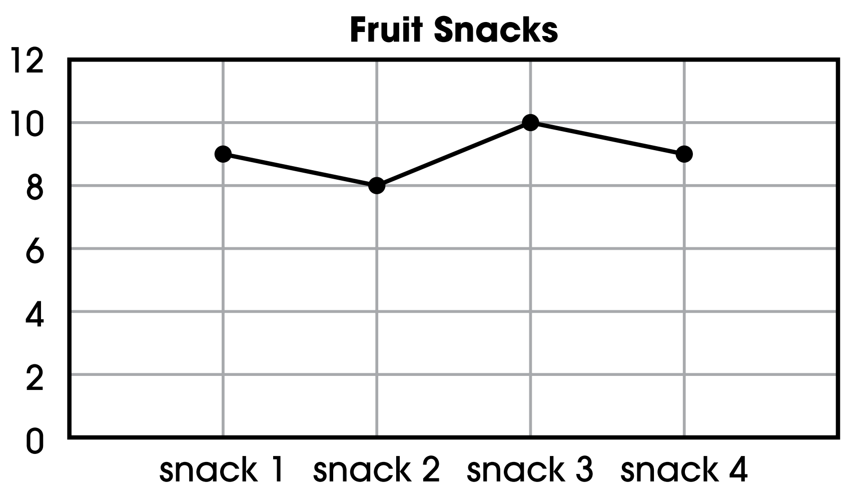
Draw an informal line of best fit based on the closeness of the data points to the line.

Extended: Determine a line of best fit based on the closeness of data points to the line.

Scaffolding Activities for the Extended Indicator

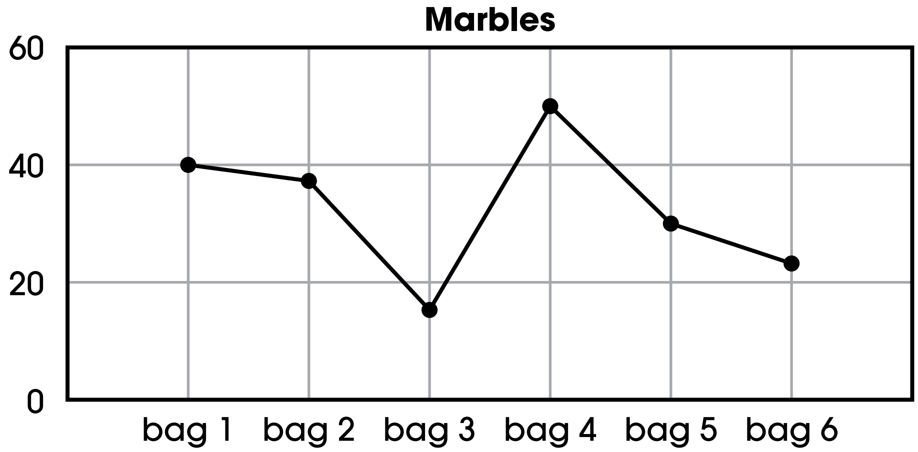
❑ **Determine whether or not data points on a line graph show a pattern or clear trend.**

- Present four fruit snack bags in which all four bags have a similar number of pieces. Determine the number of fruit snacks in each bag and identify those amounts on the line graph. Continue to demonstrate that since each of the bags of fruit snacks have similar amounts, this establishes a horizontal trend that means other bags of fruit snacks will have similar amounts of snacks.

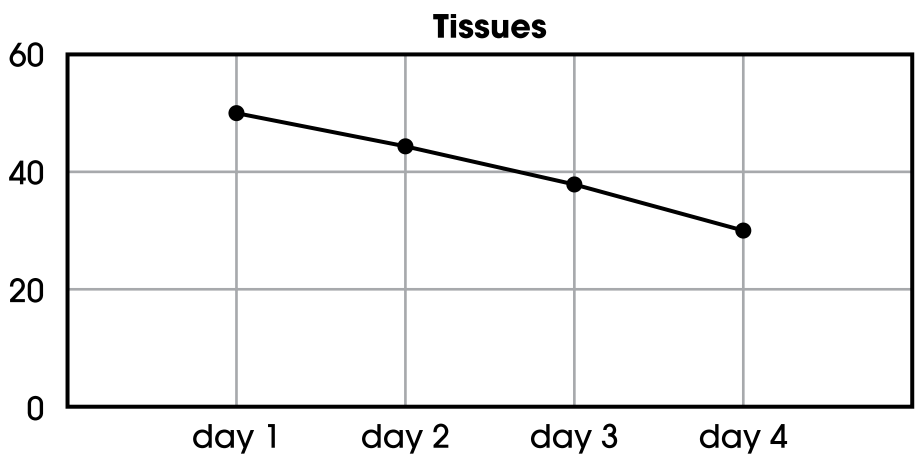


8.D.2 Analyze Data and Interpret Results

- Present four bags of marbles that have varying amounts within them. Determine the number of marbles in each bag and identify those amounts on the line graph. Demonstrate that since each of the bags of marbles have differing amounts, there is no possibility of identifying a clear pattern or trend with these data.



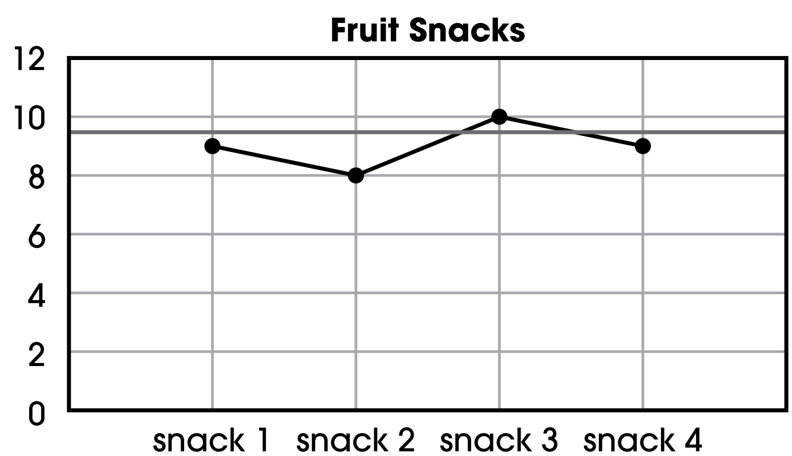
- Present a box of tissues that is used each day in the classroom. Display a graph with the starting number of tissues and the number of tissues that remain after four days. Ask students to identify whether or not the data points on the line graph show a pattern or a clear trend. Make the connection that there is a pattern of decreasing numbers of tissues based on people using them each day.



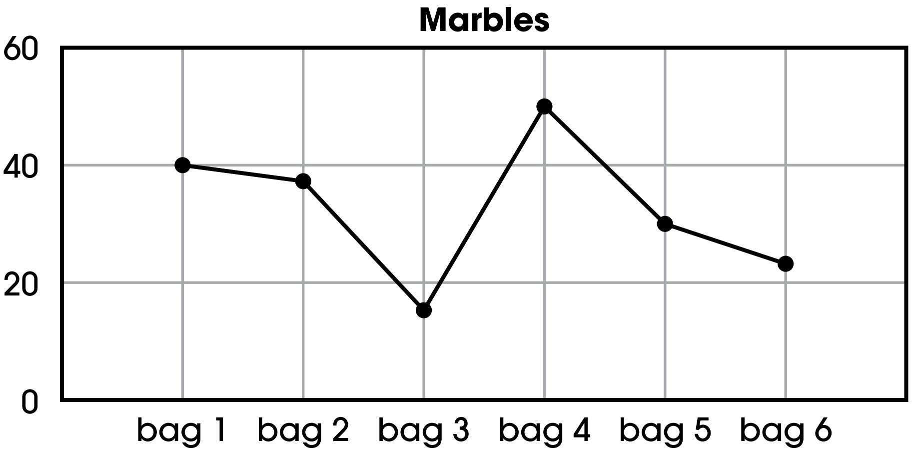
- Present a graph showing how many shoes are worn by people, beginning with one person and increasing to five people. Ask students to identify whether or not the data points on the line graph show a pattern or a clear trend.

❑ **Determine a line of best fit based on the closeness of data points to the line.**

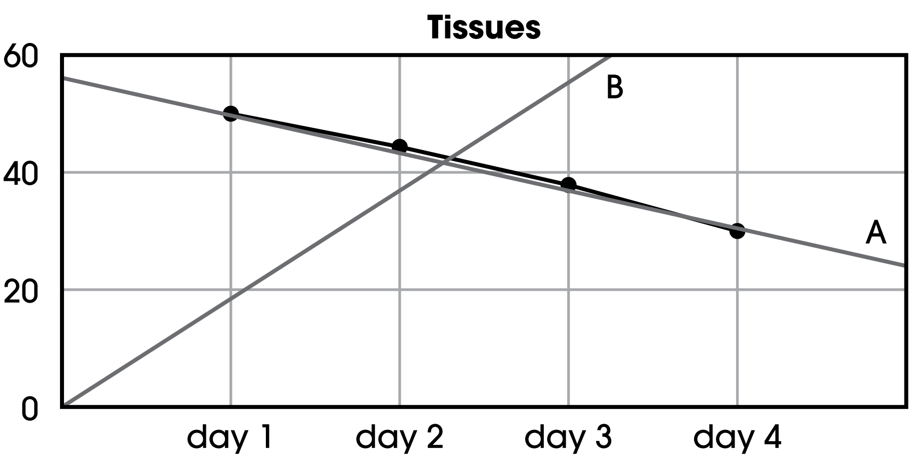
- Describe the line of best fit as a straight line that best represents the data and has the same number of points above the line as the amount below the line. It also follows the direction that the data are following.
- Present a line graph with data about the number of fruit snacks in different snack bags. Determine that each bag contains a similar number of fruit snacks and there is a similar trend between the bags. Identify the line of best fit by explaining this is the best fit because of the closeness of data points to the line. Demonstrate that the line of best fit is most accurate when the points are close to the line. Demonstrate that the line of best fit is horizontal due to the similarity in the number of fruit snacks in each individual snack bag.



- Present a line graph with data about six randomly filled bags of marbles. Determine that the number of marbles in each bag varies greatly and that there is not a consistent trend or pattern between the numbers. Demonstrate that since the data points are all over the graph, there is no line of best fit to determine how many marbles would be in the bags shown in the graph. Explain to students that there would not be a line of best fit to accurately fit the data since the all the data points would not be close to it.



- Present a line graph with data about the number of tissues within a box of tissues at the beginning of each day. Conclude that the number of tissues in the box decreases each day and there is a pattern following the number of tissues that are used each day. Ask students to identify what part of the graph is showing the line of best fit (line A or line B). Make the connection that line A is showing the line of best fit because it is following the pattern of the tissues decreasing each day and because the data points are very close to the line.



8.D.2 Analyze Data and Interpret Results

- Ask students to draw in a line of best fit for the data given when presented with a line graph.

Prerequisite Extended Indicators

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.D.1.a—Identify and compare quantities in line plots, limited to two data points.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Key Terms

data, horizontal, line of best fit, pattern, similar, trend line, vertical

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/scatter%20diagram.pdf

https://www.insidemathematics.org/inside-problem-solving/through-the-grapevine

https://im.kendallhunt.com/MS/teachers/3/6/4/preparation.html

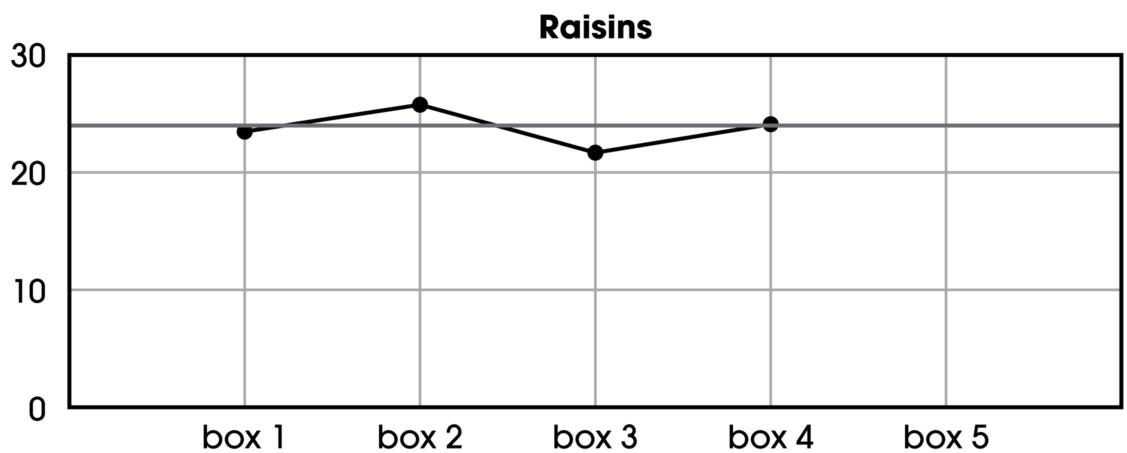
8.D.2.d

Use a linear model to make predictions and interpret the rate of change and y-intercept in context.

Extended: Use a line of best fit to make a prediction.

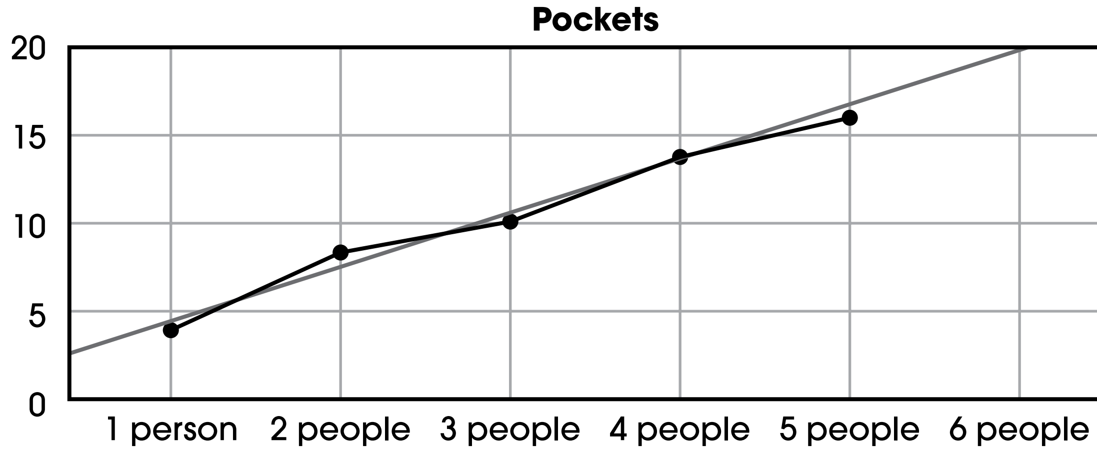
Scaffolding Activities for the Extended Indicator

- ❑ **Use a line of best fit to place a data point on a graph that follows the trend.**
  - Describe the line of best fit as a straight line that best represents the data and follows which direction the data are following. Explain that the line of best fit will be close to all the data points if it is a good fit.
  - Present a line graph with data about the number of raisins in a snack box. Determine the snack boxes each contain a similar number of raisins and that there is a horizontal line of best fit. Make a connection that a new data point could be added between the similar amounts of the other boxes of raisins. Determine that another data point could go right above or right below the line of best fit around twenty-three to twenty-six raisins. Draw in the new data point.

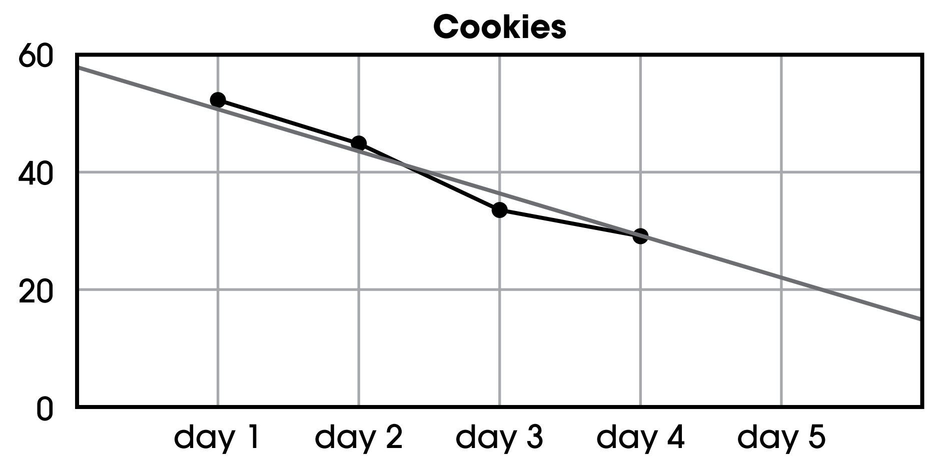


8.D.2 Analyze Data and Interpret Results

- Present a line graph with data about the number of pockets that people in a room have on their pants. Determine that the number of pockets will increase as more people enter the room. Ask students to identify a reasonable placement of the next data point when a sixth person enters the room. Make the connection that the next data point would fall right above, right below, or on the line of best fit because the pattern is increasing.



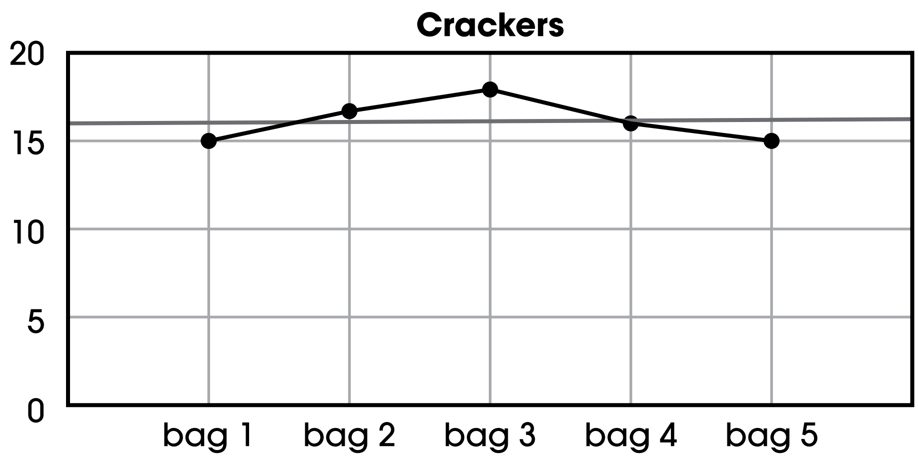
- Ask students to identify a reasonable placement of the next data point on a graph that follows a trend. For example, present a line graph with data about the number of cookies that are left after a teacher hands out snacks. Determine that the number of cookies would decrease as the cookies are handed out to classmates each day. Ask students to identify a reasonable placement of the next data point after cookies are handed out for snacks on day five.



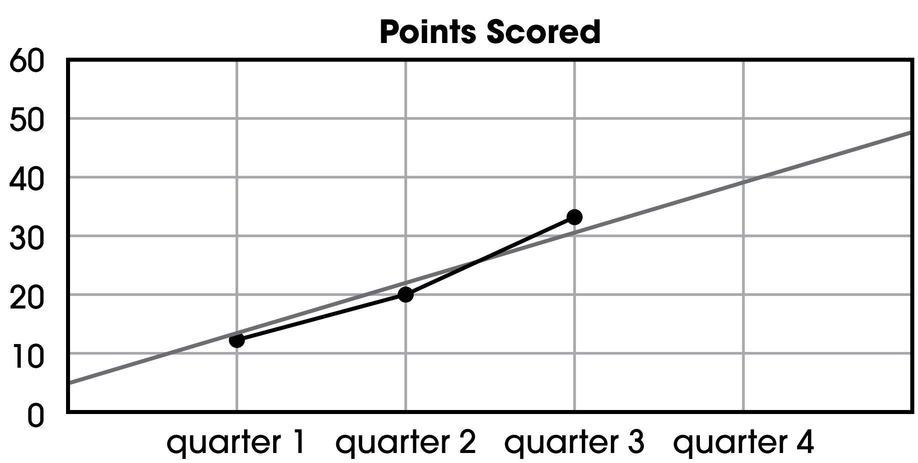
- ❑ **Use a line of best fit to make a prediction.**

  - Describe a line of best fit as a straight line that best represents the data and follows which direction the data are following. Describe a prediction as something that a person thinks might happen. The line of best fit will help provide a guide to realistic predictions for what will come next in a data set.

  - Present a line graph with data about the number of crackers in a snack bag. Determine that the snack bags each contain a similar number of crackers and that there is a horizontal line of best fit. Make a prediction that the number of crackers in the next snack bag will have a similar number of crackers. Make a prediction that there will be about sixteen crackers in the bag.

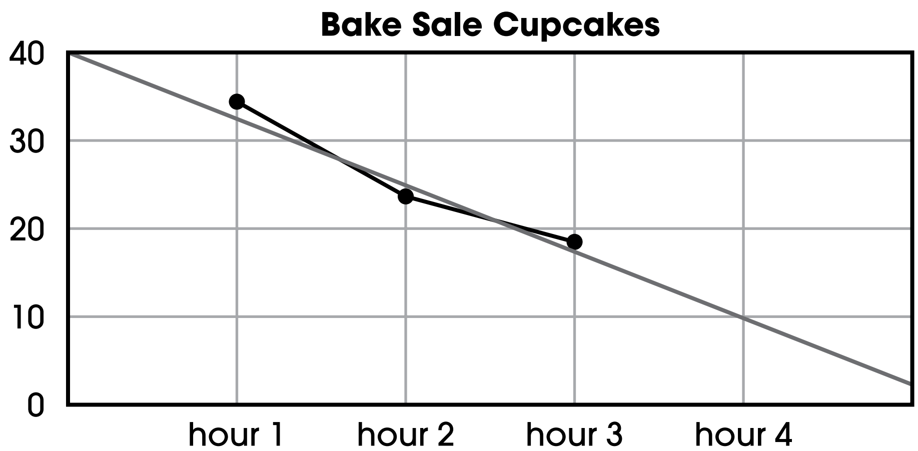


  - Present a line graph with data about points scored during a basketball game. Ask students to use the line of best fit to make a prediction about the points scored in a basketball game. Make the prediction that a higher number than the most recent data point in quarter three would be a reasonable prediction because the scores are increasing on the graph and in a game. Determine that the number of points in quarter four could be between thirty-five to fifty.



8.D.2 Analyze Data and Interpret Results

- Ask students to use the line of best fit to make a prediction. For example, present a line graph with data about the number of cupcakes that are left after cupcakes have been sold at a bake sale and then ask students to use the line of best fit to make a prediction about how many cupcakes will still be left at the end of the bake sale.



Prerequisite Extended Indicators

MAE 8.D.2.c—Determine a line of best fit based on the closeness of data points to the line.

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

MAE 4.D.1.a—Identify and compare quantities in line plots, limited to two data points.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Key Terms

data, decrease, horizontal, increase, line of best fit, pattern, prediction, similar, trend line, vertical

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/assets/inside-problem-solving/inside_problem_solving_through_the_grapevine_leveld_student_2021.pdf

https://im.kendallhunt.com/MS/teachers/3/6/5/index.html

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 8