NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 7

for Students with the Most Significant Cognitive Disabilities who take the Statewide Mathematics Alternate Assessment



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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is <u>not</u> the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and <u>American Association of Intellectual and Developmental Disabilities, 2013</u>). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (<u>American Association of Intellectual and Developmental Disabilities, 2013</u>). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider <u>all</u> of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is <u>significantly</u> below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "*IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities*" and "*Alternate Assessment Criteria/Checklist*" provide additional information on selecting appropriate statewide assessments for students with disabilities. <u>School Age Statewide Assessment Tests for Students with Disabilities</u>—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 7 Number

7.N.2 Operations

7.N.2.a

Add, subtract, multiply, and divide rational numbers (e.g., positive and negative fractions, decimals, and integers).

Extended: Add and subtract fractions and mixed numbers with like denominators up to 10 without regrouping.

Scaffolding Activities for the Extended Indicator

- Add fractions and mixed numbers with like denominators up to 10 without regrouping.
 - Use models to add fractions. Present a fraction strip and the addition sentence $\frac{3}{8} + \frac{2}{8} =$ _____. First shade 3 parts, and then shade 2 more parts.

Count the total parts shaded, 5. Write the answer to the addition sentence $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.



Present the addition sentence $1\frac{1}{4} + 1\frac{2}{4} =$ _____. Use a template and fraction pieces to model the addition problem. Start with a template with four circles divided into fourths.



Place the fraction pieces on the template to represent $1\frac{1}{4}$.



Next, add the fraction pieces that represent $1\frac{2}{4}$ to the template. Be sure to place the two-fourths in the circle that the first fourth was placed in and emphasize that there is still room for two more fourths in the circle.



Find the sum of all the pieces, 1 + 1 = 2 plus $\frac{3}{4} = 2\frac{3}{4}$.

- Ask students to use a model to add fractions and mixed numbers with like denominators without regrouping.
- Subtract fractions and mixed numbers with like denominators up to 10 without regrouping.
 - Use models to subtract fractions. Present a fraction strip and the subtraction sentence $\frac{9}{10} - \frac{2}{10} =$ _____. First shade in $\frac{9}{10}$, and then cross off two parts to subtract $\frac{2}{10}$. Count the remaining shaded parts, $\frac{7}{10}$. Write the answer to the subtraction sentence $\frac{9}{10} - \frac{2}{10} = \frac{7}{10}$.



Present the subtraction sentence $2\frac{3}{4} - 1\frac{2}{4} =$ _____. Use a template and fraction pieces to model the subtraction problem. Start with the same template with four circles divided into fourths used for the addition problem. Place the fraction pieces on the template to represent $2\frac{3}{4}$. Then take away $1\frac{2}{4}$ to model the subtraction and show the fractional pieces remaining, $1\frac{1}{4}$, as the answer.

 Ask students to use a model to subtract fractions and mixed numbers with like denominators without regrouping.

7.N.2 Operations

Prerequisite Extended Indicators

MAE 5.N.3.e—Use a visual model to add and subtract fractions with like denominators of halves, thirds, fourths, and fifths, limited to minuends and sums with a maximum of 1 whole.

MAE 4.N.3.c—Use visual models to add and subtract fractions with like denominators of halves, thirds, and fourths, limited to minuends and sums with a maximum of 1 whole.

Key Terms

addition, denominator, difference, fraction, mixed number, numerator, subtraction, sum, whole number

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-3

http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/831

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Mathematics—Grade 7 Ratios and Proportions

7.R.1 Proportional Relationships

7.R.1.a

Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table).

Extended: Determine unit rate when given a table, limited to ratios of 1:2, 1:3, 1:5, and 1:10.

Scaffolding Activities for the Extended Indicator

□ Find a missing number in a table with a ratio of 1:2.

Use manipulatives, pictures, or drawings to show a model of a 1:2 ratio. Explain that a ratio compares the values of two groups. A ratio table is used to organize quantities so patterns can be recognized, and problems can be solved. When a ratio table shows a ratio of 1:2, there is the following pattern: as the value in one group increases by 1, the value of a second group increases by 2. Present the scenario where 1 ball is needed for every 2 people to play catch, and the ratio is 1:2. The unit rate is 1 ball per 2 people. The ratio table shows the number of balls in one column and the number of people playing catch in the other. The table shows how there is 1 ball for every 2 people, there are 2 balls for 4 people, there are 3 balls for 6 people, and so on.

Balls	People
\bigcirc	££
\bigcirc	रेरेरेरे
	<u> </u>

Ratio – 1:2

Present the table with a value missing. Demonstrate how to determine the missing number of people that belongs in the table by drawing in 2 people for every ball.



Ratio – 1:2

Since the unit rate is 1 ball for every 2 people, the missing value is 8 people.

- Ask students to identify a 1:2 ratio table that represents another scenario.
- Ask students to find the missing value in a 1:2 ratio table.

□ Find a missing number in a table with a ratio of 1:3.

• Use manipulatives, pictures, or drawings to show a model of a 1:3 ratio. When a ratio table shows a ratio of 1:3, there is the following pattern: as the value of one group increases by 1, the value of the second group increases by 3. Present the scenario of a dance routine in which there is one clap for every 3 stomps. Therefore, the ratio of clapping to stomping is 1:3. Create a ratio table that shows the scenario using tick marks.

Claps	Stomps

Ratio – 1:3

Present the table with a missing value. Demonstrate how to determine the missing number of stomps by making three tick marks for every clap.

Claps	Stomps

Ratio - 1:3

Since there is 1 clap for 3 stomps, the missing value is 6.

- Ask students to identify a 1:3 ratio table that represents another scenario.
- Ask students to find a missing value in a 1:3 ratio table.

□ Find a missing number in a table with a ratio of 1:5.

• Use manipulatives, pictures, or drawings to show a model of a 1:5 ratio. When a ratio table shows a ratio of 1:5, or a unit rate of 1 student for every 5 markers, there is the following pattern: as the value of one group increases by 1, the value of the second group increases by 5. Present the scenario of each student having 5 markers. Therefore, the ratio of students to markers is 1:5. The ratio table shows the number of students in one column and the number of markers in the other. The table shows how there are 5 markers for 1 student, 10 markers for 2 students, and 15 markers for 3 students.

Students	Markers
1	
2	
3	

Ratio – 1:5

Present the table with a missing value. Demonstrate how to determine the missing number of markers by placing 5 markers for every student.

Students	Markers
1	
2	
3	

Ratio – 1:5

Since there is a unit rate of 1 student for every 5 markers, the missing value is 10 markers.

- Ask students to identify a 1:5 ratio table that represents another scenario.
- Ask students to find a missing value in a 1:5 ratio table.

□ Find a missing number in a table with a ratio of 1:10.

• Use manipulatives, pictures, or drawings to show a model of a 1:10 ratio. When a ratio table shows a ratio of 1:10, there is the following pattern: as the value of one group increases by 1, the value of the second group increases by 10.



Ratio - 1:10

Present the table with a missing value. Demonstrate how to determine the missing number of tickets by drawing a group of 10 tickets for every dollar.



Ratio - 1:10

Since 1 dollar is needed for every 10 tickets, the missing value is 30.

- Ask students to identify a 1:10 ratio table that represents another scenario.
- Ask students to find a missing value in a 1:10 ratio table.

Prerequisite Extended Indicators

MAE 6.R.1.a—Determine ratios from concrete models and drawings.

MAE 6.R.1.e—Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

Key Terms

compare, increases, pattern, ratio, ratio table, set

Additional Resources or Links

https://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/61 https://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/2157

7.R.1.b

Represent and solve authentic problems with proportions.

Extended: Given a proportional relationship that represents an authentic situation, determine the missing quantity.

Scaffolding Activities for the Extended Indicator

Recognize that proportional relationships can be determined in different ways.

• Explain to students that a proportion is a statement of equality between two ratios.

2:8 = 4:16

One way these could be read is "2 is to 8 as 4 is to 16." The two numbers in the ratio 2:8 can both be multiplied by the same number to create an equal ratio. In the case of 2:8 = 4:16 the numbers 2 and 8 were each multiplied by 2. This results in two ratios that are equal. Since the equation is a statement of equality between two ratios it is a proportion.

• Explain that proportional relationships can be determined in different ways. Present the problem shown.

Emily bought 5 pencils for \$2.00.

Each pencil cost the same amount.

How much would 10 pencils cost?

Model using division and multiplication to determine the proportional relationship between pencils and the cost. First, demonstrate finding the cost of 1 pencil by dividing \$2.00 by 5, which is \$0.40. Explain that the proportion relationship between one pencil and the cost is 1 pencil for \$0.40.

Next, explain that in order to determine the proportional relationship to solve the problem of determining how much 10 pencils will cost, the price of one pencil will need to be multiplied by 10.

Emphasize that the proportional relationship between the number of pencils and the cost was determined by using multiplication and division.

Present the same problem to students. Set up a proportional relationship of 5:\$2:00 = 10:____. Explain that to show a proportional relationship, both sides of the equation must show equality. Point to the 5 and then to the 10 in the proportional relationship. Explain that 10 is two times the value of 5. This means that \$2.00 also needs to be multiplied by 2 to complete the proportional relationship. Present the proportional relationship of 5:\$2:00 = 10:\$4.00 to students.

Emphasize to students that two different methods were used to determine the proportional relationship and they both had the same outcome.

• Present various word problems, ratios, and visual representations and model finding the proportional relationship using the different methods. For example, ask which of the following ratios has a proportional relationship with the ratio 2:5. Model different ways to determine 4:10 is the correct answer.

• Ask students to determine different ways to recognize a proportional relationship. For example, present the different ratio cards to students. Then, ask students to determine which two cards are in a proportional relationship and explain how they came to that conclusion.



Use proportional reasoning to find a missing quantity in a proportion.

• Explain to students that one way to find the missing quantity in a proportion is to calculate the unit rate. Another way to say this is to find a ratio with a denominator of 1 that is equivalent to a given ratio. For example, if a given ratio is 24:2, the unit rate can be determined by dividing the numerator and the denominator by 2 so that the denominator of the new equivalent ratio is 1. The result is 12:1, so the proportion is 24:2 = 12:1.

 Present the following problem and model how to use proportional reasoning to find the missing quantity.

"The price of a box of 24 protein bars is \$9.60. Shawna wants to buy 7 protein bars. How much will it cost Shawna to buy 7 protein bars?"

 $9.60 \div 24 = \cos t \div 7$

Proportional reasoning tells us that if you know the cost of one protein bar, you can then figure out how much 7 protein bars cost. The cost of one protein bar or the unit cost for a protein bar can be determined by dividing the price for 24 protein bars by 24.

One protein bar costs \$0.40. Multiplying this unit rate by 7 will result in the amount that Shawna will pay for the protein bars that she wants to buy.

- Present different problems and model using proportional reasoning skills to solve to find the missing quantities. Some examples are: "Liz bought 4 small picture frames for \$6.00. How much would 12 frames cost?" "A recipe calls for 2 eggs for every 3 cups of flour added. If 9 cups of flour were added, how many eggs are needed?"
- Ask students to find a missing quantity in a proportion by using proportional reasoning.

Prerequisite Extended Indicators

MAE 7.R.1.a—Determine unit rate when given a table, limited to ratios of 1:2, 1:3, 1:5, and 1:10.

MAE 6.R.1.e—Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

MAE 6.R.1.a—Determine ratios from concrete models and drawings.

Key Terms

proportion, proportional relationship, rate, ratio, unit price, unit rate

Additional Resources or Links

https://www.insidemathematics.org/classroom-videos/public-lessons/5th-grade-math-proportionsratios/proportions-planning-part-a_

https://access.openupresources.org/curricula/our6-8math/en/grade-7/unit-2/family.html

https://hub.illustrativemathematics.org/s/article/Unit-2-Introducing-Proportional-Relationships-Vimeo

7.R.1.c

Use proportional relationships to solve authentic percent problems (e.g., percent change, sales tax, mark-up, discount, tip).

Extended: Identify the percentage for an authentic discount problem, limited to 10%, 25%, and 50%.

Scaffolding Activities for the Extended Indicator

□ Identify the percent in the context of a discount problem.

- Use a story problem to describe how percent is used for a discounted price. For example, a shop has a sign in the window that reads, "All Shoes 25% Off." Discuss the meaning of 25% as the percent discount for shoes. Other examples could include 50% off clearance items or 10% off soap. Refer to other examples of signs or advertisements that use percent as a discount.
- Ask students to identify the percent sign in a number, such as 25%.
- Ask students to identify the percent discount in a figure, as shown.



- □ Identify the fractions equivalent to 10%, 25%, and 50%.
 - Use a model to demonstrate the fractions that are equivalent to 10%, 25%, and 50%. For example, the following model shows a square with 1 of 10 parts of the whole shaded and a square with 10 of 100 parts of the whole shaded. Explain that the amount shaded is the same in both models and is equal to $10\%, \frac{1}{10} = \frac{10}{100} = 10\%$.



- Use models to demonstrate that $50\% = \frac{1}{2}$ and $25\% = \frac{1}{4}$.
- Ask students to identify the fractions equivalent to 10%, 25%, and 50% using models and without using models.

• Ask students to complete a table with fractions equivalent to 10%, 25%, and 50%.

Percent Equivalencies			
Percent	10%	25%	50%
Fraction			

ont Equivalancias

□ Identify the percent for a discount problem.

Use a story problem to demonstrate how to find the percent discount. For example, tickets to a movie cost \$10, but there is a sale going on for \$1 off per ticket. Explain that \$1 off per \$10 can also be written as 10% because it is the same as the fraction ¹/₁₀, where the numerator, 1, is the discount, and the denominator, 10, is the original price.



Continue to model how to identify 10%, 25%, or 50% as the discount amount. For example, a store has a half-price sale on books. The original price of a book is \$20, and the sale price is \$10. Explain that half price is the same as the fraction $\frac{1}{2}$ off, which is 50%.

• Ask students to identify the percent for a discount problem when the fraction of the discount is given, as shown.



 Ask students to identify the percent for a discount problem when the original price and the discount amount are given, as shown.



Prerequisite Extended Indicators

MAE 6.R.1.d—Using a model, convert halves, fourths, and tenths to decimals and identify the corresponding percentages for the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

MAE 6.R.1.c—Recognize $\frac{1}{10}$ and $\frac{1}{100}$ as ratios and convert to equivalent percents.

Key Terms

discount, fraction, percent, sale

Additional Resources or Links

https://www.map.mathshell.org/lessons.php?unit=7100&collection=8

https://www.map.mathshell.org/tasks.php?unit=MA01&collection=9

https://www.engageny.org/resource/grade-7-mathematics-module-4-topic-b-lesson-7

7.R.1.d

Solve authentic problems involving scale drawings.

Extended: Given a scale drawing, identify the scale, limited to $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$.

Scaffolding Activities for the Extended Indicator

- □ Identify the scale in a scale drawing with a scale of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.
 - Use a drawing of something simple (e.g., a sketch of an animal) and a scale of ¹/₂ to create the same drawing that is ¹/₂ the size of the original. For example, use paper with grids or lines to demonstrate the scale drawing shown. Explain that the smaller penguin is ¹/₂ the size of the original penguin, so the scale of the drawing is ¹/₂.



Show a scale drawing with a scale of $\frac{1}{3}$. The smaller plant is $\frac{1}{3}$ the size of the original plant, so the scale of this drawing is $\frac{1}{3}$.



Show a variety of scale drawings using scales of $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$.

• Ask students to identify the scale in a scale drawing. For example, show students the following scale drawing and ask them to choose the correct scale: $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$.



□ Identify the measure of a scale drawing.

• Use drawings that include measurements to demonstrate finding a missing measurement in a scale drawing. For example, the original figure shown here has a height of 9 units and the scale is $\frac{1}{3}$.



The height of the scale drawing, shown with a question mark, is $\frac{1}{3}$ of the 9 units, which is 3 units.

Demonstrate finding the missing measurement using an appropriate computation method. One strategy is to draw 9 lines to represent the original figure. Next, divide the 9 lines between the three sections of the scale drawing. This could also be done with 9 manipulatives. Other strategies include counting the three sections of the drawing and then calculating $9 \div 3$, with or without a calculator.

- Continue to demonstrate finding the missing measure of a scale drawing using a variety of drawings with the original figure represented as a whole-number multiple of the denominator of the scale $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$.
- Ask students to identify the measure of a scale drawing when given the measure of the original figure and the measure of the scale drawing.

• Ask students to identify the missing measure of a scale drawing when given the scale drawing on a grid with the scale of $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ labeled and the measure of the original figure labeled.

Prerequisite Extended Indicators

MAE 5.N.3.c—Use a visual model to divide a whole number by $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ (e.g., 3 divided by $\frac{1}{2}$).

MAE 6.R.1.e—Solve authentic problems using the ratios 1:1, 1:2, 1:3, 1:5, and 1:10.

Key Terms

measure, scale, scale drawing, unit

Additional Resources or Links

https://www.map.mathshell.org/lessons.php?unit=7210&collection=8

https://www.map.mathshell.org/lessons.php?unit=7310&collection=8

https://www.engageny.org/resource/grade-7-mathematics-module-1-topic-d-lesson-16

Mathematics—Grade 7 Algebra

7.A.1 Algebraic Processes

7.A.1.c

Solve one- and two-step equations involving rational numbers.

Extended: Solve a one-step equation using multiplication.

Scaffolding Activities for the Extended Indicator

□ Identify a variable as an unknown number in a multiplication sentence.

• Show a variety of multiplication sentences with the unknown number as the product. Use real-life objects or a visual model (e.g., tally marks, array of stars, or dots) to demonstrate how to identify the missing information.

3 × 4 =	2 × 7 =	5 × 4 =
The missing number is equal to three groups of four.	The missing number is equal to two groups of seven.	The missing number is equal to five groups of four.

• Explain that sometimes the missing number is in the beginning or the middle of the multiplication sentence. Use real-life objects or a visual model (e.g., tally marks, array of stars, or dots) to demonstrate how to rephrase the multiplication sentence into a question.

5 × = 25	× 4 = 12	×3=6
What is the size of the group if five groups are needed to equal 25?	How many groups of four are needed to equal twelve?	How many groups of three are needed to equal six?

• Explain that a variable may be used instead of a blank to show a missing number. A variable is a letter that represents the unknown number. A number and a variable are often shown side-by-side, without the operator ×, to show multiplication. Repeat the process of modeling how to identify the missing information using multiplication sentences with a variable. Be sure to use examples with the variable as the product, the first factor, and the second factor, as well as with no operator between the multiplicand and the variable.

9 × <i>w</i> = 18	3 <i>n</i> = 15	$4 \times 5 = w$
2 <i>n</i> = 16	4 <i>w</i> = 8	6 × <i>n</i> = 24

• Ask students to identify the missing information in multiplication sentences containing a variable.

Give a multiplication equation using a variable.

Present the problem w × 4 = 12. Use a question model. How many groups of 4 are needed to equal 12? Use manipulatives or draw an array to solve the problem. First, make one row of 4.



Count the stars, 4, and identify that more stars are needed. With multiplication more stars are added by repeatedly adding **groups** of stars. In this case, we repeatedly add groups of 4 stars.



Continue to repeatedly add one group of 4 stars after another until the total number of stars equals 12.



Three rows (or groups) of 4 equal 12, which is the answer to the question model. Therefore, 3 is the missing number, w = 3 and $3 \times 4 = 12$.

• Repeat the process with the variable as the second factor. Present the problem 5w = 10. Use a question model. What is the size of the group if 5 groups are needed to equal 10? Use manipulatives or draw an array to solve the problem. First, make a column of 5.



7.A.1 Algebraic Processes

• Count the tokens, 5, and identify that more tokens are needed. Indicate that the size of the group must be greater than 1 because there are not enough tokens to equal 10. Increase the size of the group to 2 by adding another column of 5. Count the tokens. Now there are 10. If each group is size 2, then 5 groups makes 10.



Count the number of tokens, 10. If each group is the size of 2 tokens, then 5 groups make 10, which is the answer to the question model. Therefore, 2 is the missing number: w = 2 and $5 \times 2 = 10$.

• Ask students to use arrays to solve one-step multiplication equations containing variables.

Prerequisite Extended Indicator

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 4.A.1.b—Multiply 2s, 5s, and 10's by a single-digit number with a maximum product of 100.

Key Terms

equation, multiplication, multiply, variable

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_189_g_2_t_2. html?open=activities&from=category_g_2_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-g-lesson-28

7.A.1.d

Solve equations using the distributive property and combining like terms.

Extended: Identify equivalent expressions using the distributive property, limited to digits 1–9 (e.g., $2(3 + 4) = (2 \times 6) + (2 \times 4)$).

Scaffolding Activities for the Extended Indicator

Determine an equivalent expression without distributing.

• Use models to show that some multiplication problems can be made easier to solve by decomposing one of the terms into smaller numbers. Explain that the model shown represents the expression 4 × 7. There are 4 rows and 7 columns.



The number 7 in the model may be decomposed into smaller numbers to make it easier to solve the multiplication problem. For example, the value 7 may be "broken apart" into 5 and 2.



Explain that the 5 and 2 are added together and the sum is multiplied by 4, which is the number of rows: $4 \times (5 + 2)$.



Continue to demonstrate decomposing a variety of multiplication expressions into smaller numbers using rectangle models with unit squares.

7.A.1 Algebraic Processes

 Ask students to select a model that shows a correctly decomposed representation of a given rectangle when given two or more choices. For example, students select the model on the right that shows an equivalent decomposition of the model on the left.



Ask students to decompose an expression such as 6 × 8 into expressions such as 6 × (1 + 7), 6 × (2 + 6), etc.

□ Identify whole number expressions using the distributive property.

Use models to show how to use the distributive property to create equivalent expressions.
 For example, using the same rectangle as previously shown, represent the area of the rectangle using the decomposed expression of 5 + 2 for the side length of 7.



Explain that the expression for the area of the rectangle can also be decomposed by "distributing" the width, 4, to each value from the decomposed length.



Continue to use a variety of rectangle models to demonstrate using the distributive property to create equivalent expressions.

7.A.1 Algebraic Processes

 Ask students to identify visual models using the distributive property when given two or more choices. For example, students select the model on the right that shows an equivalent decomposition of the model on the left.



Ask students to identify equivalent whole-number expressions using the distributive property. For example, ask students to identify an equivalent expression to 3 × (5 + 2) as (3 × 5) + (3 × 2), which is the same as 15 + 6.

7.A.1 Algebraic Processes

Prerequisite Extended Indicators

MAE 5.A.1.d—Evaluate two-step numerical expressions involving addition or subtraction and multiplication using order of operations, limited to the digits 1-5 (e.g., $4 \times (5 - 2)$, $4 + 2 \times 3$).

MAE 4.A.1.b—Multiply 2s, 5s, and 10's by a single-digit number with a maximum product of 100.

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

Key Terms

distribute, distributive property, expression, multiplication

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-e-lesson-16

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-d-lesson-12

7.A.1.e

Solve one- and two-step inequalities involving integers and represents solutions on a number line.

Extended: Identify a solution to a one-step inequality involving addition, subtraction, or multiplication (e.g., n + 1 < 4, 2n > 8).

Scaffolding Activities for the Extended Indicator

Identify a solution to a one-step inequality with addition or subtraction using manipulatives.

• Use models, like manipulatives or drawings, to demonstrate finding solutions to a one-step inequality. For example, the inequality n + 3 < 9 can be represented with tokens, a card with the variable *n* on it, a card with a plus sign on it, and a card with the less than symbol on it.

Explain that the inequality will be solved by first thinking about the equation n + 3 = 9 and using a part-part-whole model. Present the model as shown. Demonstrate adding 6 tokens to 3 tokens to get 9 to determine that n = 6.

9		
n	3	

Next, explain that since the value to the left of the inequality symbol needs to be less than 9, the solution for n needs to be less than. Make changes to the part-part-whole model as shown.

total < 9	
n < 6	3

Possible solutions to this inequality are the numbers 5, 4, 3, and so on. Emphasize that an inequality can have more than one solution. Use manipulatives to demonstrate identifying one solution to an inequality. Also, demonstrate listing all the whole-number solutions to an inequality.



7.A.1 Algebraic Processes

Continue to demonstrate solving one-step addition and subtraction inequalities using manipulatives or drawings, the part-part-whole method, and cards labeled with the +, -, <, >, \leq , and \geq symbols. Be sure to emphasize that an inequality can have more than one solution. Demonstrate identifying one solution to an inequality. Also, demonstrate listing all the whole-number solutions to an inequality.

- Ask students to identify one possible solution to a one-step inequality with addition or subtraction when given the solution stated as an inequality.
- Ask students to identify all the whole-number solutions to a one-step inequality with addition
 or subtraction when given the solution stated as an inequality.

□ Identify a solution to a one-step inequality involving addition or subtraction.

Explain that an inequality can be represented on a number line. Demonstrate the solutions to the inequalities n < 7, n > 7, n ≤ 7, and n ≥ 7 on number lines as shown. Emphasize that the solutions on these number lines show **all** the possible solutions to each inequality, not just one solution.



The inequality $n - 3 \le 5$ can be represented on a number line by first finding the inequality's boundary, n = 8, since 8 - 3 = 5. Other numbers that are less than 8, such as 7, 6, or 5, can be substituted for *n*. Instead of attempting to list all the possible solutions for *n*, a number line can be used as shown.


Explain that the closed point at 8 indicates that 8 is included in the solution set, which shows $n \le 8$. The arrow for this solution points to the left of 8, since our inequality states that n is less than or equal to 8. Explain that the numbers to the left of 8 are less than it; therefore, we need to shade to the left of 8. The solutions can be verified by choosing any of the numbers included in the shaded arrow and substituting them for *n* in the original inequality. Demonstrate substituting 4 for the value of *n* as shown. The result is a true statement that 1 is less than or equal to 5, so 4 is a correct solution to $n - 3 \le 5$.

$$n - 3 \le 5$$
$$4 - 3 \le 5$$
$$1 \le 5$$

The number line can also be helpful in determining the correct solutions, since a number line can be used to count up or count down, depending on whether the inequality involves addition or subtraction. Present the inequality n + 5 > 9. To find the boundary of the inequality, start with n + 5 = 9 and demonstrate locating 9 on the number line. Next, count down 5 to find the inequality's boundary, n = 4.



The number line is used to find the inequality's boundary, 4, for *n*, but *n* is not equal to 4. If 4 + 5 = 9, then a number greater than 4 plus 5 must be greater than 9 because *n* + 5 must be greater than 9. Therefore, the solution to this inequality is *n* > 4. Demonstrate graphing the solution *n* > 4 on a number line as shown.

-											1	>
	I	1	1	1	Ψ-	1	1	1	1		T	
	0	1	2	3	4	5	6	7	8	9	10	

Continue to demonstrate solving a variety of one-step addition and subtraction inequalities using number lines and then representing the solutions on number lines.

- Ask students to identify one possible solution to a one-step inequality when given the solution set graphed on a number line.
- Ask students to identify a solution to a one-step inequality with addition or subtraction using manipulatives or a number line.

□ Identify a solution to a one-step inequality involving multiplication.

Use number lines to model graphing inequalities. Explain that inequalities can be represented on number lines. Demonstrate the solutions to the inequalities *n* < 6, *n* > 6, *n* ≤ 6, and *n* ≥ 6 on number lines as shown.



• Demonstrate that when solutions to inequalities are graphed on number lines, **all** the possible solutions to each inequality are shown or represented. Present the inequality 2n > 6. Explain that this inequality can be solved by finding the boundary of the inequality by first thinking about 2n = 6 and "2 groups of what size is equal to 6." The answer is n = 3 because $2 \times 3 = 6$. Indicate that this value of *n* can now be applied to the solution of the inequality 2n > 6 as n > 3. The solution is any number greater than 3 because for any number greater than 3, $2 \times n$ is greater than 6.

It might be helpful to use manipulatives to represent the solution. For example, show that 2 groups of 3 are equal to 6 but not greater than 6, so 3 is not a solution. However, 2 groups of 4 are greater than 6 and 2 groups of 5 are greater than 6. Be sure to emphasize that the solution to this inequality is all numbers greater than 3.

Use a number line to show all the solutions to the inequality 2n > 6.

Also, use a number line to show one solution to the inequality 2n > 6.

Continue to demonstrate solving one-step inequalities involving multiplication using manipulatives as needed and representing the solutions on number lines.

7.A.1 Algebraic Processes

• Ask students to identify a number line that represents the solution to a one-step inequality involving multiplication when given a choice of three number lines and the solution. For example, present the equation 6x < 24, the solution x < 4, and three choices of number lines as shown. Ask students which number line matches the solution x < 4.



Students can match the graph to the inequality x < 4 or test shaded numbers from the graph in the inequality. For example, graph C must be incorrect because 5 is shaded but 6 × 5 is not less than 24.

Ask students to graph the given solution to an inequality. For example, present the inequality 3y ≥ 15, the solution y ≥ 5, a number line, an open point, a closed point, and arrows pointing in both directions as shown. Ask students to graph the solution y ≥ 5.



• Model solving a one-step inequality using a bar model, algebra tiles, or connecting cubes. For example, present the inequality 2n > 12 and the figure shown. Be sure to emphasize that 2n > 12 can be solved by first finding the boundary of the inequality and thinking about 2n = 12 as represented in the bar model.



Explain that to solve for *n*, the bar model must be divided by 2.



The result shows *n* is equal to 6, which can now be represented as the solution n > 6.



7.A.1 Algebraic Processes

Then, demonstrate plotting the solution on a number line, paying particular attention to the direction of the arrow and whether the circle is open or closed.



Continue to demonstrate solving one-step inequalities involving multiplication using a bar model, algebra tiles, or connecting cubes and representing the solutions on number lines.

Ask students to graph the solution to a one-step inequality with multiplication when given the inequality and the solution represented with a bar model, algebra tiles, or connecting cubes. For example, present the inequality 2*r* < 8 and the figure shown. Ask students to graph the solution on a number line.



• Ask students to identify a solution to a one-step inequality involving multiplication using a bar model, algebra tiles, or connecting cubes.

7.A.1 Algebraic Processes

Prerequisite Extended Indicators

MAE 6.A.1.c—Use substitution to determine if a given value for a variable makes an equation true.

MAE 6.A.1.b—Given the positive integer value of the single variable, evaluate an addition or subtraction expression.

MAE 5.N.2.b—Use symbols <, >, and = to compare and order whole numbers up to 200.

MAE 4.N.1.b—Use symbols <, >, and = to compare whole numbers up to 50.

Key Terms

greater than, greater than or equal to, inequality, less than, less than or equal to, solution, variable

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1608

https://tasks.illustrativemathematics.org/content-standards/6/EE/B/8/tasks/642

7.A.2.a

Write one- and two-step equations involving rational numbers from words, tables, and authentic situations.

Extended: Identify one-step addition, subtraction, and multiplication equations that represent authentic situations.

Scaffolding Activities for the Extended Indicator

identify a one-step addition equation that represents an authentic situation.

• Use a model to demonstrate how to represent an authentic situation with an equation. Present the following scenario: "There are 10 books on a shelf, and 8 more books are added to the shelf. How many total books are on the shelf?"



Explain that the equation 10 + 8 = b, where the variable *b* is the total number of books on the shelf, can be used to solve the problem. Demonstrate how to find the solution, which is the value that can be substituted into the equation in place of the *b*. Use counting or addition to show that 10 + 8 = 18. Indicate that the answer is 18 books on the shelf, since *b* = 18.

- Demonstrate solving a variety of addition equations that represent authentic situations. Be sure to include problems in which one of the addends is missing, such as 5 + x = 27.
- Ask students to identify an addition equation that represents an authentic situation when given two or more choices of addition equations.

□ Identify a one-step subtraction equation that represents an authentic situation.

• Use an equation to model an authentic situation that involves subtraction. Present the following scenario: "There are 45 cars in a parking lot at the beginning of the day, and 35 of the cars leave at the end of the day. How many cars remain in the parking lot?" The number of cars remaining in the parking lot can be represented with the variable *c*.

Demonstrate how to solve the equation and find the answer to the problem. For this example, 35 is subtracted from 45 to get 10, so c = 10. That means there are 10 cars remaining in the parking lot.

• Demonstrate solving a variety of subtraction equations that represent authentic situations. Be sure to include problems in which the subtrahend is missing, such as 12 - d = 2.

- Ask students to identify a subtraction equation that represents an authentic situation when given two or more choices of subtraction equations.
- □ Identify a one-step multiplication equation that represents an authentic situation.
 - Use an equation to model an authentic problem that involves multiplication. Present the following scenario: "Apples cost \$2 for one pound. To make a certain recipe, 5 pounds of apples are needed. What is the total cost of the apples?" To find the total cost of the apples, use an equation like the one shown, where *a* is the total cost of the apples.

Explain that the answer to a multiplication problem is called the product. Demonstrate using an appropriate computation method to solve the problem (e.g., skip counting, repeated addition, using a calculator). For this example, the product of 2 and 5 is 10, so a = 10. The total cost of the apples is \$10.

- Demonstrate solving a variety of multiplication equations that represent real-world problems. Be sure to include problems in which a factor is missing, such as $2 \times g = 20$.
- Ask students to identify a multiplication equation that represents an authentic situation when given two or more choices of multiplication equations.

□ Identify a one-step addition, subtraction, or multiplication equation containing an unknown that represents a solution to an authentic situation.

• Use an authentic example to demonstrate an equation involving multiplication. For example, tickets to a movie cost \$4 each. How much will 3 tickets cost?

 $4 \times 3 = c$

Explain that the total cost is represented by the variable c, and since the number of tickets is 3, the cost of \$4 is multiplied by 3. If appropriate, make the connection between skip counting and multiplication to help students decide when an authentic situation involves multiplication.

• Show other one-step equations where the unknown is represented by a variable and in a variety of positions in the equation. Some examples are shown.

```
9 + a = 15
7 + 3 = b
6 - d = 0
10 - 7 = f
5 × g = 20
2 × 3 = h
```

Use a variety of relevant authentic scenarios to connect the equations to familiar situations. Use picture representations for support when appropriate.

• Ask students to identify a one-step equation that represents the solution to an authentic situation. For example, a classroom needs 9 chairs for a group to sit in. There are already 6 chairs in the classroom. How many more chairs are needed?



Students can be given a variety of equations to choose from, such as 9 - 9 = c, 6 + c = 9, and $6 \times 9 = c$. The equation in this example is identified as 6 + c = 9.

Prerequisite Extended Indicator

MAE 3.A.1.d—Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

MAE 4.A.1.e—Identify an addition or subtraction equation in an authentic mathematical situation using a variable for an unknown, limited to an unknown change or unknown result (e.g., 3 + n = 10, 12 - 6 = n).

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

MAE 7.A.1.c—Solve a one-step equation using multiplication.

Key Terms

add, difference, equation, integer, multiply, product, solution, subtract, variable

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/digging%20dinosaurs.pdf https://www.insidemathematics.org/sites/default/files/materials/diminishing%20return.pdf http://tasks.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1107

7.A.2.b

Write one- and two-step inequalities to represent authentic situations involving integers.

Extended: Identify an inequality that represents a solution to a one-step problem involving addition, subtraction, or multiplication in an authentic situation.

Scaffolding Activities for the Extended Indicator

- Identify an inequality that represents a solution to a one-step problem involving addition or subtraction in an authentic situation.
 - Explain that an inequality is a statement that represents two quantities that are <u>not</u> equal and that the solution to an inequality is any value or values that makes the inequality true.

Demonstrate how to represent an authentic situation with an inequality. Present the following scenario: "Samson baked 6 cookies. He will need more than 18 cookies to bring to the bake sale."

Explain that this information can be represented with an inequality. Explain that we know Samson needs more than 18 cookies, which can be represented in the inequality as > 18. We also know that Samson already has 6 cookies but needs to add more to have more than 18. This can be represented as the expression 6 + c, with the 6 representing the cookies Samson already has and the variable c representing the unknown number of cookies he will have to bake to get more than 18. These can be put together to form the inequality 6 + c > 18.

• Present the following scenario and inequalities to students.

Chloe has 3 bottles of water. Together, Chloe and Jake have at most 12 bottles of water. Which inequality represents a solution to this scenario?

x - 12 > 3 $x + 3 \le 12$ $12 + 3 \le x$

Model identifying the known information in the scenario. Explain that it is known that Chloe and Jake will have 12 or fewer bottles of water. Model looking at each inequality to determine which ones show less than or equal to 12. Next, explain that we know Chloe already has 3 water bottles. Model looking at the same inequality and identifying that the 3 is there, emphasizing that it continues to represent the scenario. Last, explain that it is unknown how many water bottles Jake has, but it will be added to Chloe's number of 3. Model identifying the "+ 3" in the inequality. Explain that $x + 3 \le 12$ is the inequality that represents the solution to this problem.

- Present various one-step authentic scenarios to students as well as inequalities involving addition or subtraction and then model identifying the inequality that represents the solution.
- Ask students to identify an inequality that represents a solution to a one-step problem involving addition or subtraction in an authentic situation.

□ Identify an inequality that represents a solution to a one-step problem involving multiplication in an authentic situation.

• Explain to students that multiplication can be used to solve problems involving inequalities. Demonstrate how to represent an authentic situation with an inequality. Present the following scenario: "Molly knows she will have fewer than 20 homework questions this week. She knows there are 5 days in the school week and wants to work on the homework questions each day. Molly wants to figure out how many homework questions she should answer each day."

Explain that this information can be represented with an inequality. Explain that we know Molly will have fewer than 20 homework questions, which can be represented in an inequality as < 20. We also know that there are 5 school days this week. To figure out how many homework questions she should answer each day, she needs to multiply. Since she doesn't know what number of problems it will be per day yet, it can be represented by a variable. The final inequality that represents a solution to Molly's situation can be written as $5 \times p < 20$. Emphasize that this inequality shows 5 days times the number of homework problems, p, is less than 20, which represents the same information as the scenario.

• Present the following scenario and inequalities to students.

Desmond has \$100 to spend at the bookstore. Each book costs \$20. Desmond wants to know how many books he can buy with \$100 or less. Which inequality represents a solution to this scenario?

$$b - 10 \le 100$$
 $20 \times b \le 100$ $100 \times 20 < b$

Model identifying the known information in the scenario. Explain that it is known that Desmond has \$100 to spend, so his books will need to cost \$100 or less. Model looking at each inequality to determine which one shows less than or equal to 100. Note that there are two inequalities that have this information, so more information is needed to know which one is the solution. Next, explain that we know Desmond wants books that are each \$20. Model looking at the same inequalities and identifying that the 20 is in the middle inequality and is also in the inequality on the right. Last, explain that it is unknown how many books Desmond will be able to buy, so he will need to multiply the unknown number of books by \$20. Explain that the unknown number of books is represented with the variable *b*. Explain that 20 × $b \le 100$ is the inequality that represents the solution to this problem.

- Present various one-step authentic scenarios to students as well as inequalities involving multiplication and then model identifying the inequality that represents the solution.
- Ask students to identify an inequality that represents a solution to a one-step problem involving multiplication in an authentic situation.

Prerequisite Extended Indicators

MAE 7.A.1.e—Identify a solution to a one-step inequality involving addition, subtraction, or multiplication (e.g., n + 1 < 4, 2n > 8).

MAE 6.A.2.c—Identify an inequality that represents a solution to a problem involving an authentic situation (e.g., x < 9, $x \ge 3$).

MAE 6.A.1.e—Identify a solution to an inequality on a number line from 0 to 10, limited to whole numbers (e.g., x < 9, $x \ge 3$).

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

addition, equal, greater than, inequality, less than, product, solution, subtraction, variable

Additional Resources or Links

https://curriculum.illustrativemathematics.org/MS/students/1/7/8/index.html

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-h-lesson-33

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-10

Mathematics—Grade 7 Geometry

7.G.1 Attributes

7.G.1.a

Apply properties of adjacent, complementary, supplementary, linear pair, and vertical angles to find missing angle measures.

Extended: Identify a pair of angles as complementary (equal to 90°) or supplementary (equal to 180°).

Scaffolding Activities for the Extended Indicator

- □ Identify a pair of angles as complementary (equal to 90°).
 - Explain the characteristics of a right angle and an acute angle using drawings or manipulatives. Emphasize that an acute angle is smaller than a right angle and measures less than 90 degrees. Emphasize that a right angle has two perpendicular rays and always measures 90 degrees.



• Demonstrate how a right angle can be divided into two acute angles by cutting a right angle into two parts. Repeat the demonstration by cutting different-sized parts.



• Demonstrate how two acute angles can form a right angle by using manipulatives. Explain that two acute angles that can be combined to form a right angle are called complementary angles. Contrast combining two angles that do not form a right angle.



• Explain and demonstrate that when two acute angles are combined to form a right angle, their two individual measures add up to 90 degrees making these angles complementary. Provide examples using adjacent angles and free-standing angles.



Provide a nonexample, emphasizing that it is not a right angle because the two adjacent angles do not equal 90 degrees.



 Ask students to determine whether two angles are complementary by using a reference right angle.



 Ask students to determine whether two angles are complementary using addition of measures of angles.





7.G.1 Attributes

□ Identify a pair of angles as supplementary (equal to 180°).

 Explain the characteristics of a straight angle, an obtuse angle, a right angle, and an acute angle by using drawings or manipulatives. Emphasize that a straight angle forms a straight line and measures 180 degrees, an obtuse angle is larger than 90 degrees, a right angle equals 90 degrees, and an acute angle is smaller than 90 degrees.



• Demonstrate that a straight angle can be divided into two angles in different ways (i.e., cutting it in half to make two right angles or cutting it into two pieces to form an obtuse angle and an acute angle).



• Demonstrate combining two angles to form a straight angle using manipulatives. Label the two angles as supplementary angles. Contrast combining two angles that do not form a straight angle.



Contrast combining two angles that do not form a straight angle. Emphasize that they do not form a straight angle because they do not equal 180 degrees.



 Explain and demonstrate that supplementary angles are angles whose measures add up to 180 degrees. Provide examples using adjacent angles and free-standing angles.

7.G.1 Attributes

 Ask students to identify whether two angles are supplementary using addition of measures of angles.



Prerequisite Extended Indicators

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

MAE 4.G.1.b—Classify angles as acute, obtuse, or right.

MAE 4.G.2.d—Identify benchmark angles of 90° and 180°, and relate those angle measurements to right angles, straight lines, and perpendicular lines.

Key Terms

acute angle, angle, complementary angle, obtuse angle, right angle, straight angle, supplementary angle

Additional Resources or Links

https://im.kendallhunt.com/MS/teachers/2/7/2/index.html

https://im.kendallhunt.com/MS/teachers/2/7/3/index.html

7.G.2 Coordinate Geometry

7.G.2.a

Draw polygons in the coordinate plane given coordinates for the vertices.

Extended: Given a triangle in quadrant 1 with one vertex on the origin, identify the location of one of the other vertices.

Scaffolding Activities for the Extended Indicator

□ Identify the origin on a coordinate plane.

• Use a coordinate plane to show the location of the origin on a graph.



The point where the *x*-axis and *y*-axis meet is called the origin. It can be represented by the ordered pair (0, 0). A point that is plotted at the origin looks like the point on the graph shown.



Emphasize that the origin is always located at (0, 0), even in graphs of different sizes.

• Ask students to identify the origin on a given coordinate graph.

□ Identify the vertices of a triangle on a coordinate plane.

• Use a coordinate graph to show the locations of the three vertices of a triangle.



In the triangle shown, each vertex has a point plotted, and one of the vertices is at the origin. Point out where the other two vertices, or corners, of the triangle are located. Demonstrate finding the three vertices of a triangle and identifying which vertex is at the origin using a variety of different-size triangles.

• Ask students to identify the three vertices of a triangle drawn on a coordinate graph.

□ Identify the location of a vertex of a triangle on a coordinate plane.

Use a coordinate graph to show how to find the coordinates of a given vertex of a triangle.
 For example, demonstrate identifying the coordinates of the vertex labeled P on the graph shown.



The vertex at point P has the *x*-coordinate 7 and the *y*-coordinate 1, so the ordered pair for point P is (7, 1). Show a variety of triangles graphed in the first quadrant of the coordinate plane and demonstrate finding the ordered pair for each vertex.

• Ask students to identify the coordinates of a vertex of a triangle on a coordinate plane.

Prerequisite Extended Indicator

MAE 5.G.2.a—Identify the origin, *x*-axis, and *y*-axis of a coordinate plane.

MAE 5.G.2.b—Identify the x- or y-coordinate of a point in the first quadrant of a coordinate plane.

MAE 5.G.2.a—Graph and name points in the first quadrant of a coordinate plane using ordered pairs of whole numbers.

Key Terms

coordinate plane, coordinates, origin, point, triangle, vertex, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-2/lesson.html https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-3/file/69596

7.G.3.a

Solve authentic problems involving perimeter and area of composite shapes made from triangles and quadrilaterals.

Extended: Solve authentic problems involving the perimeter and area of two adjoining rectangles by counting unit lengths and unit squares.

Scaffolding Activities for the Extended Indicator

- **G** Find the perimeter of two adjoining rectangles using grid paper.
 - Use two rectangular figures (such as cutouts or pattern blocks) that match up with whole-unit lengths on grid paper to model two adjoining rectangles.



Describe the perimeter as the distance along the outside edge of the whole shape. Make a mark on one corner of the composite shape to represent a starting point. Count each square unit along the outside of the figure to find the perimeter of the shape. One method is to use a highlighter and have students trace the perimeter of the figure as they count. Another method is to use painters' tape to form the perimeter of a rectangle on a tiled floor.



• Ask students to find the perimeter of two adjoining rectangles by counting square units.

□ Solve authentic problems involving the perimeter of two adjoining rectangles by counting unit lengths and unit squares.

Present a composite shape of two adjoining rectangles with all the side lengths labeled.
 Present this authentic problem to students:

Juan wants to move his desk. He needs to figure out the perimeter of his desk to determine whether it will fit in the new space. Here is a model of his desk. What is the perimeter of Juan's desk?



Demonstrate a counting strategy (e.g., making tally marks, using manipulatives, using a calculator) to identify each of the side lengths and find the total.

- Present various authentic problems involving the perimeter of two adjoining rectangles using both lengths and unit squares.
- Ask students to solve authentic problems involving the perimeter of two adjoining rectangles by counting unit lengths and unit squares.
- Decompose adjoining rectangles into two separate rectangles.
 - Use composite shapes made of adjoining rectangles to demonstrate how to separate the shapes into individual rectangles. For example, present adjoining rectangles such as the following:



Show multiple ways that the shape may be decomposed.



- Continue to demonstrate decomposing adjoining rectangles into two smaller rectangles by first using manipulatives (e.g., several cutouts of an adjoining rectangle in different colors that can then be cut apart or decomposed in different ways) and then progressing to adjoining rectangles drawn on grid paper that can be highlighted in various color combinations to represent the smaller rectangles.
- Ask students to decompose shapes made of adjoining rectangles into smaller rectangles.
- □ Solve authentic problems involving the area of two adjoining rectangles by counting unit lengths and unit squares.
 - Present two adjoining rectangles set on a grid as shown. Explain that the area is the number of square units that cover a shape. Present an authentic problem as shown.

Jade is creating a new walkway using square tiles. She made a model where one unit square is equal to one tile. She needs to figure out the area to determine how many tiles her walkway needs. What is the area of Jade's new walkway?



Demonstrate dividing the composite figure, or walkway, into two separate rectangles.



Demonstrate different ways to group the values to find the area of each rectangle.



Demonstrate combining the areas of the two rectangles to determine the total area of the walkway. The area of Jade's walkway is 22 square units.

10 square units + 12 square units 22 square units

- Continue to demonstrate finding the area of two adjoining rectangles using authentic problems shown on a grid using appropriate computation strategies including, but not limited to, counting individual unit squares, skip counting, using repeated addition, or using a calculator.
- Ask students to solve authentic problems involving the area of two adjoining rectangles by counting unit lengths and unit squares.

Prerequisite Extended Indicators

MAE 3.G.2.c—Find the area of a square or rectangle with whole-number side lengths by counting unit squares and showing that repeated addition is the same as multiplying the side lengths.

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

MAE 4.G.3.a—Apply perimeter formulas for rectangles to solve authentic problems.

Key Terms

add, adjoining rectangles, area, decompose, grid, perimeter, side, side length, sum, total, unit square

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-4-topic-lesson-2

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-lesson-6_

https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-c-lesson-10

https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-c-lesson-12

http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3. html?open=activities&from=category_g_2_t_3.html

(Note: Java required for website. Most recent version recommended, but not needed.)

7.G.3.c

Determine the area and circumference of circles both on and off the coordinate plane using 3.14 for the value of Pi.

Extended: Identify the center, radius, and diameter of a circle, and distinguish between the area of a circle and the circumference of a circle.

Scaffolding Activities for the Extended Indicator

Recognize the center, radius, and diameter of a circle.

• Describe a circle as a round figure with no corners. Show examples of circles. Explain that a point can be placed at the very center, or middle, of a circle. Explain to students that the center of a circle is a point that is the same distance from any point on the circle. Identify the center of a circle using examples and nonexamples. To determine whether a point is at the center of a circle, cut three pieces of string so that their lengths match the distances from the point to three different locations on the edge of the circle. If the three pieces of string are the same length, the point is at the center of the circle. If the point is not at the center of the circle.



- Present various circles to students with points located centered and not centered. Discuss other ways to tell whether the point is in the center of a drawing such as using a ruler.
- Ask students to recognize the circles that have a point on the center when presented with circles with points both centered and not centered.



 Describe the radius of a circle as the distance from the center of the circle to the edge. Demonstrate drawing the radius of a circle by first placing a point at the center of the circle and then drawing a line segment to one location on the edge of the circle with a straightedge. Demonstrate drawing the radius on several circles of different sizes in different locations on the circles. Emphasize that the radius of a circle is always a straight-line segment with one endpoint at the center of the circle and the other endpoint touching only the edge of the circle in one place.



Demonstrate nonexamples and explain why each line segment drawn is not a radius. For example, place a point that is not at the center of the circle and then draw a line segment to the edge. Explain that the line segment is not a radius because one of the endpoints is not at the center of the circle.



• Repeat the process by drawing line segments that represent other nonexamples. For example, draw a line segment from one edge of a circle to the other edge of the circle without going through the center. Explain that the line segment is not a radius because the line segment does not end at the center. Demonstrate drawing a line segment from one point on the circle through the center to the other side of the circle. Explain that the line segment is not a radius because the line segment a radius because the line segment to the other side of the circle. Explain that the line segment is not a radius because the line segment touches the edge of the circle in two places.



 Demonstrate identifying a circle with a radius. Present the following figures, and model answering a series of questions to determine whether a radius is shown on each circle. For example, ask the question, "Is there a line segment that touches the edge of the circle?" If there is, ask the questions, "Does the line segment end at the center of the circle?" and "Does the line segment touch the edge of the circle only in one location?"



• Ask students to indicate whether the line segment drawn on a circle is a radius.



• Describe the diameter of a circle as a line segment that connects two points on the circle and passes through the center of the circle. Also mention that the length of the diameter is equal to twice the radius. Demonstrate drawing the diameter of a circle by first placing a point at the center of the circle and then drawing a line segment with a straightedge that begins at one point on the circle, passes through the center of the circle, and ends at another point on the circle. Demonstrate drawing the diameter on several circles of different sizes. Emphasize that the diameter of a circle is always a straight-line segment that passes through the center of the circle and has both endpoints on the circle.



 Demonstrate nonexamples and explain why each line segment drawn is not a diameter. For example, place a point at the center of the circle and then draw a line segment that starts on the circle and ends at a different location on the circle without passing through the center of the circle. Explain that the line segment is not a diameter because the line segment does not pass through the center of the circle.



Repeat the process by drawing line segments that represent other nonexamples. For example, draw a line segment from one point on a circle to a different point on the circle so that the line segment does not pass through the center of the circle. Demonstrate drawing another line segment that begins at one point on the circle and passes through the center of the circle but ends without reaching another point on the circle. Explain that even though this line segment passes through the center of the circle, it is not a diameter because both endpoints are not on the circle.



• Demonstrate identifying a circle with a diameter. Present the following figures, and model answering a series of questions to determine whether a diameter is shown on each circle. For example, ask the question, "Is there a line segment that touches the edge of the circle at two locations?" If there is, ask the question, "Does the line segment pass through the center of the circle?"



• Ask students to recognize whether the line segment drawn on a circle is a diameter.



- Distinguish between area and circumference of circles.
 - Describe a circle as a collection of all the points that are the same distance from a point called the center. Explain that the distance around the circle is called the circumference of the circle. To determine the length of the circumference of a circle, a string could be used. Wrap a string around an object with a round base, such as a cup or a jar lid, and cut the string where one end meets the other. This string could then be stretched out straight to determine the length of the string, which would provide the approximate measure of the circumference of that circle.



- Model this process with different circular objects and emphasize that the circumference is the length around a circle.
- Ask students to identify which circle has the circumference shaded when presented with the following circles.



 Define the area of a circle as all the space inside the circle. Explain that area is a measure that describes the amount of space inside a flat (two-dimensional) object. It could be a circle, a square, a triangle, or any other closed shape. Demonstrate the area of several circles by shading or highlighting the inside of circles of different sizes.



 Demonstrate nonexamples and explain why each circle is not showing the area. For example, draw the diameter on a circle and shade only one half of the circle. Explain that only half of the area of the circle has been shaded.



• Demonstrate identifying a circle that has the area shaded when presented with circles that do not have the area shaded.

 Ask students to identify the circle that has the area shaded when presented with the following circles.



• Ask students to distinguish between circles that have only the area shaded and circles that have only the circumference shaded.



Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 3.N.2.b—Partition two-dimensional figures (circles, triangles, rectangles, and squares) into three, four, or five equal shares, and express the area of each part as a fraction of the whole using $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, or $\frac{4}{5}$.

Key Terms

area, center, circle, circumference, diameter, length, line segment, radius

Additional Resources or Links

https://curriculum.illustrativemathematics.org/MS/students/2/3/2/index.html https://www.engageny.org/resource/grade-7-mathematics-module-3-topic-c-lesson-16

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Mathematics—Grade 7

7.D.1 Data Collection and Statistical Methods

7.D.1.a

Create an investigative question and collect data.

Extended: Collect data to answer an investigative question.

Scaffolding Activities for the Extended Indicator

Distinguish between investigative questions and other types of questions.

 Explain to students that investigative questions are those that are answered using data. This is different from survey questions because survey questions are those that are asked to get the data. Demonstrate the difference between these types of questions by reading two different questions shown and explain why one is a survey question and the other is an investigative question.

Question 1: "What is your favorite ice cream flavor?" This question could be answered by everyone in a class and is a survey question because it is collecting data.

Question 2: "What is the most popular ice cream flavor for the students in your class?" This is an investigative question because it can be answered using data that have already been collected.

• Provide examples of investigative questions and questions that are not investigative questions. Remind the students that investigative questions are the type of questions that are answered using data. Read the next set of questions and discuss with the students which questions are investigative and which questions are not because they do not use data.

Investigative Questions	Other Types of Questions
What percentage of students in your school have at least one brother?	How tall are you?
What is the average number of sports students play throughout the school year?	What is your middle name?
What is the average age of dogs that live on this street?	How many pets do you have?

7.D.1 Data Collection and Statistical Methods

• Ask students to determine which of the questions shown are examples of investigative questions.

"What is the total number of hours you spent watching TV on Saturday?"

"What is the average total number of hours of TV you watch per day during a week?" ***

"What is your favorite color?"

"What is the most popular color in your class?" ***

*** (Investigative Question)

Collect data to answer an investigative question.

- Explain to students that asking survey questions results in a collection of data. Data can be collected by asking people questions or by collecting data from sources that have already asked the survey questions. For example, maps, the internet, and books are sources of data that can be used to answer investigative questions. Once the data are collected, investigative questions can be answered using the data.
- Explain to students that you want to investigate how students get to school each day. Specifically, you want to figure out how many more students take the bus than walk. Demonstrate asking each student if they walk, take the bus, or arrive in a car to school. Model placing the answers in a data table using one tally mark in the corresponding column for each student that answers.

Mode of Transportation	Nu	un	nk	e	r c	of S	Stu	Jd	er	nts
walk	Ι	I	I	I	I					
bus	Ι	I	I		I	I	I	I	I	Ι
car	Ι	I	Ι	I	I	I				

- Emphasize that now that you have data about how students get to school, you can answer the investigative question. Since there are 10 students who take the bus and 5 students who walk, there are 5 more students who ride the bus than walk. Emphasize how the data collected determined the correct answer.
- Model collecting data from students on other topics as well as collecting data from various sources to answer investigative questions.
- Discuss other types of data that could either be collected personally or obtained from other sources and have students come up with some investigative questions that could be answered using the data.
7.D.1 Data Collection and Statistical Methods

 Ask students to collect data to answer an investigative question. For example, present the following question: "Do students prefer pizza, a salad, a taco, or a sandwich for lunch?" Students can then populate the table and determine which lunch food had the most votes to answer the investigative question.

Lunch Food	Number of Students		
pizza			
salad			
taco			
sandwich			

Favorite Lunch Food

Prerequisite Extended Indicators

MAE 6.D.2.c—Find the mode and/or range of a set of ordered whole-number data.

MAE 6.D.2.d—Find the median of a set of ordered whole-number data.

MAE 5.D.2.a—Represent data on tables, pictographs, bar graphs, and line plots.

Key Terms

data, investigative question, research, survey question

Additional Resources or Links

https://illuminations.nctm.org/Search.aspx?view=search&st=d&gr=6-8

https://im.kendallhunt.com/MS/teachers/1/8/index.html

7.D.3 Probability

7.D.3.a

Find theoretical and experimental probabilities for compound independent and dependent events.

Extended: Given an event that will sometimes happen, identify the degree of likelihood of an event as more likely, equally likely, or less likely.

Scaffolding Activities for the Extended Indicator

Determine that an event can have different outcomes.

- Demonstrate scenarios with manipulatives and drawings to identify the likelihood of an outcome as a result of an event. Focus on determining whether the likelihood of the outcome is equally likely or not likely. Use a token with a different color on each side. Flip the token. Explain that flipping the token is an event and that the color that lands up is the outcome of the event. Emphasize that there are two equally likely outcomes for this event because there are two different colors represented equally on the token.
- Continue to demonstrate determining the equally likely outcomes for an event by presenting a set of manipulatives containing two items that are the same and one that is different. For example, present two green coins and one yellow coin. Explain that a green coin is more likely to be pulled from the group than the yellow coin.
- Use a spinner to demonstrate the probability of an outcome. For example, present a spinner or a drawing of a spinner with four equal sections and indicate that each outcome is equally likely.



- Continue to demonstrate determining the equally likely outcomes by using authentic scenarios (e.g., rolling a numbered cube, tossing a coin, choosing between two lunch options).
- Ask students to determine whether the outcome for an event is equally likely or not likely (e.g., drawing objects from a bag, spinning a spinner, answering a true or false question).

7.D.3 Probability

□ Identify the probability of an event as more likely, equally likely, or less likely.

 Use manipulatives to demonstrate the likelihood of an event's outcome. For example, use a chart, a bag, and six yellow, six blue, and six green marbles to create scenarios demonstrating the likelihood of the occurrences of more likely, equally likely, or less likely. Record the results in a table as shown for the following scenarios.

Turn	Yellow	Blue	Green
1			
2			
3			

To demonstrate the likelihood of more likely or less likely, put six yellow marbles and one green marble in the bag. Pick one marble at a time from the bag and record each result in the table. Return the marble to the bag each time and repeat at least three more times. Conclude that if the marbles are mostly yellow, the results will be more likely to be yellow. Extend the thinking to conclude that if there are far more yellow than green marbles, then the result of green is less likely to occur. If needed, repeat the entire demonstration with six blue marbles and one yellow marble.

To demonstrate the likelihood of equally likely, put the same numbers of blue, yellow, and green marbles in the bag. Pick one marble from the bag and record the result in the table. Return the marble to the bag each time and repeat until all the colors have been drawn. Conclude that if there are three colors of marbles in the bag and there are the same numbers of each, the outcome is equally likely for each color to be pulled.

• Demonstrate the likelihood of an event as more likely, equally likely, or less likely using drawings that represent three different scenarios. For example, present the groups of shapes shown and discuss the likelihood of drawing a square from each group. Explain that the likelihood of drawing a square is more likely if the group is mostly squares. The likelihood of drawing a square is equally likely if the group contains equal numbers of squares and triangles. The likelihood of drawing a square is less likely if the group is mostly triangles.



7.D.3 Probability

- Ask students to identify the likelihood of an event as more likely, equally likely, or less likely when shown a collection of colored manipulatives in a bag. For example, place 10 pink erasers and one green eraser in a bag and ask students to determine the likelihood of selecting each color of eraser.
- Ask students to select a scenario that represents more likely, equally likely, or less likely
 when given drawings of three different scenarios. For example, present three drawings,
 with the first drawing showing a group of seven circles and one star, the second showing
 four circles and four stars, and the third showing a group of one circle and seven stars. Ask
 students to identify the likelihood of selecting a circle from each drawing.

Prerequisite Extended Indicators

MAE 6.D.3.a—Identify a list of possible outcomes for a simple event, limited to four possible outcomes.

MAE 6.D.3.c—Identify the probability of an event as always, sometimes, or never.

Key Terms

always, equally likely, event, less likely, likelihood, more likely, never, outcome, probability

Additional Resources or Links

https://im.kendallhunt.com/MS/teachers/2/8/1/index.html

https://im.kendallhunt.com/MS/teachers/2/8/2/index.html

https://www.mathlearningcenter.org/sites/default/files/pdfs/VM1-Guide-Web.pdf

(Lesson 12; Focus Teacher Activity, pg. 139.)

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Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 7



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