

NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 4

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment



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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska’s College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following “e.g.” in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is “characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains” (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student’s cognitive functioning is significantly below age expectations and has an impact on the student’s ability to function in multiple environments (school, home, and community).
- The student’s demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student’s curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education’s technical assistance documents “***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***” and “***Alternate Assessment Criteria/Checklist***” provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher’s perspective, these first suggestions are examples of “I do.” The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with “Ask students to . . .” and are examples of moving from “I do” activities to “we do” and “you do” activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 4

Number

4.N.1 Numeric Relationships

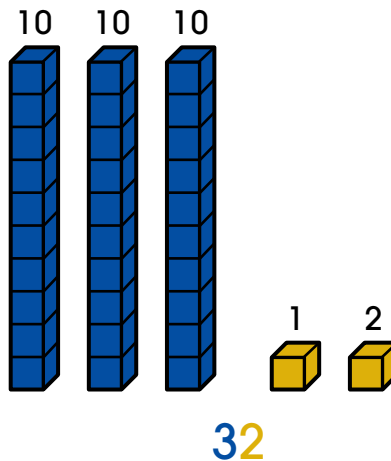
4.N.1.a

Read, write, and demonstrate multiple equivalent representations for whole numbers up to 1,000,000 and decimals to the hundredths using visual representations, standard form, and expanded form.

Extended: Identify representations of whole numbers up to 100.

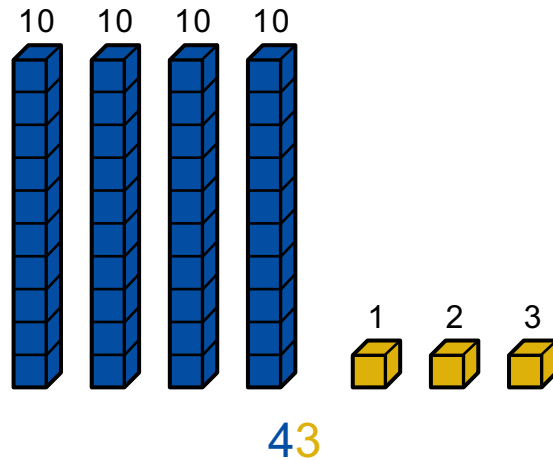
Scaffolding Activities for the Extended Indicator

- ☐ **Represent the standard form of numerals 0–100 with base-ten blocks.**
 - Use base-ten blocks to demonstrate that each base-ten rod represents a group of 10 and that each block represents one. Demonstrate that the blue 3 in the number 32 represents 3 groups of 10, or 30, and that the yellow 2 represents a group of 2.

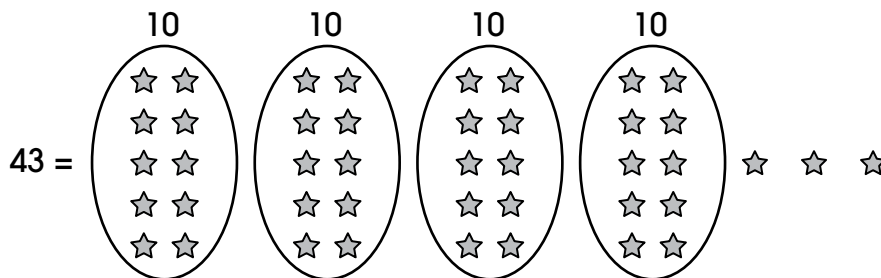


4.N.1 Numeric Relationships

- Ask students to identify the number represented by a visual model. For example, ask students, “What number is represented by this visual model?” Use place-value templates or multiple-choice options for support as needed.



- Present a number from 0 to 100 in standard form. Ask students to represent the number using base-ten blocks and a place-value mat.
- ☐ **Identify the numeral equivalent to a quantity of objects from 0 to 100 decomposed into groups of 10.**
- Use objects to demonstrate grouping for counting. For example, show students the following model and explain that there are 10 stars inside each circle and that every circle with stars represents the number 10. Demonstrate that the number 43 is being represented in this visual and that rather than counting each individual star, students may count the groups together. Skip count the four groups, 10, 20, 30, 40, and then count on to 41, 42, 43.



- Ask students to determine the whole number represented by a quantity of objects decomposed into groups of 10.
- Present a number from 0 to 100 in standard form. Ask students to represent the number by decomposing the group of objects into groups of 10 or by identifying a visual representation of the objects decomposed into groups of 10.

4.N.1 Numeric Relationships

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

base-ten block, group, skip count

Additional Resources or Links

<https://www.engageny.org/resource/grade-1-mathematics-module-6-topic-b-lesson-3>

<https://www.engageny.org/resource/grade-1-mathematics-module-6-topic-b-lesson-4>

4.N.1 Numeric Relationships

4.N.1.b

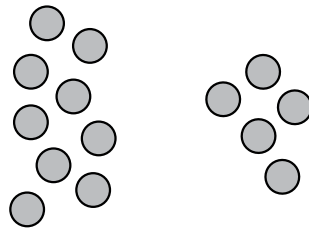
Represent and justify comparisons of whole numbers up to 1,000,000 and decimals through the hundredths place using number lines and reasoning strategies.

Extended: Use symbols $<$, $>$, and $=$ to compare whole numbers up to 50.

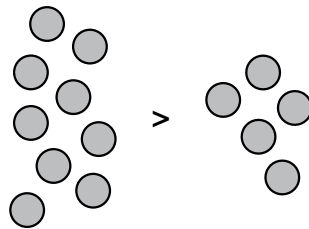
Scaffolding Activities for the Extended Indicator

□ **Use $<$, $>$, and $=$ symbols to visually compare quantities.**

- Present objects in two groups. The quantities in the two groups should be different enough that it is possible to recognize which group has more or fewer objects without counting.



Ask questions about the quantities of the two groups. Does each group have the same amount? Which group has more? Which group has less? Demonstrate using the inequality symbols to indicate which group has more and which group has less. Indicate how the symbols always “open” to the larger quantity.

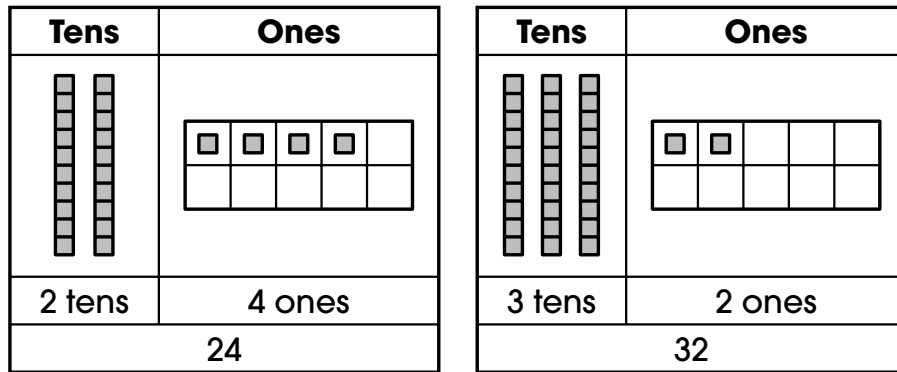


- Ask students to compare various quantities of objects by using the symbols.
- Ask students to compare various quantities by supplying an initial quantity and then having students set a second quantity that is to be greater or less than the given quantity.

4.N.1 Numeric Relationships

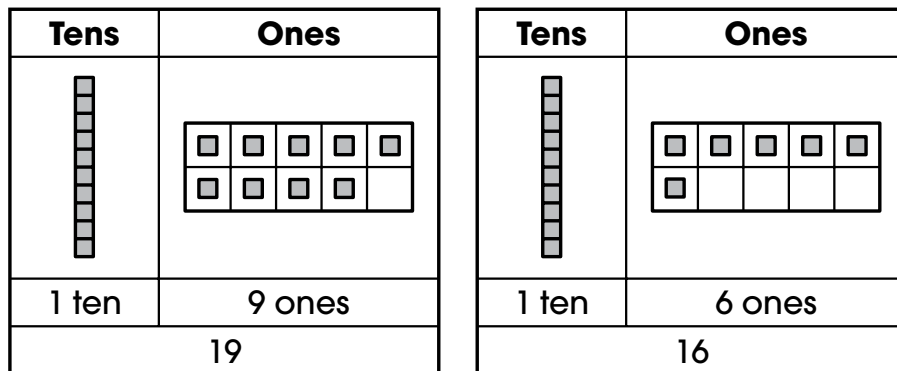
□ Use base-ten blocks and base-ten mats to compare whole numbers up to 50.

- Use base-ten blocks and mats to demonstrate how to compare the value of whole numbers. Indicate that the 2 in the tens column represents 2 tens in 24 and the 3 in the tens column represents 3 tens in 32. Since 2 tens is less than 3 tens, 24 is less than 32.



$$24 < 32$$

- Use base-ten blocks and mats to demonstrate how to compare values in the ones place. For example, create a comparison of the values 19 and 16.



$$19 > 16$$

In this case, the tens place-values are equal. However, there are 9 ones in the number 19, and there are 6 ones in the number 16. Since 9 ones is greater than 6 ones, 19 is greater than 16. Be sure to include examples of equal whole numbers, such as 12 and 12.

- Ask students to compare other whole numbers up to 50 with different values in the tens place, such as 35 and 18.
- Ask students to compare other whole numbers up to 50 with the same number of tens, such as 25 and 27, and equivalent whole numbers up to 50, such as 27 and 27.

4.N.1 Numeric Relationships

Prerequisite Extended Indicator

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

Key Terms

amount, compare, equal to, greater than, less than, ones, quantity, symbols, tens

Additional Resources or Links

<https://www.engageny.org/resource/grade-1-mathematics-module-4-topic-b-lesson-7>

<http://tasks.illustrativemathematics.org/content-standards/1/NBT/B/3/tasks/1102>

4.N.1 Numeric Relationships

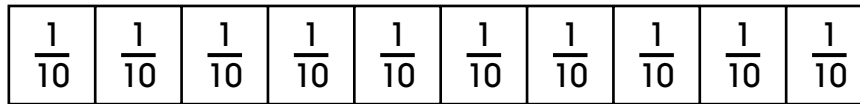
4.N.1.d

Use decimal notation for fractions with denominators of 10 or 100 (e.g., $\frac{43}{100} = 0.43$).

Extended: Use decimal notation for fractions from 0 to 1 with a denominator of 10 (e.g., $\frac{2}{10} = .2$), and identify those decimals on a number line from 0 to 1.

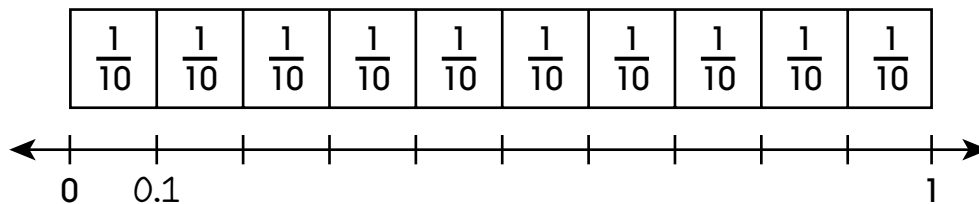
Scaffolding Activities for the Extended Indicator

- Identify that a whole can be divided into ten equally sized pieces and those pieces are called tenths.**
 - Use real objects to demonstrate that a whole can be divided into ten equally sized pieces called tenths. This can be done by cutting straws, modeling with base-ten blocks, or using other objects.



For students who understand that halves and fourths are formed when a whole is divided into two and four equally sized pieces, relate that skill to tenths.

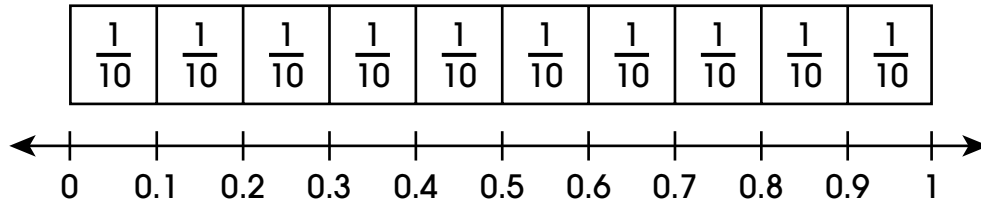
- Ask students to count the number of pieces of a whole that has been divided into tenths. Then ask students to identify various amounts (e.g., two-tenths, six-tenths).
- Equate the language of tenths to decimal numbers. For example, three-tenths is 0.3.**
 - Connect the ten equally sized pieces of the whole to a number line from 0 to 1, with decimal numbers to represent the tenths.



Begin with a number line that shows 0 and 1 with no numbers below the 9 inner tick marks. Write 0.1 and say, “One-tenth” then write $0.1 = \frac{1}{10}$ to show students the equivalence. Write 0.2 and say, “Two-tenths” then write $0.2 = \frac{2}{10}$ to show students the equivalence. Continue with various decimals and fractions to connect the equally sized pieces with the concept of tenths as decimal numbers.

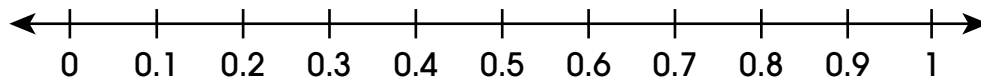
4.N.1 Numeric Relationships

- Ask students to count the number of pieces of a whole in a real object that has been divided into tenths and placed above a number line from 0 to 1, labeled with tenths. Then ask students to identify numbers on the number line.



□ Identify tenths on a number line from 0 to 1 without using manipulatives.

- Demonstrate that a number line from 0 to 1 can be divided into ten equally sized intervals (or sections) and each of those intervals is called a tenth.



The numbers below each tick mark are decimal numbers, and each represents how many tenths there are: one-tenth or 0.1, two-tenths or 0.2, and so on.

- Ask students to identify tenths on a number line from 0 to 1 when given the name (e.g., seven-tenths) or the decimal number (e.g., 0.7).

Prerequisite Extended Indicator

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.N.2.c—Represent halves and wholes on a number line.

Key Terms

decimal, interval, number line, section, tenth

Additional Resources or Links

<https://www.mathlearningcenter.org/apps/number-line>

<https://tasks.illustrativemathematics.org/content-standards/4/NF/C/6>

4.N.2 Fractions and Decimals

4.N.2.a

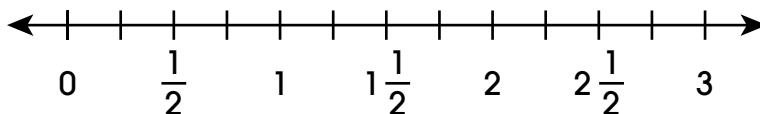
Explain and demonstrate how a mixed number is equivalent to a fraction greater than one and how a fraction greater than one is equivalent to a mixed number using visual fraction models and reasoning strategies.

Extended: Compare and order mixed numbers with denominators up to 5.

Scaffolding Activities for the Extended Indicator

□ **Identify mixed numbers with denominators up to 5 on a number line.**

- Use a number line to show the location of mixed numbers with halves. Present a number line as shown with tick marks at each fourth and labels for the wholes and halves.



The mixed numbers on this number line are $1\frac{1}{2}$ and $2\frac{1}{2}$. Model identifying the mixed numbers. Explain that mixed numbers have a fraction after the whole number. Model pointing to the whole number and then to the fraction that follows. Emphasize that the fraction $\frac{1}{2}$ on a number line is between 0 and 1 and extending the number line past 1 to include 2 and 3 will also include the mixed numbers $1\frac{1}{2}$ and $2\frac{1}{2}$.

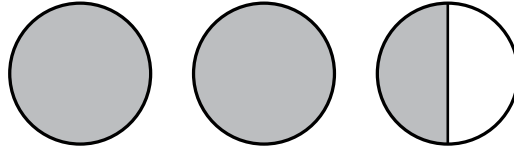
Repeat this process with different number lines with mixed numbers with tick marks at increments of halves, thirds, fourths, and fifths.

- Ask students to identify the whole numbers on a number line.
- Ask students to identify the mixed numbers on various number lines with tick marks at increments of halves, thirds, fourths, or fifths.

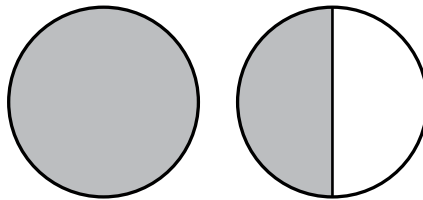
4.N.2 Fractions and Decimals

☐ Identify mixed numbers with denominators up to 5 in a model.

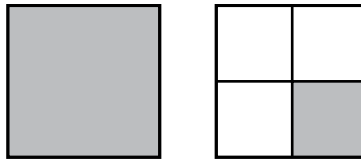
- Use models to show the value of mixed numbers. Explain that a shaded circle represents one whole and that two shaded circles represent the whole number 2. Indicate the model for $2\frac{1}{2}$ as 2 whole circles shaded and $\frac{1}{2}$ of another circle shaded.



Indicate the model for $1\frac{1}{2}$.



Use the same method of modeling to show mixed numbers with fourths. For example, the large square shown represents one whole, so the entire figure shows $1\frac{1}{4}$.

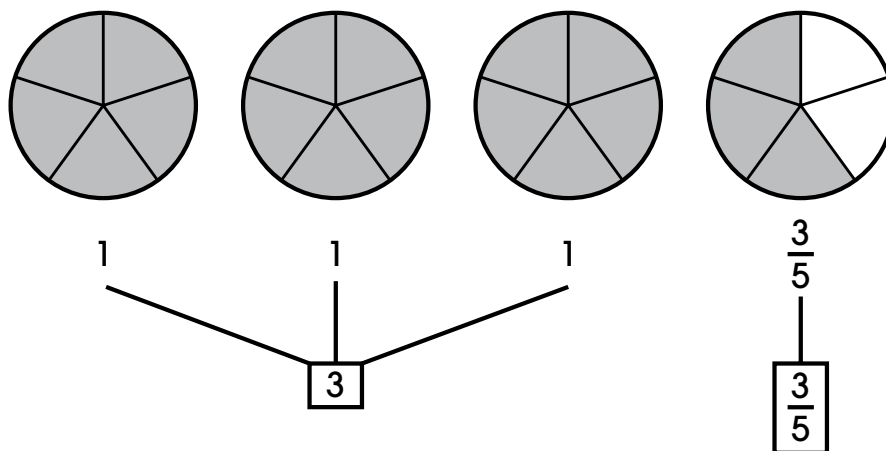


Use the same large square as the whole and model other mixed numbers, such as $2\frac{3}{4}$.



4.N.2 Fractions and Decimals

Model counting the whole(s) of a mixed number and then model counting the fraction that will follow the whole number in a mixed number. Explain that the model shown has 3 circles representing 3 wholes and one circle with 3 out of 5 parts shaded, so the mixed number is $3\frac{3}{5}$.



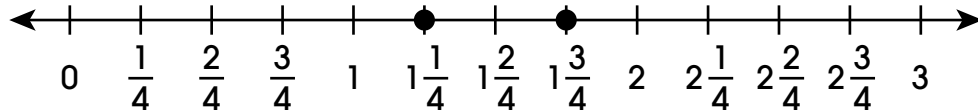
Continue to show a variety of models (using shapes such as circles, squares, and rectangles) to represent mixed numbers with denominators up to 5.

- Ask students to use a model to identify mixed numbers with denominators up to 5.

4.N.2 Fractions and Decimals

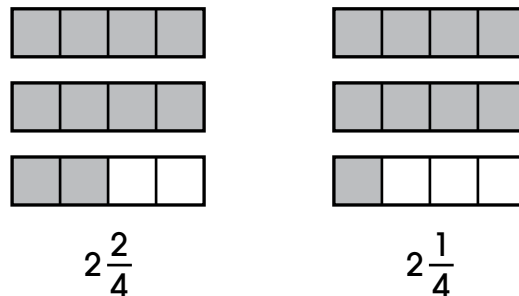
□ Compare mixed numbers with denominators up to 5.

- Use a number line to show how to compare mixed numbers. Present a number line as shown with points at the locations $1\frac{1}{4}$ and $1\frac{3}{4}$. Explain that numbers on a number line go from left to right, getting greater the farther to the right they are located. So, $1\frac{3}{4}$ is greater than $1\frac{1}{4}$ because $1\frac{3}{4}$ is farther to the right.



Continue to demonstrate comparing other mixed numbers with like denominators up to 5 using number lines with halves, thirds, fourths, and fifths labeled.

- Ask students to compare mixed numbers with halves, thirds, fourths, or fifths on a number line.
- Use a model to compare mixed numbers. Explain that each group shown contains four rectangles and that each group is divided into four equal pieces called fourths.



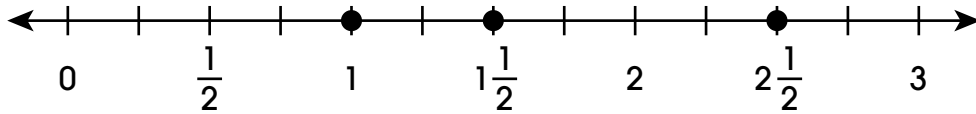
Emphasize that each model has 2 fully shaded wholes, but the first model has a third group with only 2 fourths shaded and the second model has a third group with only 1 fourth shaded. So, $2\frac{2}{4}$ is greater than $2\frac{1}{4}$. Show a variety of models of mixed numbers with denominators up to 5.

- Ask students to use models to compare mixed numbers with like denominators of halves, thirds, fourths, and fifths.

4.N.2 Fractions and Decimals

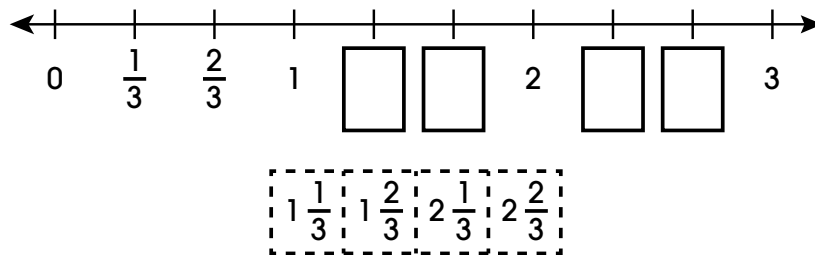
□ Order mixed numbers with denominators up to 5.

Use a number line to show how to order mixed numbers from least to greatest. Present a number line as shown with points located at 1, $1\frac{1}{2}$, and $2\frac{1}{2}$. Explain that to order the numbers from least to greatest, the points should be read from left to right. For this example, the order is 1, $1\frac{1}{2}$, $2\frac{1}{2}$.

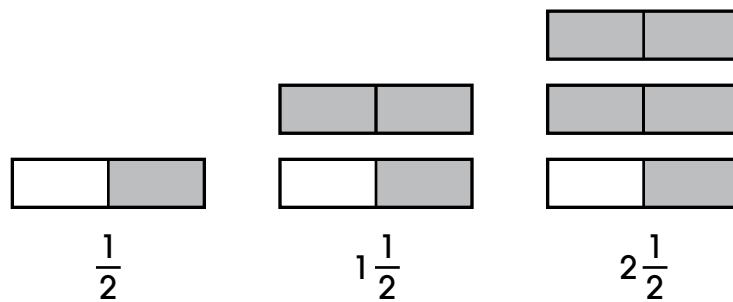


Continue to demonstrate ordering mixed numbers with like denominators by using various number lines with halves, thirds, fourths, and fifths labeled.

- Provide a number line with missing mixed fractions as shown. Model counting from left to right on the number line and placing mixed fractions in their correct position. Model checking your work by ensuring the mixed fractions are in order from least to greatest.



- Use a model to order mixed numbers from least to greatest. Present models for $2\frac{1}{2}$, $1\frac{1}{2}$, and $1\frac{1}{2}$ in random order and then demonstrate ordering the mixed numbers from least to greatest, as shown.



Continue to demonstrate using models to order mixed numbers with like denominators using basic shapes divided into halves, thirds, fourths, and fifths.

- Ask students to order mixed numbers with like denominators up to 5 using a number line.
- Ask students to order mixed numbers with like denominators up to 5 using models.

4.N.2 Fractions and Decimals

Prerequisite Extended Indicators

MAE 3.N.2.c—Represent halves and wholes on a number line.

MAE 3.N.2.e—Given a model, represent a whole number (1, 2, or 3) as a fraction with a denominator of 2, 3, or 4.

MAE 3.N.2.f—Use a model to compare unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Key Terms

fifth, fourth, fraction, greatest, half, least, less, mixed number, more, third, whole number

Additional Resources or Links

<https://apps.mathlearningcenter.org/>

http://nlvm.usu.edu/en/nav/frames_asid_159_g_2_t_1.html?from=category_g_2_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

<http://tasks.illustrativemathematics.org/content-standards/3/NF/A/3/tasks/875>

4.N.3 Operations with Fractions

4.N.3.c

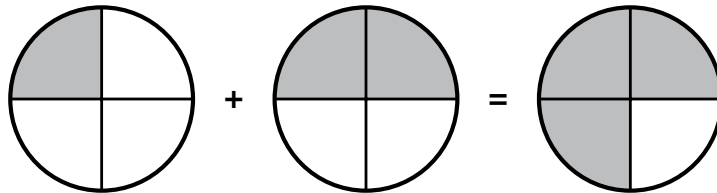
Add and subtract fractions and mixed numbers with like denominators.

Extended: Use visual models to add and subtract fractions with like denominators of halves, thirds, and fourths, limited to minuends and sums with a maximum of 1 whole.

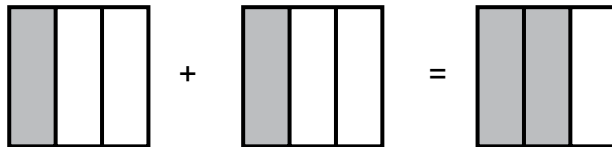
Scaffolding Activities for the Extended Indicator

□ Add fractions with like denominators using a visual model.

- Use models to demonstrate adding fractions with like denominators. For example, to add $\frac{1}{4} + \frac{2}{4}$, circle models can be used, with each circle representing one whole and each whole divided into 4 equal parts. The shaded parts of the circles represent the addition problem $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$.



Continue to demonstrate adding other fractions with like denominators using a variety of shapes and models. The model shown can be used to demonstrate $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.



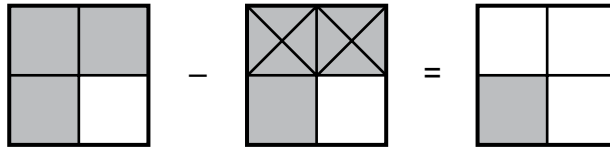
Demonstrate adding fractions with models that represent adding halves to halves, thirds to thirds, fourths to fourths, and so on. Emphasize that the denominator always stays the same in the sum and that the numerator in the sum is the sum of the numerators that are given.

Ask students to add fractions with like denominators using a model.

4.N.3 Operations with Fractions

□ Subtract fractions with like denominators using a visual model.

- Use models to demonstrate subtracting fractions with like denominators. Present the model shown to demonstrate $\frac{3}{4} - \frac{2}{4}$. Each group represents one whole and is divided into 4 equal parts. To subtract, 2 of the shaded fourths are removed from the 3 fourths, represented by the crossed-off fourths in the second group. The shaded part of the last group shows that the difference, which is the remaining shaded part of the group that isn't crossed out, is 1 fourth, so $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$.



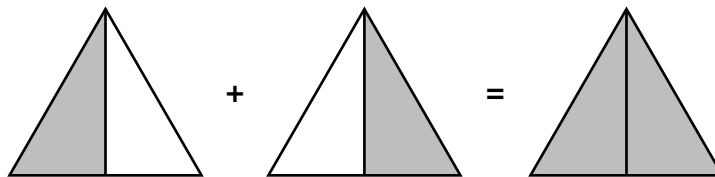
Emphasize that the denominator does not change when subtracting (or adding) fractions. The answer still has a 4 in the denominator, and the numerator is the difference of the given numerators.

Demonstrate subtracting fractions with models that represent subtracting halves from halves, thirds from thirds, and fourths from fourths. Emphasize that the denominator always stays the same in the difference and that the numerator is the difference of the numerators that are given. When appropriate, progress to subtracting fractions without using a model.

- Ask students to subtract fractions with like denominators using a model.

□ Add and subtract fractions with a whole using a visual model.

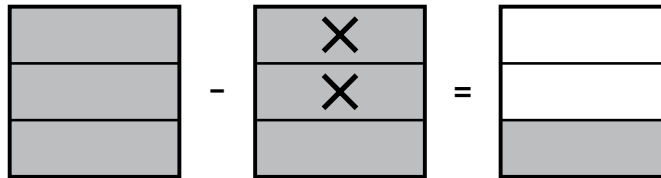
- Use models to demonstrate adding fractions to equal one whole. Present the model shown to demonstrate $\frac{1}{2} + \frac{1}{2}$. Each triangle represents one whole, so the shaded parts represent $\frac{1}{2} + \frac{1}{2} = 1$.



Show models of halves, thirds, and fourths that sum to 1 whole. Emphasize the pattern showing that when the numerator and the denominator are the same, the fraction is equal to 1 (i.e., $\frac{2}{2} = 1$, $\frac{3}{3} = 1$, and $\frac{4}{4} = 1$).

4.N.3 Operations with Fractions

Use models to demonstrate subtracting fractions from one whole. Present the model shown to demonstrate $\frac{3}{3} - \frac{2}{3}$. The first group represents one whole, and the second group shows subtracting $\frac{2}{3}$, so the model represents $\frac{3}{3} - \frac{2}{3} = \frac{1}{3}$.



Show models of halves, thirds, and fourths subtracted from one whole.

- Ask students to add and subtract fractions with a whole using a model.

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 3.N.2.e—Given a model, represent a whole number (1, 2, or 3) as a fraction with a denominator of 2, 3, or 4.

MAE 3.N.2.f—Use a model to compare unit fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

MAE 4.N.2.a—Compare and order mixed numbers with denominators up to 5.

Key Terms

add, denominator, difference, fourth, fraction, half, numerator, subtract, sum, third, whole

Additional Resources or Links

<https://www.engageny.org/resource/grade-4-mathematics-module-5>

<https://www.insidemathematics.org/common-core-resources/3rd-grade>

4.N.3 Operations with Fractions

4.N.3.d

Solve authentic problems involving addition and subtraction of fractions and mixed numbers with like denominators.

Extended: Use visual models to solve authentic problems involving addition and subtraction of fractions with like denominators of halves, thirds, and fourths, limited to minuends and sums with a maximum of 1 whole.

Scaffolding Activities for the Extended Indicator

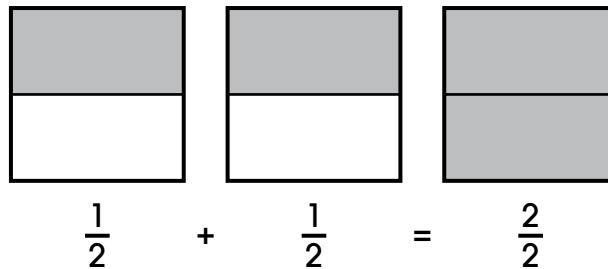
□ **Solve authentic addition problems with halves, thirds, and fourths.**

- Use models or manipulatives to demonstrate adding fractions from authentic problems with halves, thirds, and fourths. Present the word problem and the model shown. Explain that the model shows a square to represent the whole garden, and the shaded parts are each $\frac{1}{2}$ of the whole garden. Two-halves, $\frac{2}{2}$ is the same as one whole, so it can be written as the digit 1. Sammy's whole garden is planted because $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$.

Sammy plants $\frac{1}{2}$ of his garden with carrots.

He also plants $\frac{1}{2}$ of his garden with onions.

How much of Sammy's garden is planted?



Continue to demonstrate how to solve a variety of real-world problems adding halves, thirds, and fourths using models or manipulatives. Be sure to include problems with sums of $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ to emphasize 1 whole.

- Ask students to solve addition real-world problems with halves, thirds, and fourths using models or manipulatives.

4.N.3 Operations with Fractions

□ Solve authentic subtraction problems with halves, thirds, and fourths.

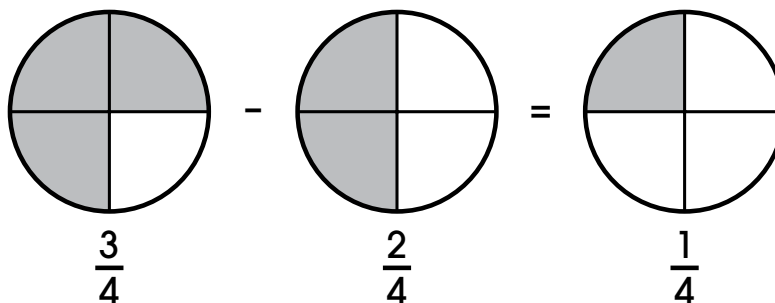
- Use models or manipulatives to demonstrate subtracting fractions in authentic problems with halves, thirds, and fourths. Present the word problem and model shown. Explain that the model shows a circle to represent one whole inch of snow, and the shaded parts show how many fourths of an inch of snow were on the ground to start, which is $\frac{3}{4}$ inch. The model of $\frac{2}{4}$ shows how much snow melted. The amount of snow still on the ground after some melted is $\frac{1}{4}$ inch shown by the model that has 1 out of 4 parts shaded.

Raisa is recording the amount of snow that melted today.

In the morning, there was $\frac{3}{4}$ inch of snow.

By the afternoon, $\frac{2}{4}$ inch of snow had melted.

How much snow is left?



Continue to demonstrate solving a variety of authentic subtraction problems using halves, thirds, and fourths using models and manipulatives. Be sure to include problems with minuends of $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ to emphasize 1 whole.

- Ask students to solve authentic subtraction problems with halves, thirds, and fourths using models or manipulatives.

4.N.3 Operations with Fractions

Prerequisite Extended Indicators

MAE 4.A.1.f—Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

MAE 4.N.3.c—Use visual models to add and subtract fractions with like denominators of halves, thirds, and fourths, limited to minuends and sums with a maximum of 1 whole.

Key Terms

add, denominator, digit, fourth, half, numerator, subtract, sum, third, whole

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/856>

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-d-overview/file/77296>

4.N.4 Factors and Multiples

4.N.4.a

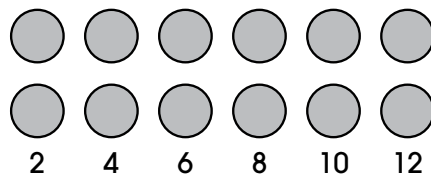
Determine whether a given whole number up to 100 is a multiple of a given one-digit number.

Extended: Count by 2s, 5s, and 10s with numbers, models, or objects up to 50.

Scaffolding Activities for the Extended Indicator

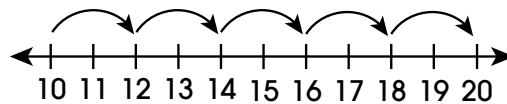
□ Count by twos.

- Use objects to demonstrate counting by twos. This can also be called skip counting and is a faster way to count. For example, pair up objects to help skip count by two.

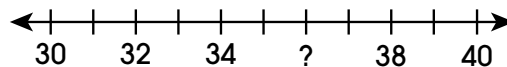


Indicate that counting by twos uses the even numbers: 2, 4, 6, 8, and so on.

- Use a number line to demonstrate counting by twos. For example, draw arrows on a number line from 10 to 20 that show skip counting by two. Indicate that the pattern that results in landing on every other number (or skipping one number in between) is 10, 12, 14, 16, 18, 20.



Show skip counting by two on a variety of number lines. When the pattern becomes more familiar, leave a number out of the pattern and demonstrate how to find the missing number.



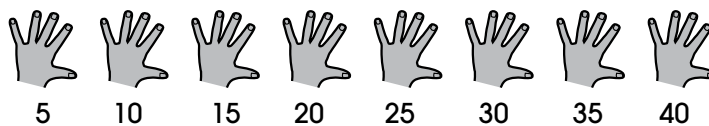
Counting by twos may also be modeled using a hundreds chart up to 40. Note that skip counting by twos results in all even numbers, which are shaded in the chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

- Ask students to count by twos with numbers, models, or objects up to 50.

□ **Count by fives.**

- Use objects to show counting by fives. For example, the five fingers on a hand can be used as shown.



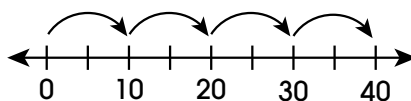
This can also be modeled using a number line or a hundreds chart up to 40. Be sure to explain the pattern of the last digit always being 0 or 5 when skip counting by five.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

- Ask students to count by fives with numbers, models, or objects up to 50.

□ **Count by tens.**

- Use models to demonstrate counting by tens. Skip counting by ten is the fastest way to count to 40, since the “jumps” made are the largest. Indicate that only four numbers are needed when counting to 40 by tens.



Show the same pattern on a hundreds chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Connect counting by ones (1, 2, 3, 4) to counting by tens (10, 20, 30, 40) by pointing out that a zero is just added to the end of the number. This is the same pattern as multiplying a single digit by ten.

- Ask students to count by tens with numbers, models, or objects up to 50.

4.N.4 Factors and Multiples

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 4.N.1.a—Identify representations of whole numbers up to 100.

MAE 4.N.4.b—Identify numbers 1–20 as odd or even, and identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

Key Terms

fives, skip count, tens, twos,

Additional Resources or Links

<https://www.mathlearningcenter.org/apps/number-line>

<http://tasks.illustrativemathematics.org/content-standards/2/NBT/A/2/tasks/1309>

4.N.4 Factors and Multiples

4.N.4.b

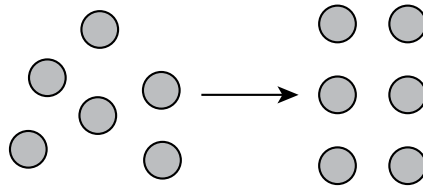
Determine factors of any whole number up to 100 and classify a number up to 100 as prime or composite.

Extended: Identify numbers 1–20 as odd or even and identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

Scaffolding Activities for the Extended Indicator

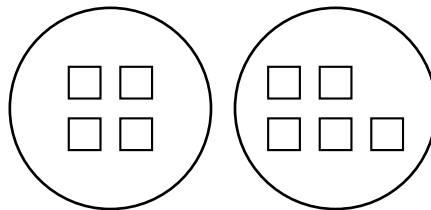
☐ Identify odd and even numbers up to 20.

- Use manipulatives to model decomposing objects into pairs. For example, count out a set of 6 tokens. Then demonstrate sorting those tokens into groups of 2, or pairs. Explain that since all 6 tokens are grouped in sets of 2, or pairs, the number 6 is even.



Repeat the process with 7 tokens. Explain that since there is 1 token remaining after all the pairs are made, the number is odd. Continue with other numbers, always making as many pairs as possible to determine whether the number is odd or even.

- Use manipulatives sorted into two groups to demonstrate whether a number is even or odd. For example, sort 9 objects into two groups. Explain that since the number of objects is not the same in each group and it is not possible to make the groups equal (and keep the objects whole), 9 is an odd number. Explain and demonstrate that an even number of objects could be sorted into two equal-sized groups.

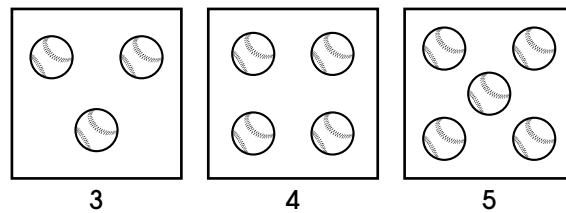


4.N.4 Factors and Multiples

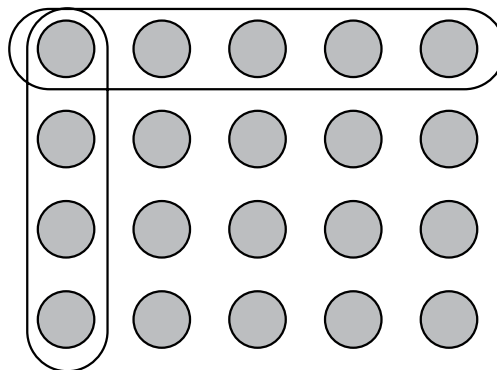
- Ask students to sort groups of objects into pairs and into two groups. Create a table to organize students' results about whether the numbers 1–20 are odd or even. Describe even and odd number patterns, emphasizing that every other number is odd (or even) and that even numbers are used when counting by twos. Also discuss the pattern of the last digit of a number indicating whether it is even or odd.

odd	even
1	2
3	4
5	6
7	8
9	10

- Model how to identify odd and even numbers when given a picture representation of the number of objects. For example, present the following groups of baseballs. Model identifying the pairs and determining whether the number is odd or even. If necessary, circle the pairs to help clarify when there is an extra baseball that could not be grouped into a pair. Progress to identifying odd and even numbers without picture representations when appropriate.



- Ask students to identify odd and even numbers with and without picture representations.
- Identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20 when given an array.**
- Use an array to demonstrate how to find the factors of 20 by circling one row and one column. Indicate that the quantities in the circled row and circled column can be counted to find the factors.



There are 20 total shapes in the array, and it has 4 rows and 5 columns, so this array of 20 has the factors 4 and 5.

Continue modeling with different arrays of 20 until all factors are identified.

4.N.4 Factors and Multiples

- Ask students to identify the factors of 20 using a different array of 20. Ask students to identify the number of stars in one row and one column to find the factors 2 and 10.



- Ask students to identify the factors when given a variety of arrays of sizes 4, 6, 8, 9, 10, 12, 15, and 20.
- **Identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20 when given a multiplication sentence.**
- Use a multiplication sentence to demonstrate how to find the factors of 15. Indicate that the factors are the numbers being multiplied and the product or answer is 15.

$$\textcircled{5} \times \textcircled{3} = 15$$

The factors in the multiplication sentence are 5 and 3.

- Ask students to identify the factors when given a variety of multiplication sentences (e.g., $2 \times 2 = 4$, $1 \times 6 = 6$, $2 \times 4 = 8$, $3 \times 3 = 9$, $2 \times 5 = 10$, $3 \times 5 = 15$, $10 \times 2 = 20$).

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.N.1.b—Compare and order whole numbers 1–20 using number lines or quantities of objects.

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.A.1.h—Multiply 1 and 2 by multiples of 10 with a maximum product of 100.

Key Terms

array, column, equal, even, factor, multiplication sentence, odd, pair, pattern, product, row

Additional Resources or Links

<https://tasks.illustrativemathematics.org/content-standards/2/OA/C/3/tasks/1418>

<https://tasks.illustrativemathematics.org/content-standards/2/OA/C/3/tasks/620>

<http://tasks.illustrativemathematics.org/content-standards/4/OA/B/4/tasks/938>

http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html

(Note: Java required for website. Most recent version recommended, but not needed.)

Mathematics—Grade 4

Algebra

4.A.1 Operations and Algebraic Thinking

4.A.1.a

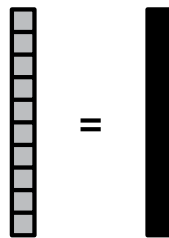
Add and subtract multi-digit numbers using an algorithm.

Extended: Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

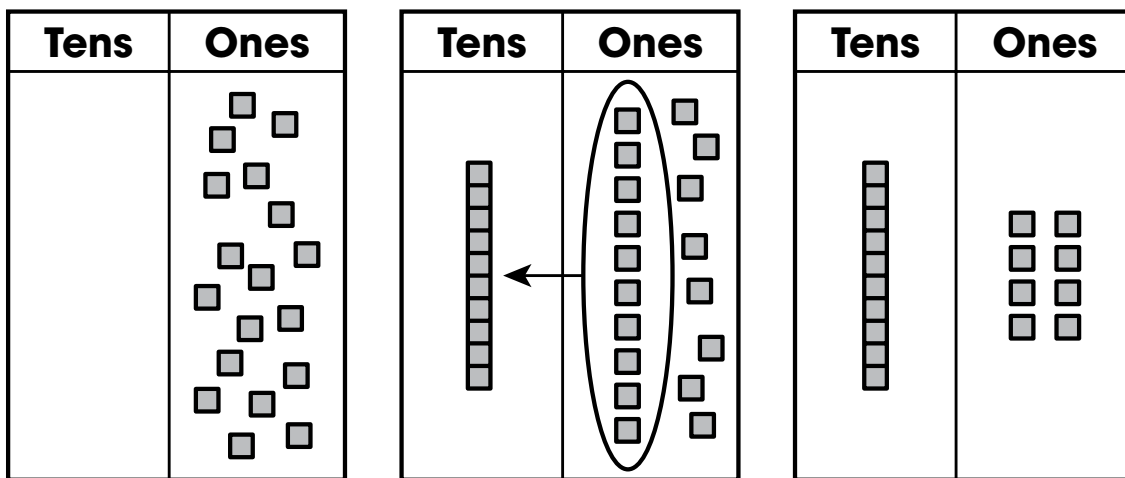
Scaffolding Activities for the Extended Indicator

Add a two-digit number to a one-digit number with regrouping.

- Use a place-value mat and base ten blocks to teach regrouping values of 10 or more into tens and ones. Compare 10 unit cubes to a tens rod to demonstrate that 10 unit cubes are equal to the tens rod.



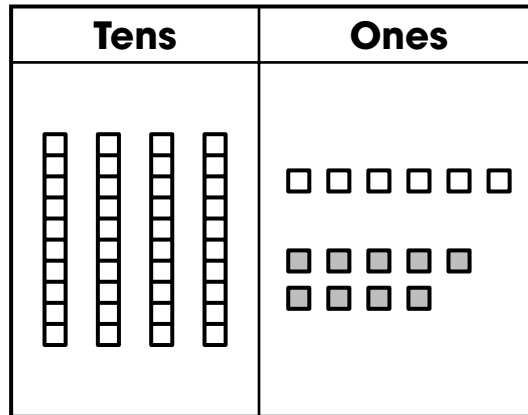
Next, demonstrate that 18 unit cubes are equal to 1 tens rod and 8 unit cubes. First, place 18 unit cubes in the ones column. Next, exchange 10 unit cubes for a tens rod. Continue regrouping values of 10 or more to demonstrate the exchange of 10 unit cubes for a tens rod.



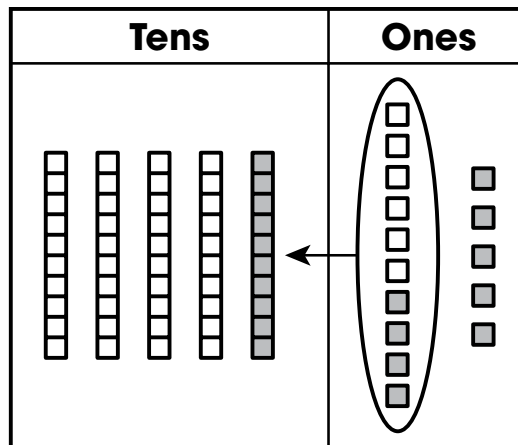
- Ask students to regroup values of 10 or more into tens and ones using a place-value mat and base ten blocks.

4.A.1 Operations and Algebraic Thinking

- Use a place-value mat and base ten blocks to teach addition with regrouping. Present the problem $46 + 9 = \underline{\quad}$. Demonstrate the value 46 on the place-value mat. Then place 9 unit cubes in the ones column in the lower half to represent the value of the second addend.



Indicate that the total in the ones column is larger than 10. Demonstrate that 10 unit cubes can be exchanged for a tens rod, and that rod will be placed in the tens column.



- Indicate that there are now 5 tens and 5 ones, which is written as 55. Complete the number sentence $46 + 9 = 55$.
- Ask students to add a one-digit number to a two-digit number with regrouping using a place-value mat and base ten blocks. Use sticky notes labeled with tens and ones (left and right) to teach addition with regrouping using the standard algorithm. Present the problem $37 + 6$ in standard vertical form on a piece of oversize grid paper with the tens column and the ones column labeled.

Tens	Ones
3	7
+	6

4.A.1 Operations and Algebraic Thinking

- Add the digits in the ones column. Write the number 13 on a sticky note that has been labeled with a tens column and a ones column.

Tens	Ones
1	3

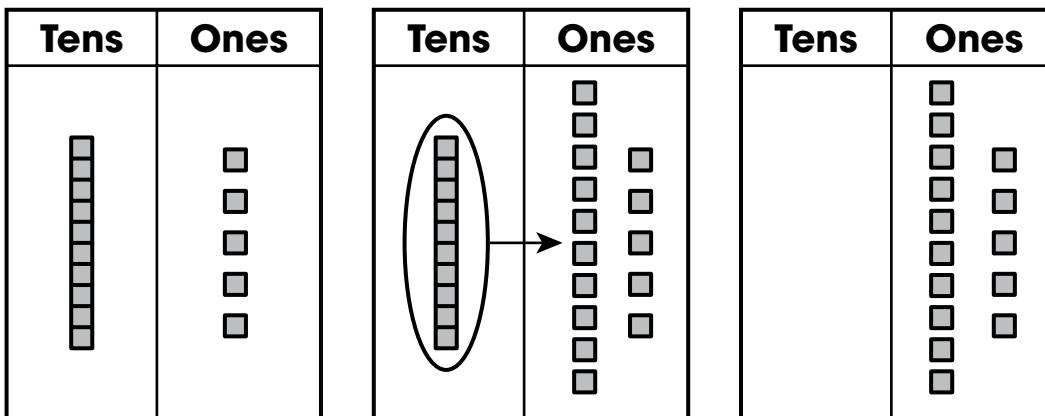
Cut the sticky note in half. Place the right half under the ones column of the problem. Place the left half above the tens column of the problem.

Tens	
1	

Tens	Ones
3	7
+	6
	Ones 3

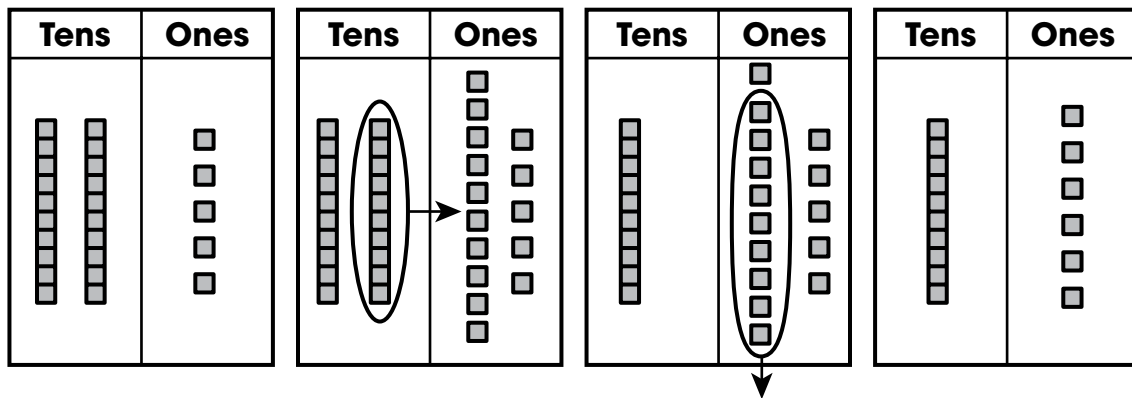
Add the tens column to complete the problem, showing that $37 + 6 = 43$. Gradually fade out the use of cutting the sticky note and placing the two halves to using the information on the sticky note to record the ones value in the answer and the tens value above the tens column. Transition next to leaving the column labels off the sticky note. Finally, omit the column labels on the problem and use the sticky note as scratch paper.

- Ask students to use a regrouping strategy to add a one-digit number to a two-digit number.
- **Subtract a one-digit number from a two-digit number with regrouping.**
- Demonstrate that 15 (1 ten and 5 ones) can also be represented as 15 ones. First, place 1 tens rod and 5 unit cubes on a place-value mat. Next, exchange the tens rod for 10 unit cubes and place all the unit cubes under the ones column. Continue to demonstrate with numbers such as 24 and 37.



4.A.1 Operations and Algebraic Thinking

- Use a place-value mat and base ten blocks to teach subtraction with regrouping. Present the problem $25 - 9 = \underline{\quad}$. Represent 25 on a place-value mat with base ten blocks. Present the dilemma of needing to take away 9 ones and having only 5. Demonstrate exchanging a tens rod for 10 unit cubes. Demonstrate removing (subtracting) 9 from the 15 unit cubes. Count the remaining rods and blocks to find the solution to the problem. Complete the number sentence: $25 - 9 = 16$.



- Ask students to subtract a one-digit number from a two-digit number with regrouping using a place-value mat and base ten blocks.
- Use oversize grid paper with a tens column and a ones column labeled to teach subtraction of a one-digit number from a two-digit number with regrouping. For example, present the problem $72 - 5 = \underline{\quad}$. Next, present the dilemma of having only 2 ones and needing to take away 5. Demonstrate regrouping. Finally, subtract the ones column and the tens column to find the solution of the problem. Complete the number sentence: $72 - 5 = 67$.

Tens	Ones
7	2
-	5

Tens	Ones
6	$10 + 2 = 12$
-	5

Tens	Ones
6	12
-	5
6	7

- Ask students to use a grid and a regrouping strategy to subtract a one-digit number from a two-digit number.

4.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 4.N.4.a—Count by 2s, 5s, and 10s with numbers, models, or objects up to 50.

MAE 4.N.1.a—Identify representations of whole numbers up to 100.

Key Terms

groups, multiply, ones, place-value, product, same, tens

Additional Resources or Links

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-e-lesson-14>

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-f-lesson-19>

4.A.1 Operations and Algebraic Thinking

4.A.1.b

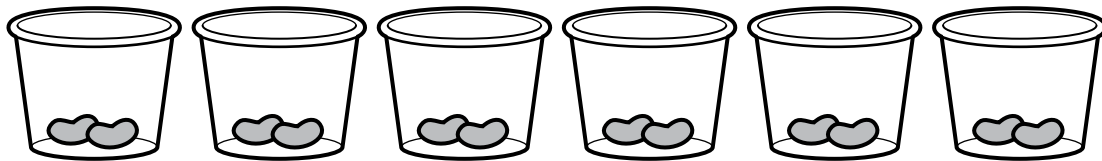
Multiply up to a four-digit whole number by a one-digit whole number and multiply a two-digit whole number by a two-digit whole number, using strategies based on place-value, properties of operations, and algorithms.

Extended: Multiply 2s, 5s, and 10's by a single-digit number with a maximum product of 100.

Scaffolding Activities for the Extended Indicator

☐ Multiply 2 by a single-digit number.

- Explain that 6×2 means 6 groups of 2. Use manipulatives in groups of 2 to model 6×2 . Demonstrate finding the total (product) by counting the manipulatives or skip counting by twos and record the answer, $6 \times 2 = 12$.

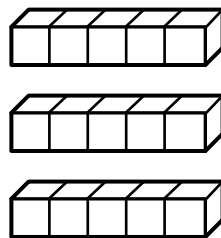


Next, write the expression 2×6 and indicate that this math expression means 2 groups of 6. Show the same manipulatives in 2 groups of 6. Count the manipulatives to find the total (product) and record the answer, $2 \times 6 = 12$. Reinforce the idea that the product (answer) was the same for 6×2 and 2×6 . Continue using manipulatives to demonstrate 2 multiplied by a single-digit number when 2 is the first factor and when 2 is the second factor (for example, 2×4 as 2 groups of 4, and 4×2 as 4 groups of 2).

- Ask students to multiply a single-digit number by 2 using manipulatives in groups of 2.

☐ Multiply 5 by a single-digit number.

- Explain that 3×5 means 3 groups of 5. Use manipulatives in groups of 5 to model 3×5 . Demonstrate finding the total (product) by counting the manipulatives or skip counting by fives and record the answer, $3 \times 5 = 15$.



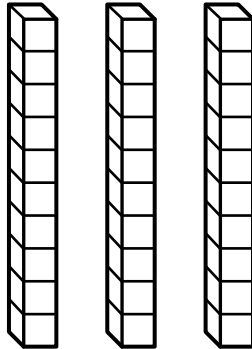
Repeat the process to model the expression 5×3 as 5 groups of 3 using the same manipulatives and resulting in the same answer.

- Ask students to multiply a single-digit number by 5 using manipulatives in groups of 5.

4.A.1 Operations and Algebraic Thinking

□ Multiply 10 by a single-digit number.

- Explain that 3×10 means 3 groups of 10. Use manipulatives in groups of 10 to model 3×10 . Demonstrate finding the total (product) by counting the manipulatives, skip counting by tens, or using repeat addition and record the answer, $3 \times 10 = 30$.



When appropriate, describe the relationship between multiples of 10 times a single-digit number (i.e., know that $10 \times 3 = 30$ based on notions of place-value).

- Ask students to multiply a single-digit number by 10 using manipulatives in groups of 10 or by applying the concept of place-value.

Prerequisite Extended Indicators

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 4.N.4.a—Count by 2s, 5s, and 10s with numbers, models, or objects up to 50.

MAE 4.N.1.a—Identify representations of whole numbers up to 100.

Key Terms

groups, multiply, ones, place-value, product, same, tens

Additional Resources or Links

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-e-lesson-14>

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-f-lesson-19>

4.A.1 Operations and Algebraic Thinking

4.A.1.c

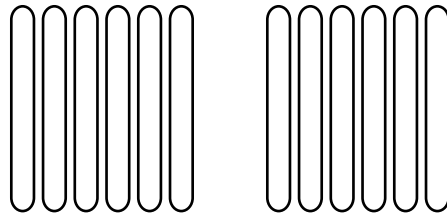
Divide up to a four-digit whole number by a one-digit divisor with and without a remainder using strategies based on place value.

Extended: Identify division equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent division without a remainder, limited to groups up to 20.

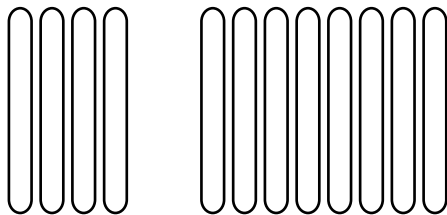
Scaffolding Activities for the Extended Indicator

□ Identify equal-size groups of real-life objects.

- Use manipulatives to show up to 20 objects in groups. For example, present two groups of 6 craft sticks each. Count the number of craft sticks in each group and indicate that the amount is the same for each group; therefore, equal-size groups are shown.



Next, rearrange the same 12 craft sticks into two groups of unequal size. Count the number of craft sticks in each group and indicate that the groups do not have the same amount; therefore, the groups are not of equal size.



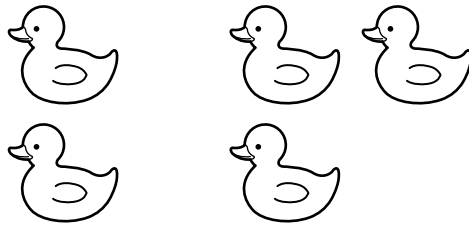
Repeat the process with the same 12 craft sticks, making three groups of equal size and then three groups of unequal size, as well as four groups and six groups, since 12 can be divided by all those numbers evenly. Continue to demonstrate using a variety of objects and varying the total number of objects from 2 to 20.

- Ask students to identify whether two groups of real-life objects are of equal size. For example, present a group of 3 markers and a group of 5 markers and ask students whether equal-size groups are shown.

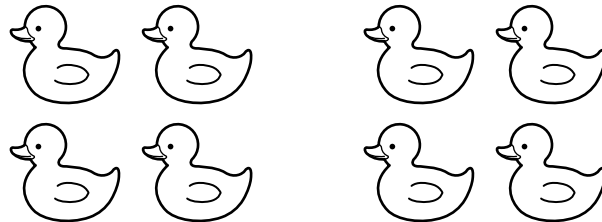
4.A.1 Operations and Algebraic Thinking

- Ask students to identify equal-size groups of real-life objects. For example, present groups of objects as shown and ask which option has equal-size groups. Students should determine that option B has equal-size groups.

Option A

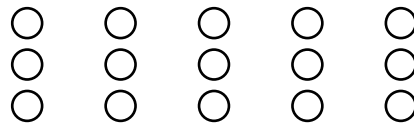


Option B

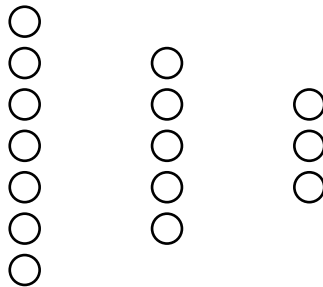


□ Identify equal-size groups in a drawing.

- Use drawings to show up to 20 objects in groups. For example, present circles in groups as shown. Count to find the size of each group and determine that the groups are of equal size.



Draw the same number of circles in three groups to show groups of different sizes. Count how many circles are in each group and determine that the groups are not of equal size.

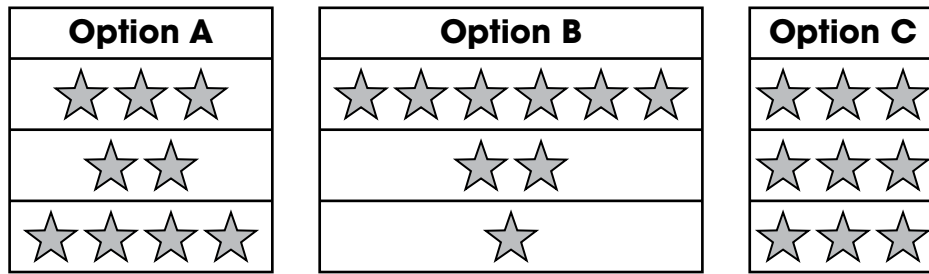


Continue to demonstrate with a variety of drawings, using shapes or tally marks, to show groups of equal sizes and groups of different sizes.

- Ask students to identify whether two groups in a drawing are of equal size. For example, present a drawing of 6 triangles in one group and 3 triangles in another group and ask students whether equal-size groups are shown.

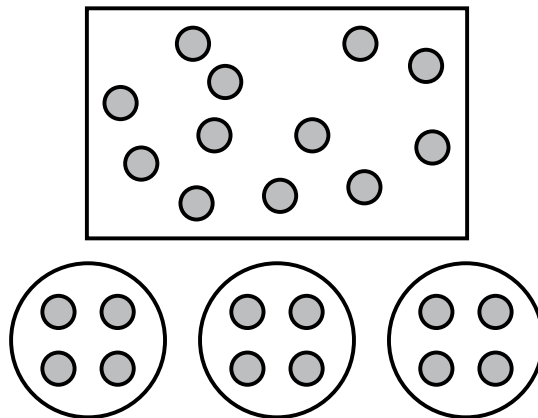
4.A.1 Operations and Algebraic Thinking

- Ask students to identify equal-size groups of objects in a drawing. For example, show students the following figure and ask which option has groups of equal size. Students should determine that option C has equal-size groups.



- Identify a division equation that represents a set of objects sorted into groups of equal size.

- Use manipulatives to show a group of 12 objects. Also show 12 objects in 3 smaller groups of equal size. Explain that both displays show the same number of objects.



Explain that we are going to use the math operation of division to show how many objects are in each group. Having the objects placed into equal groups, will help us to determine. Refer to the set of 12 objects that are arranged in 3 equal groups of 4 and ask the following questions. How many total objects are there? How many groups are there? How many objects are in each group? Demonstrate how to use this information to fill in the template for the division equation.

$$\begin{array}{c}
 \underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \\
 \swarrow \quad \downarrow \quad \searrow \\
 \text{How many} \quad \text{How many} \quad \text{How many} \\
 \text{total objects?} \quad \text{groups?} \quad \text{objects in} \\
 \hspace{10em} \text{each group?}
 \end{array}$$

Repeat the same process with other examples of objects arranged in groups of equal size.

- Ask students to identify the division equation for images of objects in equal groups. Be sure to practice all division equations of groups up to 20.

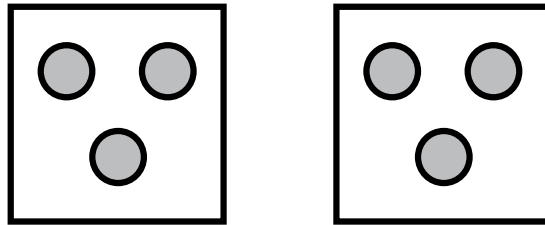
4.A.1 Operations and Algebraic Thinking

□ Use groups of equal size to represent a division equation.

- Write the equation $6 \div 2 = 3$ on the template indicating the total number of objects, how many groups there are, and how many objects are in each group.

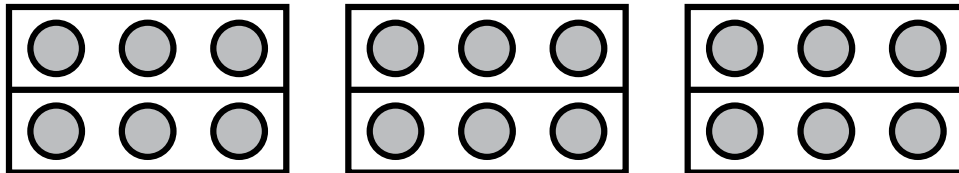
$$\begin{array}{ccc} \frac{6}{/} & \div & \frac{2}{|} = \frac{3}{\backslash} \\ \text{How many} & & \text{How many} & & \text{How many} \\ \text{total objects?} & & \text{groups?} & & \text{objects in} \\ & & & & \text{each group?} \end{array}$$

Demonstrate dividing the total number of objects into two groups of 3. Then demonstrate counting how many objects are in each group.



Do this for several equations with up to 20 objects total. Then progress to working through the division equation without the use of the template. Demonstrate the process with objects and drawings of objects.

- Ask students to use groups of equal size to represent division equations that are written on a template. Then progress to division equations that are not written on the template.
- Ask students to identify a division equation that matches a given picture. For example, give students the picture shown, with three possible equations to choose from.



- A. $12 \div 6 = 2$
- B. $18 \div 6 = 3$
- C. $8 \div 2 = 4$

4.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicators

MAE 4.N.4.b—Identify numbers 1–20 as odd or even and identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

Key Terms

amount, array, division, division sentence, equal, equation, group, total

Additional Resources or Links

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-b-lesson-4>

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-c-lesson-10>

4.A.1 Operations and Algebraic Thinking

4.A.1.d

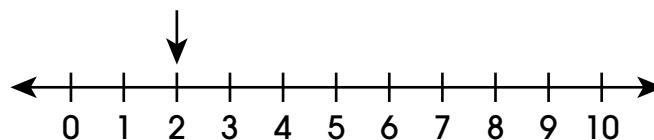
Determine the reasonableness of whole number products and quotients using estimations and number sense.

Extended: Round one- and two-digit whole numbers to estimate two-digit products.

Scaffolding Activities for the Extended Indicator

□ Round a single-digit number to 0 or 10 using a number line.

- Discuss the concept of rounding. Explain that sometimes an exact number is not needed, only an estimate or about how much is needed. Provide relevant real-world examples of using an estimate or approximate number.
- Use a number line to demonstrate that the location of a single-digit number on a number line can be closer to 0 or 10.

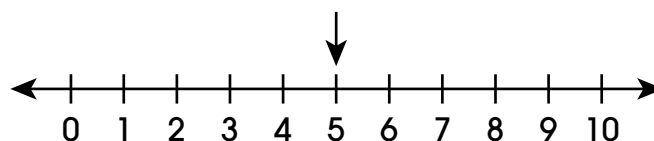


The number 2 is closer to 0 than 10, so it rounds to 0.



The number 7 is closer to 10 than 0, so it rounds to 10.

- Explain that the number 5 always rounds to 10, even though it is the same distance from 10 as it is from 0.

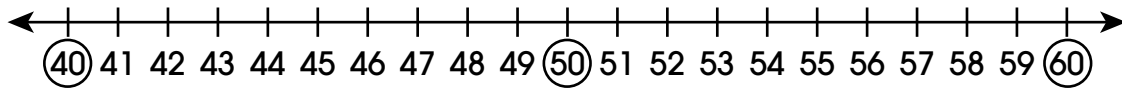


- Ask students to round a single-digit number to 0 or 10 when given a number on a number line.

4.A.1 Operations and Algebraic Thinking

□ Round a two-digit number to the nearest ten using a number line.

- Demonstrate how to find the multiples of 10 on a number line. Any two-digit number that is a multiple of 10 will end with 0.



Relate skip counting by 10 to finding the multiples of 10 on a number line.

- Ask students to identify all the multiples of 10 when provided with a number line with two-digit numbers.
- Ask students to round various two-digit numbers to the nearest ten using a number line. Be sure to include numbers with a 5 in the ones place.

□ Round one- and two-digit whole numbers to estimate two-digit products.

- Demonstrate how to find the multiples of 10 on a number line. Any two-digit number that is a multiple of 10 will end with 0.
- Explain that multiplying a single-digit number by 10 results in a multiple of 10, similar to skip counting by tens. For example, $3 \times 10 = 10 + 10 + 10 = 30$. It may also be helpful to make a table, like the one shown, to demonstrate the pattern of multiplying by multiples of 10.

	$\times 10$	$\times 20$
1	10	20
2	20	40
3	30	60
4	40	80
4	40	80
5	50	100

The table can be developed further for all the single-digit numbers and other multiples of 10.

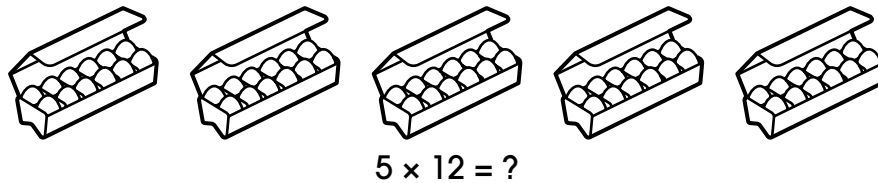
- Use the rounding of factors to estimate the product of two numbers. Demonstrate that a multiplication problem can be estimated by first finding the multiple of 10 closest to one of the given factors. For example, to multiply 21 by 3, first round 21 to 20.

$$21 \times 3 = \underline{\quad} \text{ is close to } 20 \times 3 = 60.$$

Explain that rounding the factor helps to estimate the product because multiplying by a multiple of 10, in this case 20, is easier to do. Show students a variety of multiplication problems involving two-digit numbers multiplied by a one-digit number. Ask students to use rounding to estimate the products. Be sure to include numbers with a 5 in the ones place.

4.A.1 Operations and Algebraic Thinking

- Ask students to estimate multiplication results to the nearest 10. For example, present the following picture.



Then give students three equations for the possible estimate of the product: $5 \times 10 = 50$, $5 \times 20 = 100$, and $10 \times 10 = 100$. Students should identify $5 \times 10 = 50$ as the best estimate for this product.

Prerequisite Extended Indicators

MAE 3.A.1.f—Identify multiplication equations, and use models (e.g., number lines, repeated addition, equal groups, arrays) to represent multiplication, limited to groups up to 20.

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 4.N.4.b—Identify numbers 1–20 as odd or even and identify the factors of 4, 6, 8, 9, 10, 12, 15, and 20.

MAE 4.A.1.b—Multiply 2s, 5s, and 10's by a single-digit number with a maximum product of 100.

Key Terms

estimate, factor, greater, multiple, multiply, nearest, number line, round, skip counting, tens place

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/3/NBT/A/1/tasks/1805>

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB2SUP-A4_NumPIVal-201304.pdf

<https://www.engageny.org/resource/grade-3-mathematics-module-3-topic-f-lesson-19>

4.A.1 Operations and Algebraic Thinking

4.A.1.e

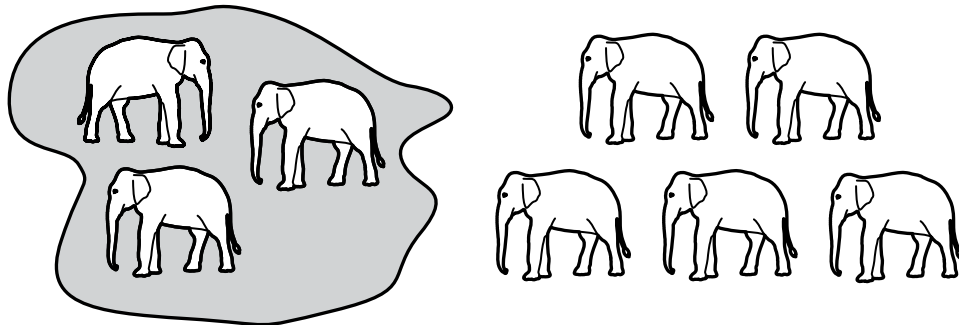
Create a simple algebraic expression or equation using a variable for an unknown number to represent an authentic mathematical situation (e.g., $3 + n = 15$, $81 \div n = 9$).

Extended: Identify an addition or subtraction equation in an authentic mathematical situation using a variable for an unknown, limited to an unknown change or unknown result (e.g., $3 + n = 10$, $12 - 6 = n$).

Scaffolding Activities for the Extended Indicator

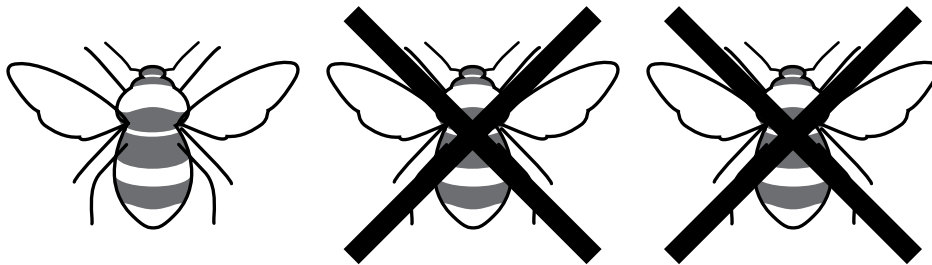
□ Identify an equation that represents an authentic mathematical problem.

- Use a real-world scenario to model an addition equation. For example, there are 3 elephants in the water, and 5 more elephants join them.



This problem represents an addition equation because more elephants join the other elephants in the water. There are now $3 + 5 = 8$ elephants in the water. Remind students that words like “more” are clues that it is an addition problem. Also note that each side of the equal sign represents the same amount, since 3 plus 5 is the same as 8.

- Use a real-world scenario to model a subtraction equation. For example, there are 3 bees sitting on a flower, and 2 bees fly away.



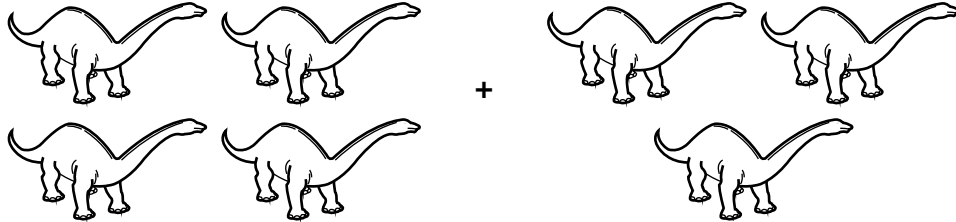
This problem represents a subtraction equation because bees leave the flower. There is now $3 - 2 = 1$ bee on the flower. Remind students that words like “away” are clues that it is a subtraction problem. Again, note that each side of the equal sign represents the same amount, since 3 minus 2 is the same as 1.

- Ask students to determine if a real-world scenario involves addition or subtraction. Remind students to look and listen for the terms “more”, “less”, “add”, “take away,” and so on because those words are clues to the correct operation.

4.A.1 Operations and Algebraic Thinking

□ Identify an equation that represents an authentic mathematical situation with an unknown change or result that is represented using a variable.

- Use a real-world scenario to explain that an addition or subtraction problem can have missing numbers that need to be found using math knowledge. Explain that those missing numbers can be represented with a variable, which is a letter. For example, Jason had 4 dinosaur toys, and he gets 3 more dinosaur toys. How many dinosaur toys does he have now?



The picture shows 4 dinosaur toys being added to 3 dinosaur toys, which can be represented with the equation $4 + 3 = d$. The variable, d , can be used to represent the total number of dinosaurs Jason has now. In this scenario, the variable d is equal to 7.

- Use a variety of objects and real-world scenarios to demonstrate other addition and subtraction equations where one of the numbers is missing and is represented with a variable. Be sure to have a variable representing the unknown number when writing out each equation. Some examples are shown.

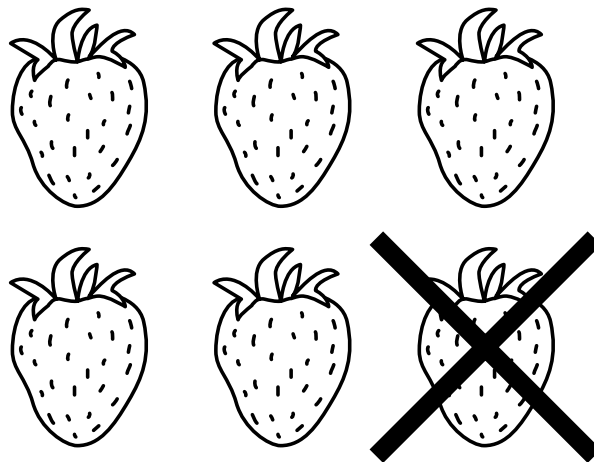
$$5 + b = 8$$

$$7 - 2 = z$$

$$y + 6 = 13$$

$$4 - x = 0$$

- Ask students to identify an equation, using a variable, that represents a given scenario or picture. For example, Sheree has 6 strawberries in her lunchbox and eats 1 of them. How many strawberries does she have left in her lunchbox?



Students should identify $6 - 1 = y$ and $6 - 1 = 5$ as equations that represent this scenario.

4.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 3.A.1.d—Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

Key Terms

addition, authentic problem, equation, operation, subtraction, variable

Additional Resources or Links

<https://www.engageny.org/resource/grade-1-mathematics-module-1/file/109001>

<http://tasks.illustrativemathematics.org/content-standards/1/OA/D/8/tasks/4>

4.A.1 Operations and Algebraic Thinking

4.A.1.f

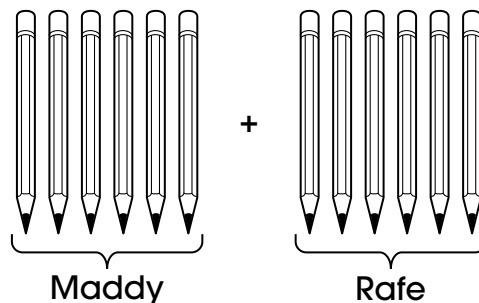
Solve one- and two-step authentic problems using the four operations including interpreting remainders and the use of a letter to represent the unknown quantity.

Extended: Solve one-step authentic problems involving addition and subtraction and including the use of a letter to represent an unknown quantity, limited to two-digit addends and minuends.

Scaffolding Activities for the Extended Indicator

- **Solve one-step authentic addition or subtraction problems using visual models and a letter to represent an unknown quantity.**
 - Use manipulatives or drawings to demonstrate creating a visual model that represents an authentic addition or subtraction problem. Present the authentic scenario and figure shown below. Demonstrate identifying that this is an addition equation. Demonstrate creating an equation to solve with the use of a letter as the unknown quantity. Write $6 + 6 = p$. Demonstrate counting or adding $6 + 6$ to find the value of p , which is a total number of 12 pencils.

Maddy has 6 pencils. Rafe has 6 pencils.
How many pencils do they have altogether?



Continue to demonstrate solving a variety of addition and subtraction problems using visual models that represent authentic scenarios, using a letter to represent the unknown quantity.

- Ask students to identify the correct model for a one-step real-world addition problem when given two or more choices of models. Repeat for a subtraction problem.
- Ask students to solve a one-step authentic addition problem using a visual model and a letter to represent an unknown quantity. Repeat for a subtraction problem.

4.A.1 Operations and Algebraic Thinking

□ Solve one-step authentic addition or subtraction problems using equations and a letter to represent an unknown quantity.

- Use a template to create equations that represent authentic addition or subtraction problems. Present the following scenario.

Brett makes 36 muffins for a bake sale.

He sells 20 muffins.

Brett wants to find out how many muffins, m , are left over.

How many muffins remain?

Present a template to demonstrate organizing the information in the real-world problem.

	+		=	
What happened first?		What happened next?		How did it end?

Demonstrate writing 36 in the first box to represent the 36 muffins Brett made. Emphasize that selling the muffins indicates that this is a subtraction problem. Circle subtraction on the template, and then write 20 for the number of muffins sold. Last, model calculating the answer using an appropriate computation method (e.g., standard algorithm, calculator, manipulatives). Identify that the answer to the equation $36 - 20 = m$, is 16.

36	+	⊖ 20	=	16
What happened first?		What happened next?		How did it end?

Continue to demonstrate solving a variety of addition and subtraction problems presented in authentic scenarios using equations and a variable to represent the unknown quantities.

- Ask students to identify the correct equation that represents a one-step addition problem when given an authentic scenario and a choice of two or more equations. Repeat for a one-step subtraction problem.
- Ask students to solve a one-step authentic addition problem using an equation when given a scenario and a letter to represent the unknown quantity. Repeat for a one-step subtraction problem.

4.A.1 Operations and Algebraic Thinking

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 3.A.1.d—Solve one-step authentic addition and subtraction problems using the digits 0–9, limited to problems with an unknown change or unknown result.

Key Terms

add, difference, equation, subtract, sum, quantity, variable

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/968>

<https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-d-overview/file/77296>

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Mathematics—Grade 4

Geometry

4.G.1 Shapes and Their Attributes

4.G.1.a

Identify, create, and describe points, lines, line segments, rays, angles, parallel lines, perpendicular lines, and intersecting lines.

Extended: Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

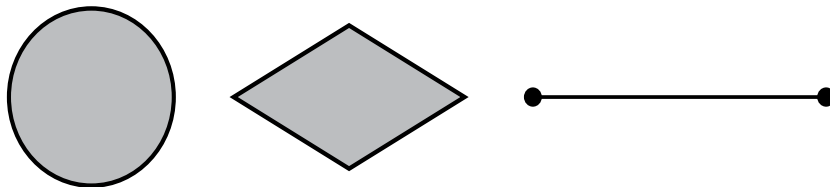
Scaffolding Activities for the Extended Indicator

☐ Identify points, line segments, lines, and rays.

- Explain that a point is an exact location in space and is represented by a “dot.” Demonstrate placing a point on a piece of paper.
- Explain that a line segment is a straight row of points that begin at one point and end at another point. Demonstrate drawing a line segment by making a row of dots and explain that it is represented by drawing a solid line from one point to another. Demonstrate line segments in various orientations.



- Ask students to identify a line segment from a group of geometric figures.



- Ask students to identify the endpoints of a line segment.

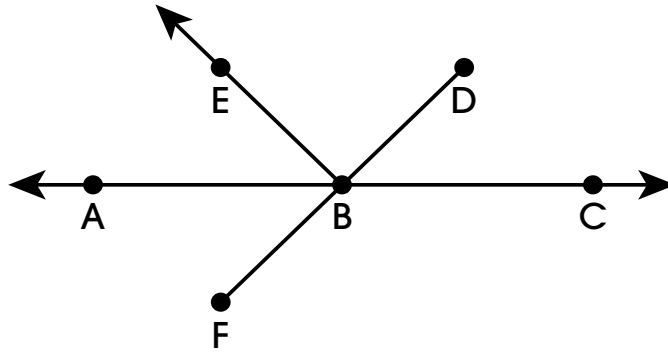


- Ask students to identify line segments represented in the classroom (edges of books or desks, pencils, rulers, etc.)
- Explain that a line is a straight row of points that goes on forever in both directions. Explain that it is represented by drawing a line segment with arrows on each end to indicate that it goes on indefinitely in both directions. Demonstrate lines in various orientations.

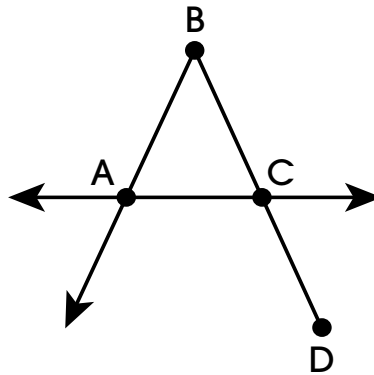


4.G.1 Shapes and Their Attributes

- Explain that a ray is a straight row of points that begin at one point and goes on forever in the other direction. Explain that it is represented by drawing a line segment with an end point at one end, and an arrow on the other end, as shown with ray BE below.

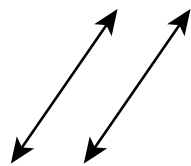


- Continue identifying the points, line segments, line, and ray on the given drawing and others.
- Ask students to identify points, line segments, lines, and rays from a given, labelled drawing.

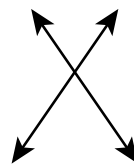


□ Identify parallel lines and intersecting lines.

- Explain the difference between parallel lines and intersecting lines. Parallel lines never touch and are always the same distance apart. Intersecting lines touch or cross each other.



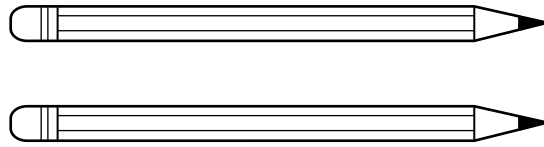
parallel lines



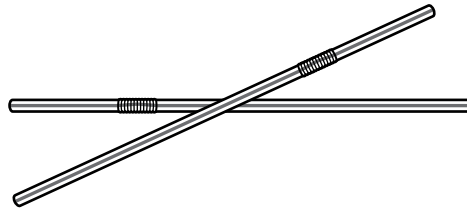
intersecting lines

4.G.1 Shapes and Their Attributes

- Use objects (e.g., pencils, pipe cleaners, spaghetti noodles, straws) to represent parallel lines and emphasize that parallel lines are always the same distance apart and never touch. (Note: The objects suggested resemble line segments more than lines, but they are used here and in the next bullet as models to represent lines.)



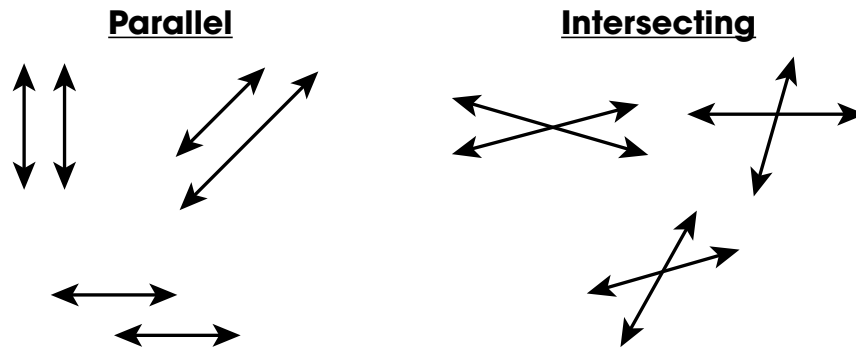
- Use objects (e.g., pencils, pipe cleaners, spaghetti noodles, straws) to represent intersecting lines and emphasize that intersecting lines touch or cross each other.



parallel lines

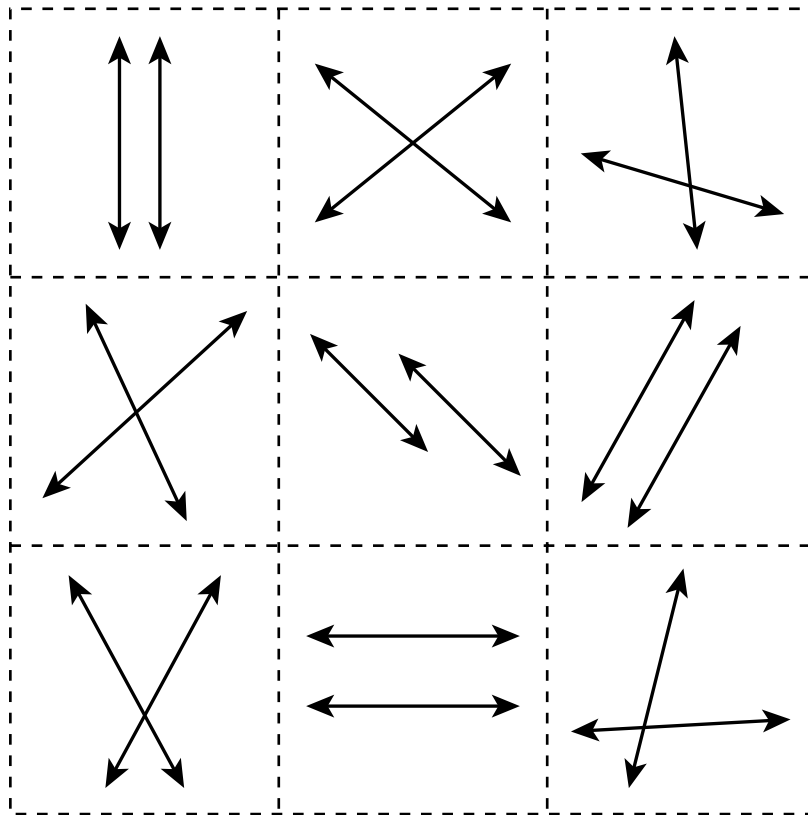
intersecting lines

- Ask students to use objects to represent parallel lines.
- Ask students to use objects to represent intersecting lines.
- Show images of parallel and intersecting lines and ask students to identify lines that are parallel and lines that are intersecting.



4.G.1 Shapes and Their Attributes

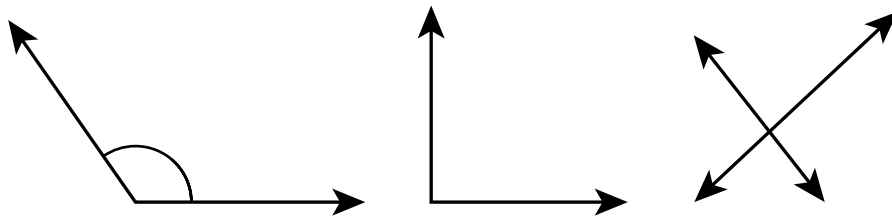
- Ask students to sort pairs of parallel and intersecting lines in a two-column chart with one column labelled parallel and one column labelled intersecting.



- Ask students to identify a pair of lines as parallel or intersecting given a drawing of two lines.

□ Identify angles.

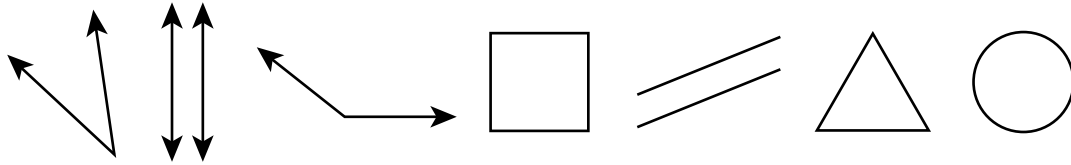
- Explain that angles form where two rays share a common endpoint and where two lines or line segments intersect. Sometimes angles are marked with a symbol (indicate the arc symbol).



- Explain that angles can be found in real life. Model identifying various angles around the classroom. Most shapes have angles, so any shape found in the classroom can be used to demonstrate finding angles (e.g., corner of a book or door, where the floor meets the wall).

4.G.1 Shapes and Their Attributes

- Model identifying which figures have angles.



- Ask students to identify figures with angles.

Prerequisite Skills

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

Key Terms

angle, endpoint, indefinitely, intersect, line, line segment, location, parallel, point, rays

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/4/G/A/1/tasks/1263>

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-overview>

4.G.1 Shapes and Their Attributes

4.G.1.b

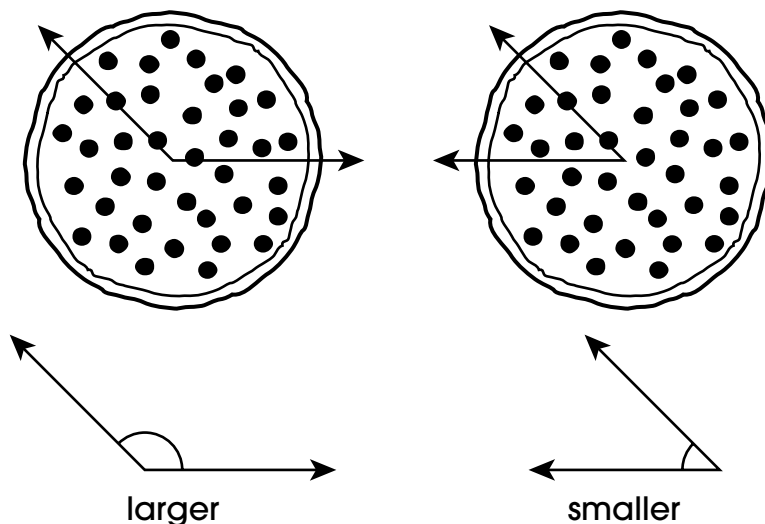
Justify the classification of angles as acute, obtuse, or right.

Extended: Classify angles as acute, obtuse, or right.

Scaffolding Activities for the Extended Indicator

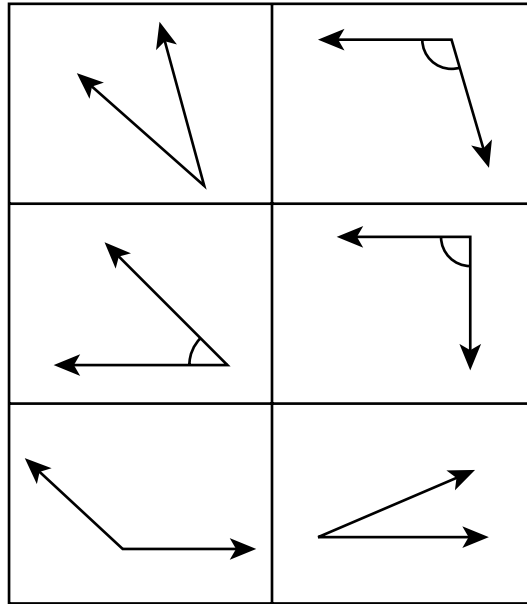
□ Compare larger and smaller angles.

- Demonstrate and explain that the size of angles can be compared by looking at the distance between the two rays that form the sides of each angle. When the rays are close together it is a small angle, and when the rays are farther apart, it is a large angle.
- Explain that this can be demonstrated by putting the angle at the center of a circle and noting how much of the circle the angle takes up. Think of it like slices of pizza—the larger angle makes a bigger slice.



4.G.1 Shapes and Their Attributes

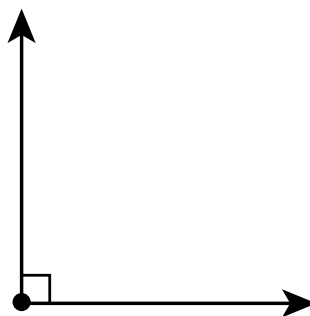
- Use manipulatives to compare angles. For example, use two strips of paper attached at the end of each piece with a brass paper fastener, bendable straws, or pipe cleaners to model comparing two angles to determine which angle is larger or smaller.



- Ask students to compare two angles and identify which angle is larger or smaller.

□ Identify a right angle.

- Present a diagram showing a right angle. Explain that a right angle is a special angle where the two rays make a square corner. Point out the two rays and the point where they meet. Explain that sometimes right angles are marked with a square symbol that represents the right angle.



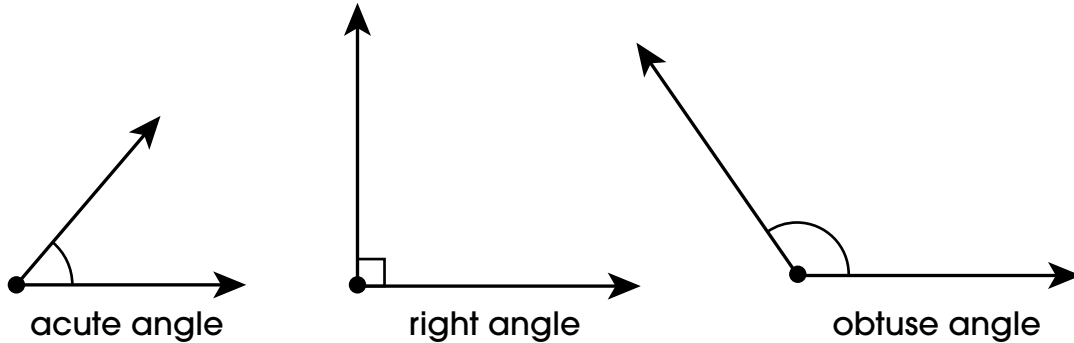
right angle

- Demonstrate right angles using authentic examples (e.g., hands of a clock at nine o'clock, corner of a book cover, piece of paper, desk, door).
- Ask students to determine if an angle constructed by the teacher is a right angle.
- Ask students to construct right angles using manipulatives such as popsicle sticks, straws, pipe cleaners, etc.

4.G.1 Shapes and Their Attributes

□ Classify angles as acute, obtuse, or right.

- Present drawings of an acute angle, a right angle, and an obtuse angle. Emphasize that the right angle has two rays that form a square corner where they meet. Explain that an acute angle has two rays that are closer together than those that form a right angle. Explain that an obtuse angle has two rays that are farther apart than those that form a right angle.



- Compare angles by using both diagrams and manipulatives such as craft sticks, straws, pipe cleaners, etc. to demonstrate that acute angles are smaller than right angles. In the same way, demonstrate that obtuse angles are larger than right angles. Use manipulatives and diagrams to present a variety of acute and obtuse angles to compare to a given right angle.
- Provide a chart to students with columns for right, acute, and obtuse angles. Provide students with a variety of picture representations of various right, acute, and obtuse angles. Model sorting the representations into categories.
- Ask students to identify acute angles using picture representations.
- Ask students to identify obtuse angles using picture representations.
- Ask students to identify right angles using picture representations.

Prerequisite Extended Indicator

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

Key Terms

acute, angle, larger, line, line segment, obtuse, right angle, smaller

Additional Resources or Links

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-1>

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-2>

4.G.1 Shapes and Their Attributes

4.G.1.c

Justify the classification of two-dimensional shapes based on the presence or absence of parallel and perpendicular lines or the presence or absence of specific angles.

Extended: Classify quadrilaterals based on the presence or absence of parallel and perpendicular lines and the presence or absence of right angles.

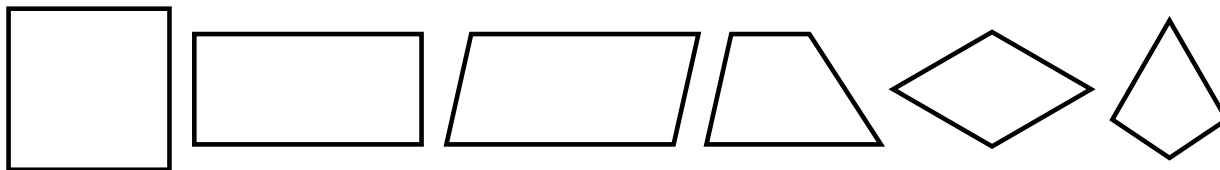
Scaffolding Activities for the Extended Indicator

□ Identify a quadrilateral as a shape with four sides.

- Present a circle, quadrilateral, and triangle. Demonstrate that shapes have different characteristics such as sides and vertices or neither. Explain that shapes that have four sides are called quadrilaterals.



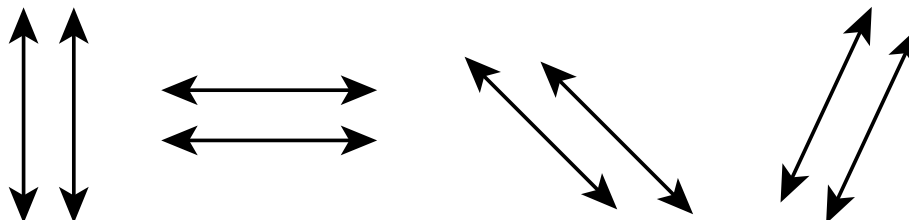
- Demonstrate that quadrilaterals have four sides but that the shapes may not all look the same by presenting quadrilateral cutouts for tactile exploration.



- Ask students to identify quadrilaterals from a group of various shapes.
- Ask students to sort a group of various shapes into two groups: quadrilaterals and not quadrilaterals.

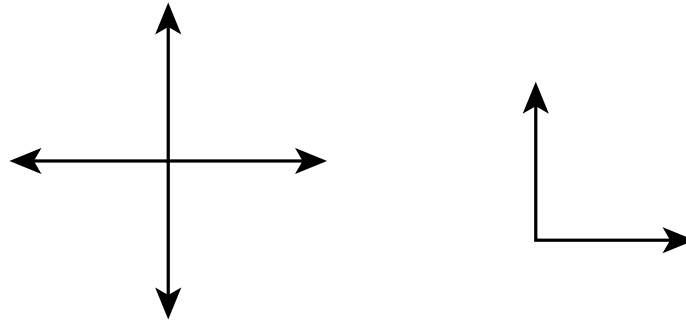
□ Identify parallel and perpendicular lines in a quadrilateral.

- Using diagrams and manipulatives, present examples of parallel lines. Explain that parallel lines never meet. Demonstrate that parallel lines can have various orientations.



4.G.1 Shapes and Their Attributes

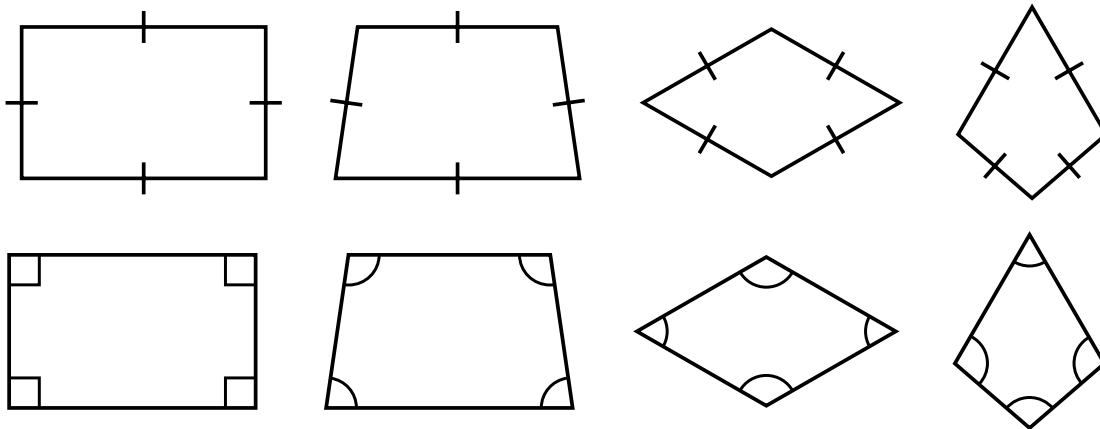
- Using diagrams and manipulatives, present examples of perpendicular lines. Explain that perpendicular lines meet at one point and form right angles. Demonstrate that perpendicular line segments can make a corner.



- Explain that quadrilaterals have four sides and four angles. Demonstrate that parallel and perpendicular line segments can form a quadrilateral using diagrams or manipulatives. Using diagrams and manipulatives, demonstrate that pairs of parallel line segments can form a quadrilateral.
- Ask students to identify parallel lines when given a quadrilateral drawing.
- Ask students to identify perpendicular lines when given a quadrilateral drawing.

□ Identify right angles and non-right angles in a quadrilateral.

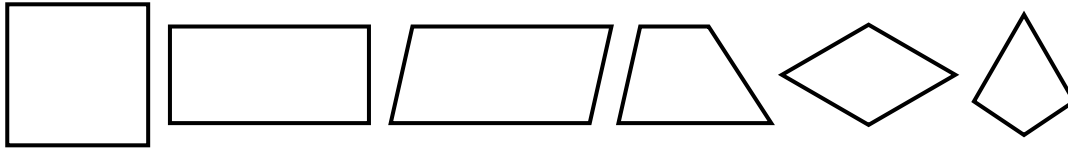
- Remind students that quadrilaterals have four sides and four angles. Use diagrams to show the sides and angles of a quadrilateral.



- Ask students to identify angles on various quadrilaterals.

4.G.1 Shapes and Their Attributes

- Explain that quadrilaterals can have right angles and non-right angles. Remind students that right angles in quadrilaterals will make a square corner, and all other angles are not a right angle. Construct various four-sided shapes using manipulatives such as paper strips, craft sticks, pipe cleaners, etc. Work with students to determine which angles on the quadrilateral are right angles or non-right angles.



- Ask students to identify right angles in a quadrilateral.
- Ask students to identify non-right angles in a quadrilateral.

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

MAE 4.G.1.b—Classify angles as acute, obtuse, or right.

Key Terms

angle, line segment, non-right angle, parallel, perpendicular, quadrilateral, right angle, sides

Additional Resources or Links

<https://tasks.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1275>

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Shape-Tool/>

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-d-lesson-15>

4.G.1 Shapes and Their Attributes

4.G.1.d

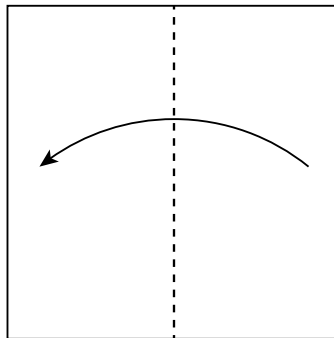
Recognize, draw, and justify lines of symmetry in two-dimensional shapes.

Extended: Identify lines of symmetry in two-dimensional shapes.

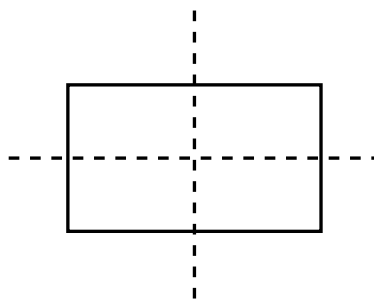
Scaffolding Activities for the Extended Indicator

□ Identify lines of symmetry in folded paper shapes.

- Fold a piece of rectangular paper evenly in half. Indicate that each section is one-half. Cut the paper along the fold line and place the halves on top of each other to demonstrate that the pieces are the same size and the same shape. Explain that because the two pieces are the same size and shape, the fold line separated the rectangle into two equal pieces, called halves.



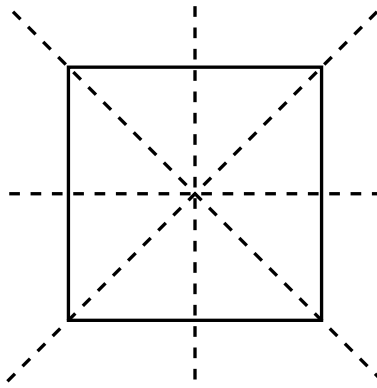
- Explain that rectangles have 2 lines of symmetry. Fold and identify each line of symmetry with 2 rectangles, each folded along a different line of symmetry. Demonstrate how the two halves match when folded together, thus representing a line of symmetry. Demonstrate a counterexample, by folding a rectangle diagonally. Show how the two halves do not match and explain that this is not a line of symmetry.



2 lines of symmetry

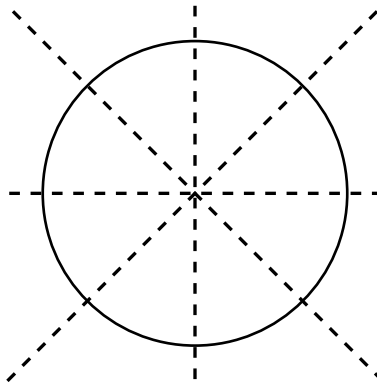
- Explain that squares have 4 lines of symmetry. Fold and identify each line of symmetry with 4 squares, each folded along a different line of symmetry. Demonstrate how the two halves match when folded together, thus representing a line of symmetry.

4.G.1 Shapes and Their Attributes



4 lines of symmetry

- Explain that circles have many lines of symmetry. Fold and identify several lines of symmetry with circles. Demonstrate how the two halves match when folded together, thus representing a line of symmetry. Repeat process with various two-dimensional shapes.



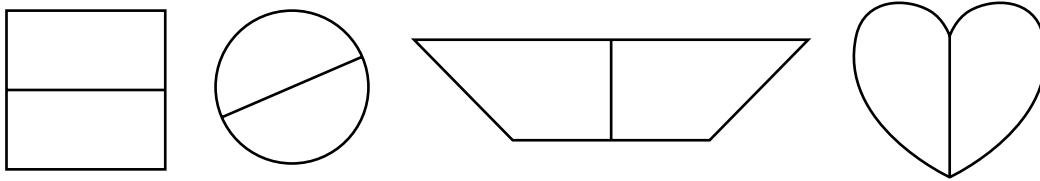
infinite lines of symmetry

- Ask students to demonstrate their understanding by sorting folded two-dimensional shapes into two categories to identify folds along lines of symmetry and folds not along lines of symmetry.

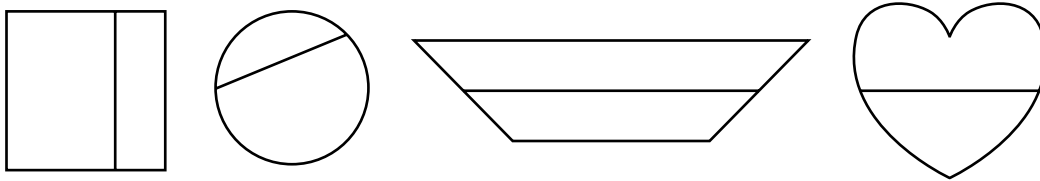
4.G.1 Shapes and Their Attributes

□ Identify lines of symmetry drawn on shapes.

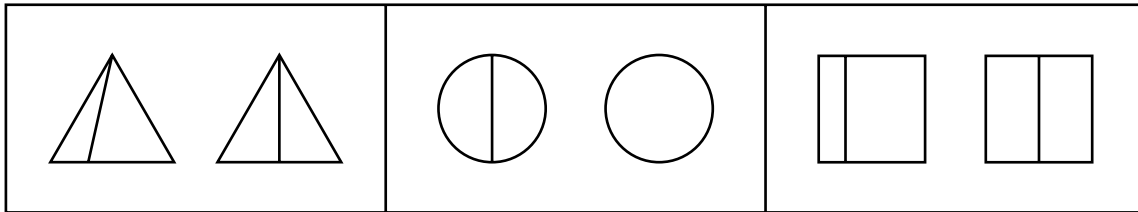
- Show examples of two-dimensional shapes that are separated into halves by a line of symmetry.



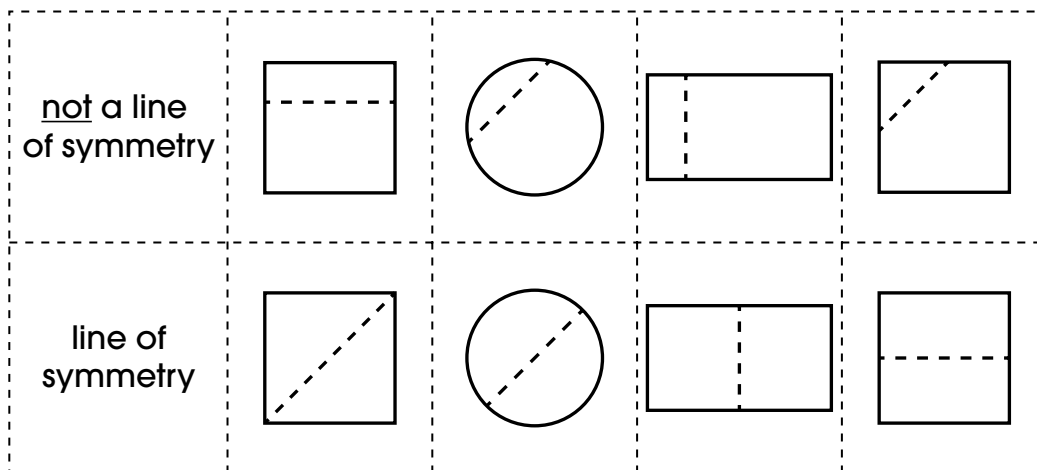
- Show similar nonexamples in which the line drawn does not separate the shape into halves.



- Ask students to identify which shape in a set of two shapes is separated by a line of symmetry.

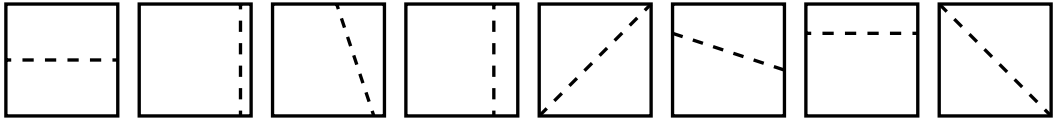
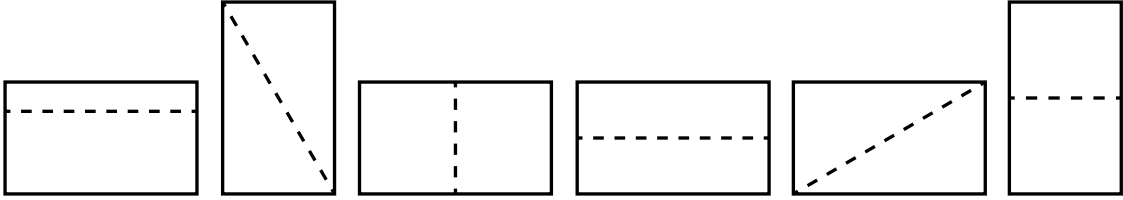
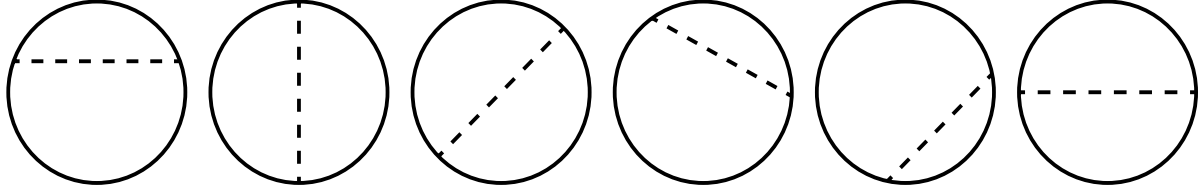


- Use a similar chart with shapes presorted or ask students to help you sort similar shapes. Explain that the line drawn on the shapes is like the fold line. If the line divides the shape into two equal pieces (in half), then it is a line of symmetry. If the line does not divide the shape into halves, then it is not a line of symmetry. Repeat process with various two-dimensional shapes.



4.G.1 Shapes and Their Attributes

- Ask students to identify all the shapes showing lines of symmetry in each row of the table as shown.

Squares	
Rectangles	
Circles	

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

Key Terms

equal, halves, line, shape, size, sort, symmetry

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/4/G/A/3>

<https://www.insidemathematics.org/sites/default/files/materials/quilt%20making.pdf>

4.G.2 Measurement

4.G.2.a

Identify and use the appropriate tools, operations, and units of measurement, both customary and metric, to solve authentic problems involving time, length, weight, mass, and capacity.

Extended: Identify and use the appropriate units of measurement to solve authentic problems involving time, length, weight, and liquid volume, using customary units.

Scaffolding Activities for the Extended Indicator

☐ Identify appropriate units of measurement, using customary units.

- Explain to students that the length of what is being measured determines the unit of measurement that should be used to describe the length. Inform students that length can be measured in inches, feet, yards, and miles. Explain that inches are used to measure small lengths and yards are used to measure large lengths.
- Hold a paper clip next to a ruler in front of the students. Remind students that a ruler is equal to 12 inches, which equals one foot. Explain that since the paper clip is smaller than the ruler, it makes more sense to identify inches as the appropriate unit of measurement. Hold the ruler next to the whiteboard (or another larger object) in the classroom. Explain that since the whiteboard is larger than the ruler, it makes more sense to identify feet as the appropriate unit of measurement. Continue this process with various objects that are smaller and larger than a ruler, determining whether inches or feet is the appropriate unit of measurement.
- Provide students with a T-chart for sorting as shown. Provide students with a list of objects that can be found in the classroom (some smaller and some larger than a ruler) and a ruler. Model using the ruler to determine whether inches or feet would be the appropriate unit of measurement for the object. Demonstrate how to place/write the name of the object in the correct column of the T-chart.

Inches	Feet

paper clip	backpack
door	pen
calculator	desk

4.G.2 Measurement

- Ask the students to identify the appropriate units of measurement for the list of objects and complete the chart.
 - Repeat these steps using measurements of other units of length, time, weight, and liquid volume until all customary units have been covered.
- **Solve authentic problems involving units of measurement.**
- Demonstrate solving authentic problems involving units of measurement in the natural environment throughout the school day. For example, when you are nearing the end of a math lesson, inform students that there isn't a lot of time left in class. Model looking at the clock and determining that since there is less than an hour, minutes is the correct unit of measurement. Using the clock, model how to determine how many minutes are left in the math class. Repeat this modeling process in the natural environment frequently and with various units of measurement (e.g., liquid volume during cooking group, length when walking to the lunchroom).
 - Ask students to solve authentic problems involving units of measurement in the natural environment throughout the school day.
 - Create a problem as shown and display in front of students. Inform students that when solving problems involving units of measurement there are two things that need to be noted, the numbers used to solve the problem and the units in which they are measured. Circle the numbers in the problem. Underline the units of measurement in the problem. Explain that you will be creating a subtraction equation using the numbers 5 and 1 and that the answer will be measured in the unit of cups. Model putting the 5 and 1 in the circles and then solving the problem and writing the numerical answer of 4 in the circle. Model writing the unit of measurement label in the underlined area of the answer template.

Emily is filling her water bottle. It can hold 5 cups of water. It already has 1 cup of water. How many more cups of water does Emily need to fill her water bottle?

$$\bigcirc - \bigcirc = \bigcirc \underline{\hspace{1cm}}$$
$$\textcircled{5} - \textcircled{1} = \textcircled{4} \underline{\text{ cups}}$$

- Use the same template and repeat these steps to model solving authentic problems using various types of units of measurement, using customary units. Involve students by having them identify the numbers, identify the unit of measurement, and solve the problem.
- Ask students to solve authentic problems involving time, length, weight, and liquid volume, using customary units.

4.G.2 Measurement

Prerequisite Extended Indicators

MAE 3.G.3.a—Identify the appropriate tool to use to solve authentic problems involving length, weight, and liquid volume.

MAE 3.G.3.b—Measure the length of an object to the nearest inch.

MAE 3.G.4.b—Solve authentic problems involving addition and subtraction of time intervals to find elapsed time, limited to whole-number hours.

Key Terms

cups, feet/foot, fluid ounces, hours, inches, length, minutes, ounces, pounds, seconds, time, units of measurement, weight

Additional Resources or Links

<https://www.mathlearningcenter.org/educators/free-resources/lessons-publications/bridges-1st-edition-activities/3>

(Measurement, Set D4: Capacity in U.S. Customary Units, pdf)

<https://illuminations.nctm.org/Search.aspx?view=search&type=ls&st=m&gr=3-5>

4.G.2 Measurement

4.G.2.c

Generate simple conversions from a larger unit to a smaller unit within the customary and metric systems of measurement.

Extended: Generate simple conversions from larger units to smaller units, using weeks/days, years/months, hours/minutes, or feet/inches.

Scaffolding Activities for the Extended Indicator

☐ Recognize that measurements can be given in various unit sizes.

- Explain that different types of measurements can be described in various unit sizes. Present the visual model of one year being equivalent to twelve months as shown.

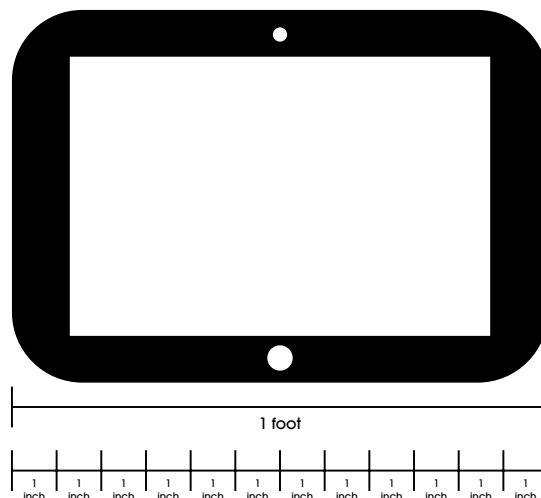


Point to the top row. Explain that the top row represents the larger measurement unit of one year. Point to the bottom row. Explain that the bottom row represents the smaller measurement unit of twelve months. Point to both rows. Explain that the visual shows that twelve months is equal to one year.

- Ask students to point to the larger unit of measurement on a simple conversion model. Ask students to point to the smaller unit of measurement on a simple conversion model.
- Repeat the process using various unit sizes, including weeks/days, years/months, hours/minutes, and feet/inches.

☐ Convert from a larger unit to a smaller one, using a visual model.

- Present the visual model of the tablet to students. Present the following question: “What is the length of the tablet, in inches?”



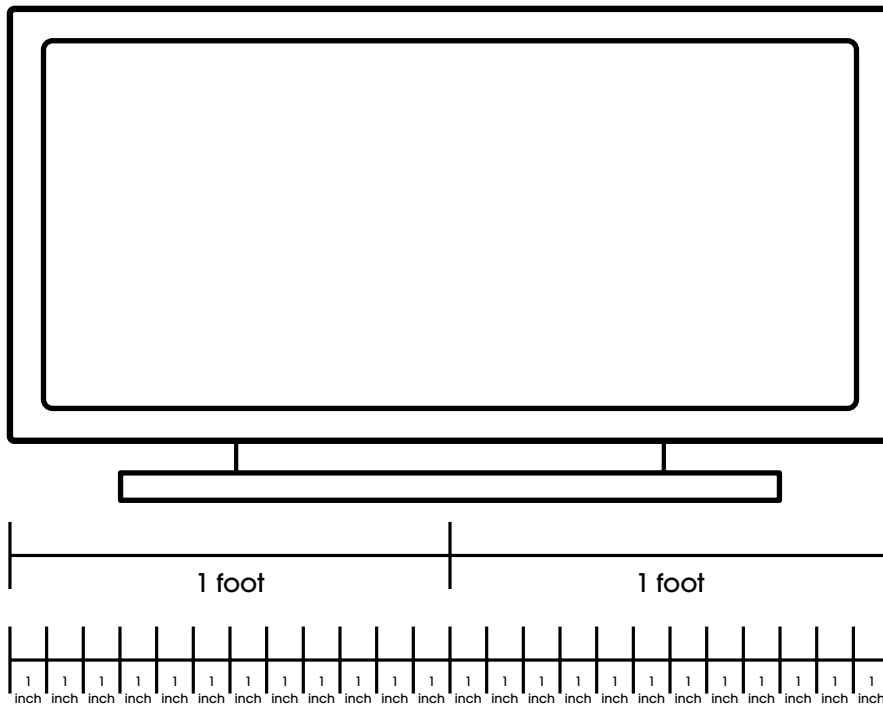
4.G.2 Measurement

Explain to students that it is known that the tablet has a measurement of one foot. Point to the 1-foot line segment.

State that you need to convert the larger unit of feet to a smaller unit of inches to get the answer. Model counting each inch on the line segment below the visual. Explain that one foot is equal to 12 inches, so the length of the tablet is 12 inches. Model writing in 12 inches as shown.

$$\text{tablet} = 1 \text{ foot} = \underline{12} \text{ inches}$$

- Present the following information to students and say, “This television has a length of two feet.”



- Ask students to answer the question “What is the length of the television, in inches?” using the visual model. Indicate the visual model and model counting the number of inches and stating that the television is 24 inches long.
 - Repeat the process using different unit sizes and different simple unit conversions, including weeks/days, years/months, hours/minutes, and feet/inches.
 - Ask students to convert larger units to smaller units using visual models.
- Convert from a larger unit to a smaller one.**
- Present the Conversion Table to students as shown.

4.G.2 Measurement

Conversion Table
1 week = 7 days

- Point to the table and say that one week is equal to seven days. Present the question “How can I figure out how many days are in 2 weeks?” Conversion from a larger unit to a smaller units means we will multiply. Model setting up the equation and working through it as shown.

$$1 \text{ week} = 7 \text{ days}$$

$$2 \text{ weeks} = 2 \times 7 \text{ days}$$

$$2 \text{ weeks} = 14 \text{ days}$$

(Note: Students may use a calculator or multiplication table as needed.)

Repeat process with various simple conversions from larger units to smaller units involving units of measurements.

- Ask students to convert weeks to days, years to months, hours to minutes, and feet to inches.

Prerequisite Extended Indicators

MAE 3.N.1.a—Read, write, and demonstrate whole numbers 1–20 that are equivalent representations, including visual models, standard forms, and word forms.

MAE 3.G.4.b—Solve authentic problems involving addition and subtraction of time intervals to find elapsed time, limited to whole-number hours.

MAE 4.A.1.a—Add and subtract numbers with regrouping, limited to two-digit addends and minuends.

MAE 4.A.1.b—Multiply 2s, 5s, and 10's by a single-digit number with a maximum product of 100.

Key Terms

convert, day, inch, foot, larger unit, minute, month, multiply, second, smaller unit, week

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_272_g_2_t_4.html?open=instructions&from=search.html?qt=ffet

(Note: Java required for website. Most recent version recommended, but not needed.)

<https://www.engageny.org/resource/grade-2-mathematics-module-2-topic-c-lesson-7>

4.G.2 Measurement

4.G.2.d

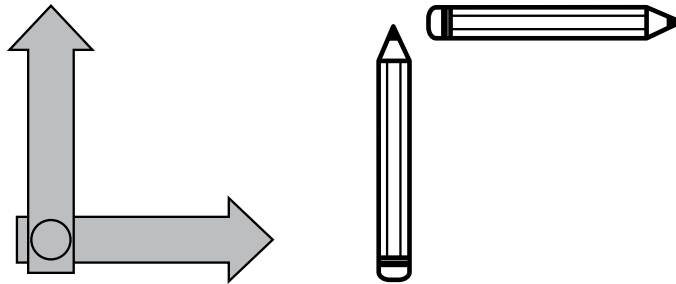
Measure angles in whole number degrees using a protractor and relate benchmark angle measurements to their rotation through a circle (e.g., $180^\circ = \frac{1}{2}$ of a circle).

Extended: Identify benchmark angles of 90° and 180° , and relate those angle measurements to right angles, straight lines, and perpendicular lines.

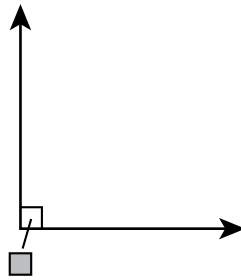
Scaffolding Activities for the Extended Indicator

□ Identify a 90° angle as a right angle.

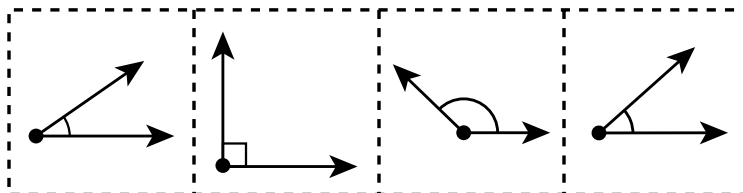
- Demonstrate a 90-degree angle using manipulatives for the two rays. Use cardboard/paper arrows connected by craft sticks, pencils, etc.



- Explain that 90-degree angles are special angles that look like the corners of a square and therefore they are often, but not always, marked with a square at the vertex. This special symbol is used to identify the angle as 90 degrees. Demonstrate the right-angle symbol by putting a small square into the vertex of a right angle formed by manipulatives or drawn.



- Explain that right angles are found in many places in the real world. Demonstrate right angles in the classroom by pointing out the corners of door frames, windows, ceiling and/or floor tiles, cabinet doors, etc.
- Ask students to identify an angle of 90 degrees when presented with at least three angles of various measures.

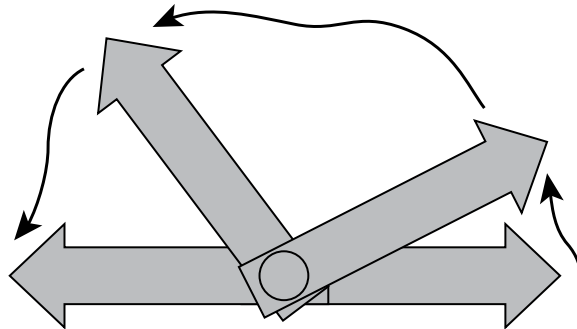


- Ask students to construct a right angle using manipulatives.

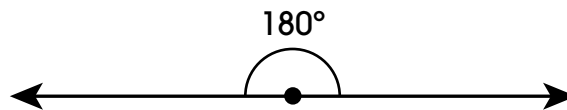
4.G.2 Measurement

□ Identify a 180° angle as a straight line.

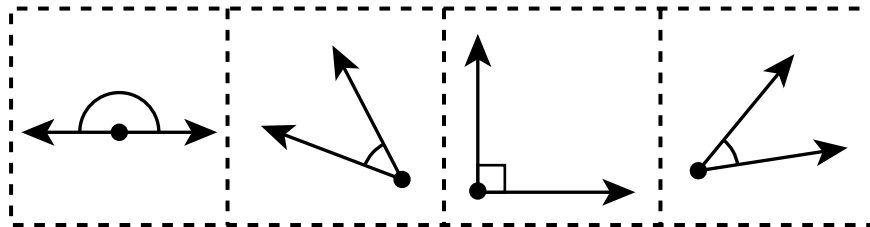
- Using a manipulative, demonstrate that angles can increase in measure from 0 degrees to 180 degrees (e.g., two paper strips connected with a brad fastener, craft sticks, pipe cleaners, pencils).



- Explain that the two rays of a 180-degree angle form a straight line.



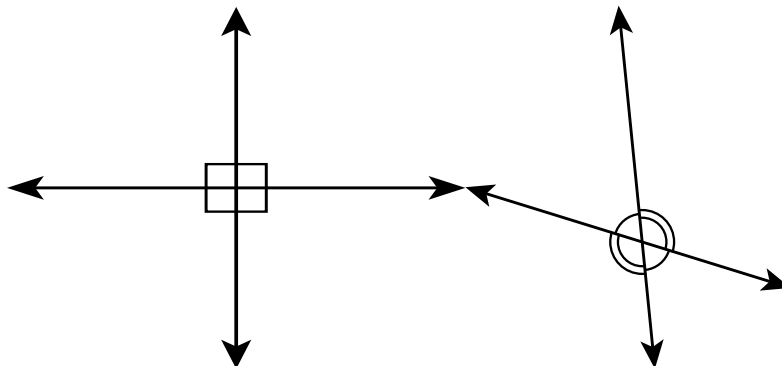
- Ask students to identify a 180-degree angle when presented with different angles.



- Ask students to construct a 180-degree angle with manipulatives.

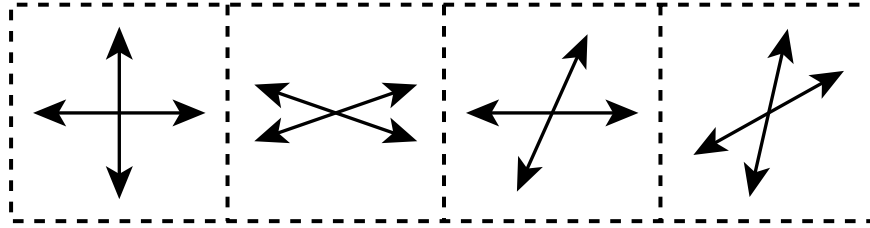
□ Identify perpendicular lines.

- Use manipulatives to demonstrate intersecting lines. Use manipulatives to demonstrate perpendicular lines. Explain that intersecting lines can intersect at different angles other than 90 degrees. Explain that perpendicular lines intersect at one point and form four right angles.
- Contrast the angles formed by perpendicular lines with the angles formed by two intersecting lines that are not perpendicular. Contrast the symbol for 90-degree angles and the symbol for angle measurement on intersecting lines that are not perpendicular.



4.G.2 Measurement

- Ask students to identify perpendicular lines from a group of various intersecting lines.



- Ask students to construct perpendicular lines using manipulatives.

Prerequisite Extended Indicators

MAE 3.G.1.a—Identify two-dimensional shapes, circles, triangles, rectangles, or squares.

MAE 4.G.1.a—Identify points, lines, line segments, rays, angles, parallel lines, and intersecting lines.

MAE 4.G.1.b—Classify angles as acute, obtuse, or right.

Key Terms

angle, degrees, intersecting lines, measurement, perpendicular lines, right angle, straight angle, straight line, symbol

Additional Resources or Links

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-2>

<https://www.engageny.org/resource/grade-4-mathematics-module-4-topic-lesson-3>

<https://curriculum.illustrativemathematics.org/k5/teachers/grade-4/unit-7/lesson-10/lesson.html>

<https://tasks.illustrativemathematics.org/content-standards/4/G/A/2/tasks/1273>

4.G.3 Area and Perimeter

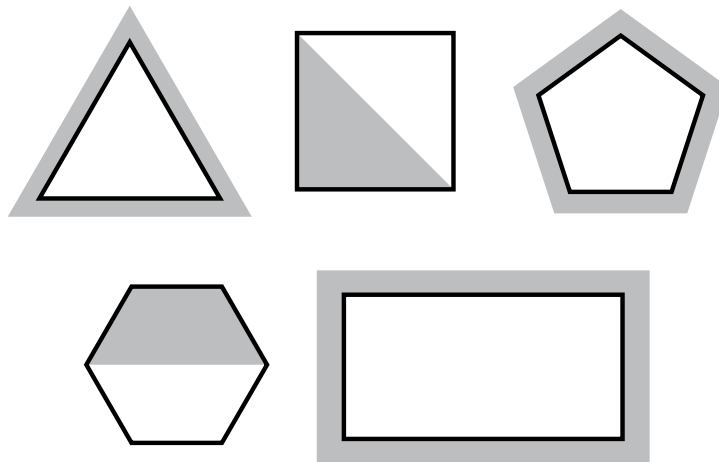
4.G.3.a

Apply perimeter and area formulas for rectangles to solve authentic problems.

Extended: Apply perimeter formulas for rectangles to solve authentic problems.

Scaffolding Activities for the Extended Indicator

- **Identify perimeter as the distance around the outside of a polygon.**
 - Explain that the perimeter is the distance around the outside of a shape. Find various items throughout the classroom that are polygons. For example, a book. Hold up the book and model tracing around the outside of the book. Explain that you are tracing the perimeter of the book.
 - Ask students to take turns tracing the perimeter of different polygons throughout the classroom.
 - Present a large visual model of a rectangle to students. Shade the outside border of the rectangle and explain that this shading represents the perimeter of the rectangle. Present another rectangle to the students. Shade the inside of the rectangle as a non-example of perimeter. Remind students that perimeter is the distance around all the sides of the polygon.
 - Present an assortment of polygons with various shading as shown.

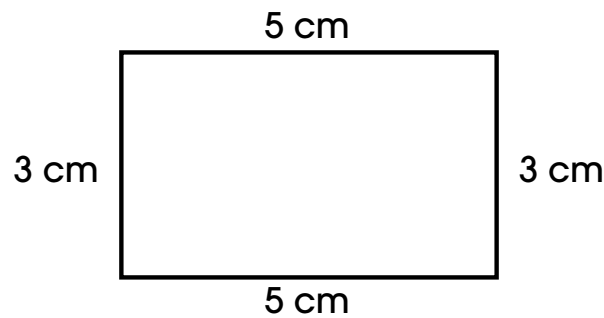


- Ask students to identify which polygons have their perimeter shaded.

4.G.3 Area and Perimeter

□ Use formulas to find the perimeter of rectangles.

- Use a rectangle with the lengths and widths labeled as shown. Explain that each side of the rectangle has a measurement and that they can be added together to find the perimeter.



Model counting each side of the rectangle and state that the rectangle has four sides. To find the perimeter of a rectangle, use the formula where l is the length of the rectangle and w is the width of the rectangle. Model drawing the formula using four blank spaces to represent each side of the rectangle as shown. Then model filling in each blank space with the lengths and widths of the rectangle. Explain that the lengths are the same for opposite sides of rectangles and that the widths are the same for opposite sides. Model solving the addition equation and identifying the unit of measurement. Complete the modeling by stating that the perimeter, or distance around the rectangle, is 16 cm.

$$\text{perimeter} = l + w + l + w$$

$$\text{perimeter} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$\text{perimeter} = \underline{5} + \underline{3} + \underline{5} + \underline{3}$$

$$\text{perimeter} = \underline{16 \text{ cm}}$$

- Inform students that another way to find the perimeter is to use a multiplication formula.

$$\text{perimeter} = 2 \times l + 2 \times w$$

$$\text{perimeter} = \underline{2} \times \underline{5} + \underline{2} \times \underline{3}$$

$$\text{perimeter} = \underline{10} + \underline{6}$$

$$\text{perimeter} = \underline{16 \text{ cm}}$$

Inform students that the repeated addition formula and multiplication formula can both be used to find the perimeter of rectangles.

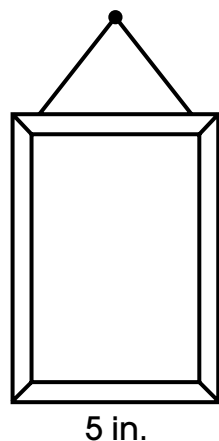
- Ask students to find the perimeters of rectangles of various sizes using the formulas.

4.G.3 Area and Perimeter

- Present an authentic perimeter problem to students. Model using the information presented in the problem and the visual of the rectangle to find the measurements of the sides of the rectangle. Work with students to enter side measurements into the repeated addition formula and solve the problem.

Jenny wants to glue ribbon around a picture frame. She needs to find the perimeter of her picture frame. The picture frame is 5 inches long and 7 inches wide.

How many inches of ribbon does Jenny need to cover the perimeter of the frame?



$$\text{perimeter} = l + w + l + w$$

$$\text{perimeter} = 2 \times l + 2 \times w$$

- Repeat the process with multiple rectangles using both the repeated addition formula and the multiplication formula to find the perimeter.
- Ask students to find the perimeter of rectangles of various sizes in authentic problems by using the repeated addition formula for perimeter.
- Ask students to find the perimeter of rectangles of various sizes in authentic problems by using the multiplication formula for perimeter.

Prerequisite Extended Indicators

MAE 3.A.1.a—Add and subtract without regrouping, limited to maximum sum and minuend of 20.

MAE 3.A.1.h—Multiply 1 and 2 by multiples of 10 with a maximum product of 100.

MAE 3.G.2.a—Find the perimeter of a square or rectangle given the side lengths and a visual model.

Key Terms

distance, formula, length, perimeter, polygon, rectangle, shape, side, width

Additional Resources or Links

<https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-c-lesson-12>

<https://apps.mathlearningcenter.org/geoboard/>

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Mathematics—Grade 4

Data

4.D.1 Data Collection

4.D.1.a

Generate and represent data using line plots where the horizontal scale is marked off in appropriate units—whole numbers, halves, fourths, or eighths.

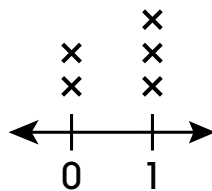
Extended: Identify and compare quantities in line plots, limited to two data points.

Scaffolding Activities for the Extended Indicator

☐ Identify components of a line plot.

- Present a line plot as shown, and explain that a line plot is a way to organize data using a number line. A line plot is a graph that shows how many (the frequency) by using dots or x's above the numbers on a number line. Identify the different components of the line plot, including the number line, the title, the x's showing frequency, and the label. Repeat this for a variety of line plots.

Families with Pets



Number of Pets

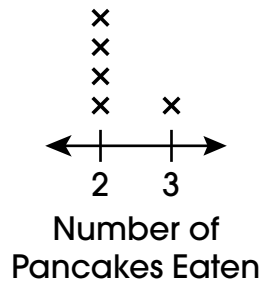
- Ask students to identify the components of a line plot (e.g., title, number line, label, frequency).

4.D.1 Data Collection

□ Interpret information in a line plot using two data points.

- Present the line plot shown and explain that the line plot is about a pancake party. The numbers below the line plot show how many pancakes were eaten, and the x's represent how many people ate that number of pancakes. Model interpreting the x's on the line plot. This line plot shows that four people each ate two pancakes and one person ate three pancakes. More people ate two pancakes than ate three pancakes.

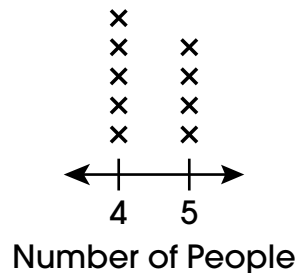
Pancake Party



Continue to model interpreting information on a variety of line plots, with a focus on identifying the frequency and comparing the two data points.

- Ask students to interpret information on a line plot. For example, present the line plot shown and ask questions such as “How many families have a household size of four?” or “Do more families have a household size of four or five?”

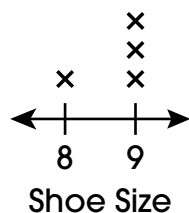
Household Size



- Ask students to compare information on multiple line plots. For example, present the line plots shown and ask questions such as “Which line plot shows two people with a shoe size of eight?” or “Which line plot shows more people with a shoe size of eight?”

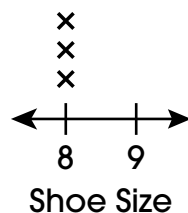
A

Aimee's Friends' Shoes



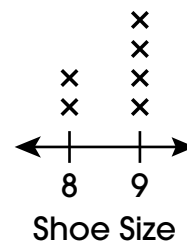
B

Aimee's Friends' Shoes



C

Aimee's Friends' Shoes



4.D.1 Data Collection

Prerequisite Extended Indicators

MAE 3.N.1.b—Compare and order whole numbers 1–20, using number lines or quantities of objects.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

Key Terms

data, frequency, label, line plot, number line, title

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB2SUP-D2_MeasLengCust-201304.pdf

<https://www.mathlearningcenter.org/apps/number-line>

4.D.2 Analyze Data and Interpret Results

4.D.2 Analyze Data and Interpret Results

4.D.2.a

Solve authentic problems and analyze data involving addition or subtraction of fractions presented in line plots.

Extended: Solve problems with addition or subtraction of whole numbers using information from pictographs, bar graphs, and line plots.

Scaffolding Activities for the Extended Indicator

- **Solve a problem with addition or subtraction of whole numbers using information from a pictograph.**
 - Explain that pictographs can be used to help display data visually and can be used to learn more specific information. Present the scenario and pictograph shown.

A teacher wants to know what students in his class like to do as a favorite activity when not at school. The results about the favorite activities are shown in the pictograph.

Favorite Activity

Reading	☆ ☆ ☆ ☆
Dancing	☆ ☆ ☆
Games	☆ ☆ ☆ ☆ ☆

Key
☆ = 2 students

4.D.2 Analyze Data and Interpret Results

- Present the following question to students: How many students chose reading and dancing as their favorite activity?

Model working through the problem using the following method to figure out the answer:

1—Determine what information is needed to answer the question. For example, you will need to know that each star in the pictograph is equal to 2 students. We know this because the key tells us the value of each star.

2—Find the needed information on the pictograph. For example, find the reading and dancing rows in the pictograph.

3—Determine the arithmetic operation to use (add or subtract). Model reading the question and identifying that addition will be used to find the answer.

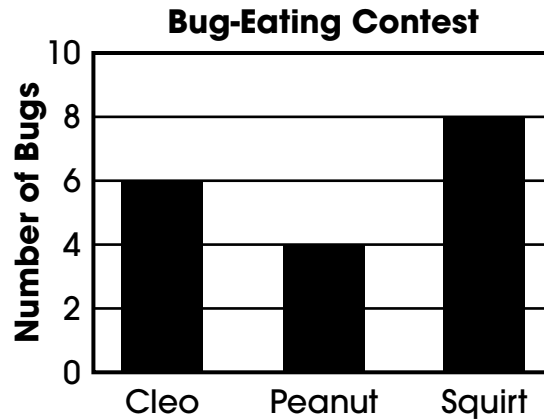
4—Carry out the operation to get the answer. Model counting by twos to get the total number of students who chose reading as their favorite activity and then counting by twos to get the total number of students who chose dancing as their favorite activity. Model adding the two numbers together to get the answer. $8 + 6 = 14$ students.

- Repeat this process by presenting various addition and subtraction problems.
- Ask students to solve problems using addition and subtraction of whole numbers presented in a pictograph.

4.D.2 Analyze Data and Interpret Results

- Solve a problem with addition or subtraction of whole numbers using information from a bar graph.
 - Solve a word problem using addition and subtraction of values presented in a bar graph. For example, present the scenario and bar graph shown.

Three geckos decided to have a bug-eating contest to see which gecko could eat the most bugs in one minute. The results from the contest are shown in the bar graph.



Use the bar graph to answer the following questions.

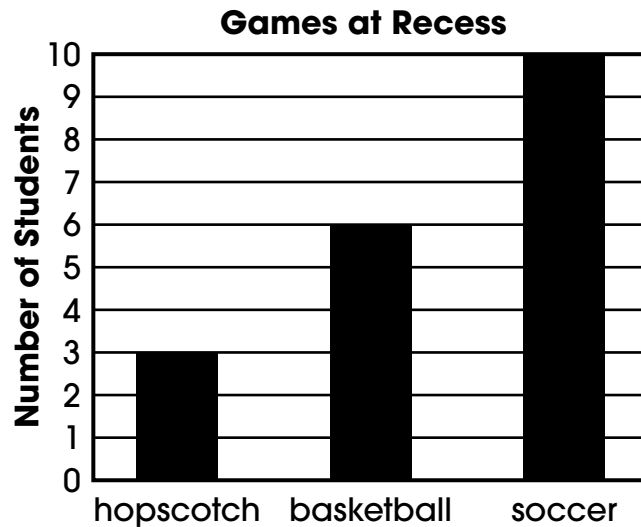
- ▶ How many more bugs did Cleo eat than Peanut ate?
- ▶ Squirt ate more bugs than Cleo and Peanut put together. Is this statement true or false?
- ▶ Which two geckos ate exactly 10 bugs altogether?
- ▶ Demonstrate following a sequence of steps to answer the questions:

- 1—Determine what information is needed to answer the question.
- 2—Find the needed information on the bar graph.
- 3—Determine the arithmetic operation to use (add or subtract).
- 4—Carry out the operation to get the answer.

4.D.2 Analyze Data and Interpret Results

- Ask students to solve problems using addition and subtraction of values presented in a bar graph with a scale of 1. For example, present the scenario and bar graph shown.

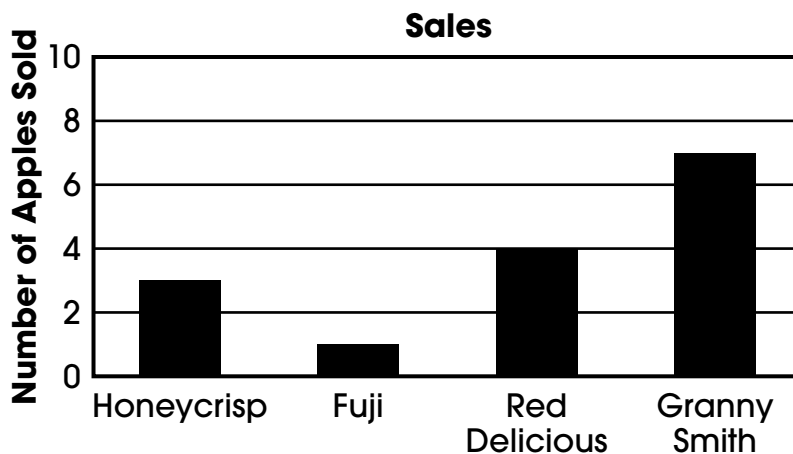
Students were playing games at recess. Here is a bar graph showing the number of students playing each game.



Use the bar graph to answer the following questions.

- ▶ How many more students were playing soccer than hopscotch?
 - ▶ How many students were playing basketball and soccer?
- Ask students to solve problems using addition and subtraction of values presented in a bar graph with a scale other than 1. For example, present the scenario and bar graph shown.

A farmer sold different varieties of apples at a farmers' market. The results of the farmer's sale are shown in the bar graph.



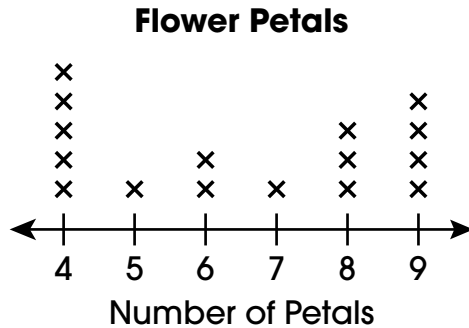
Use the bar graph to answer the questions.

- ▶ How many more Granny Smith apples were sold than Red Delicious apples?
 - ▶ How many Honeycrisp and Fuji apples were sold altogether?

4.D.2 Analyze Data and Interpret Results

☐ **Solve a problem with addition or subtraction of whole numbers using information from a line plot.**

- Present a line plot as shown. Indicate the title, label, and values shown on the line plot. Explain that a group of students counted the number of petals on flowers and recorded the results on the line plot. Demonstrate using information from the line plot to solve addition and subtraction problems. “How many more flowers have 4 petals than have 9 petals? How many flowers, in total, have 8 petals or 9 petals?”



- Ask students to record information from the line plot and then interpret the information to solve a problem. Use the line plot referenced above and present a template as shown or workspace for students to record answers to a series of questions. “How many flowers have 4 petals? How many flowers have 5 petals? How many flowers have 4 or 5 petals?” Students need to complete the addition problem to answer the final question.

_____ flowers with 4 petals

_____ flowers with 5 petals

How many flowers have 4 or 5 petals?

Repeat this process for a subtraction problem. “How many flowers have 8 petals? How many flowers have 7 petals? How many more flowers have 8 petals than have 7 petals?”

- Ask students to solve a problem with addition or subtraction using information from a line plot.

4.D.2 Analyze Data and Interpret Results

Prerequisite Extended Indicators

MAE 4.D.1.a—Identify and compare quantities in line plots, limited to two data points.

MAE 3.N.1.b—Compare and order whole numbers 1–20, using number lines or quantities of objects.

MAE 3.G.4.b—Solve authentic problems involving addition and subtraction of time intervals to find elapsed time, limited to whole-number hours.

MAE 3.D.1.b—Identify characteristics (e.g., title, labels, horizontal axis, quantities) on a line plot.

MAE 3.D.2.a—Identify and compare quantities in pictographs and bar graphs.

Key Terms

addition, bar graph, data, frequency, label, line plot, number line, pictograph, subtraction, title

Additional Resources or Links

<http://tasks.illustrativemathematics.org/content-standards/2/MD/D/10/tasks/506>

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB2SUP-D2_MeasLengCust-201304.pdf

(line plots on pages 41 – 66)

<https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-f-lesson-25/file/73291>

Alternate Mathematics
Instructional Supports
for
NSCAS Mathematics Extended Indicators
Grade 4



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