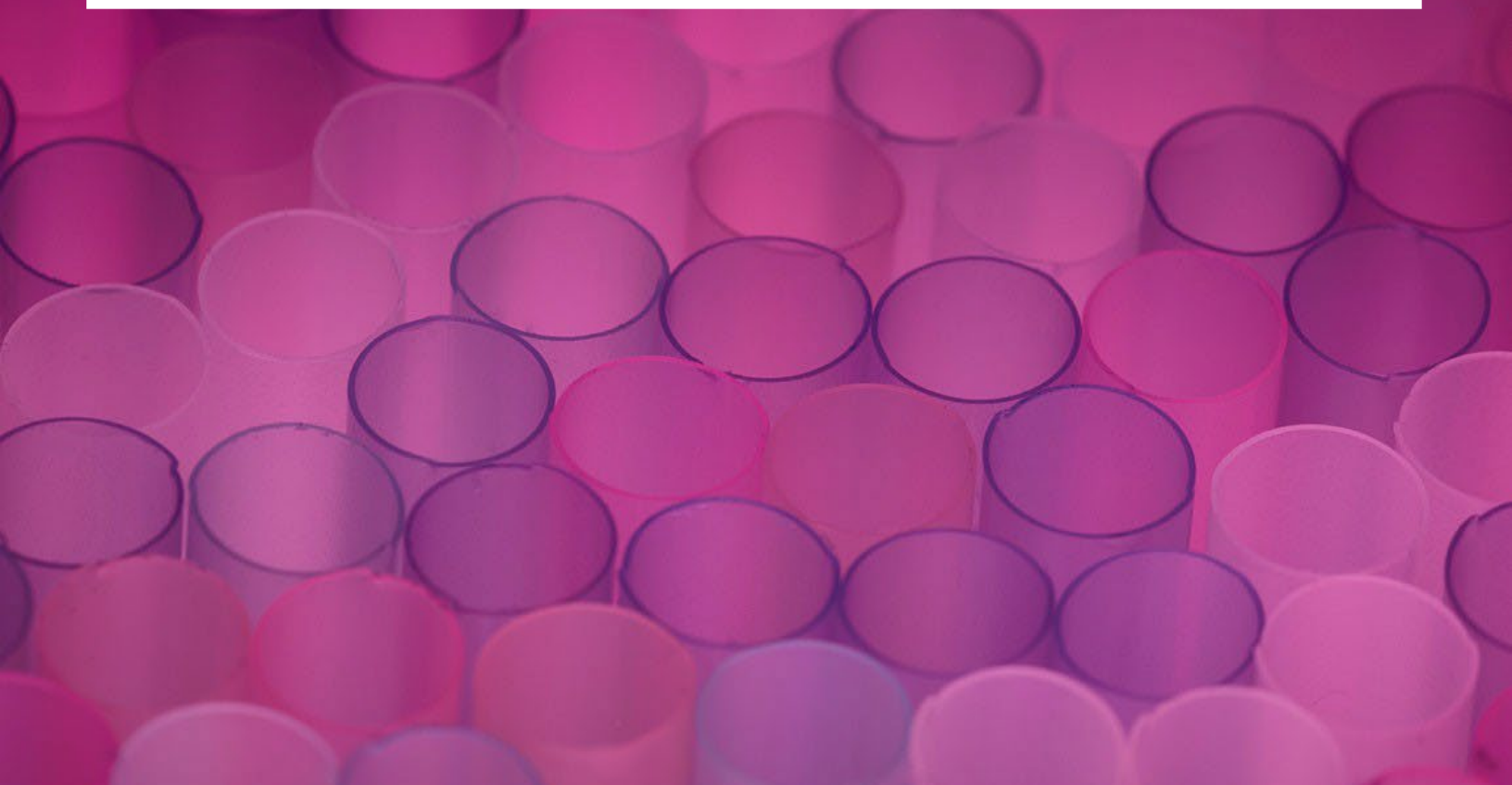
TEACHING STUDENTS WITH SPECIFIC LEARNING DISABILITIES



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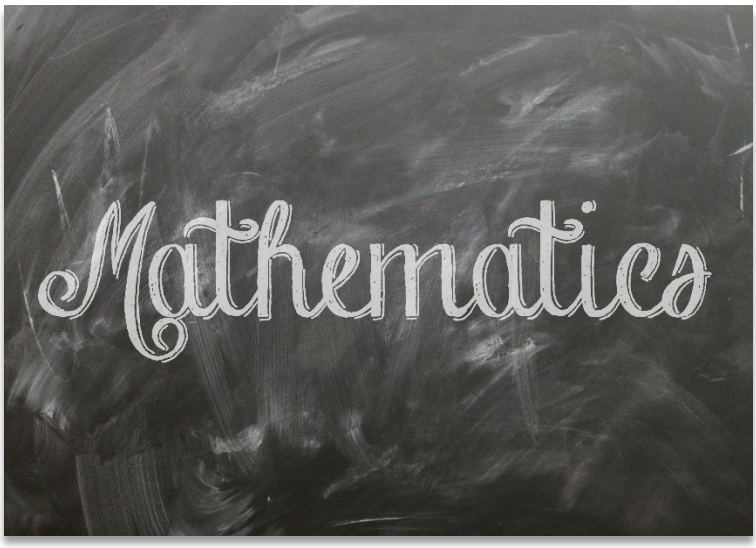


A Mathematician is someone who uses an extensive knowledge of mathematics in their work, typically Mathematicians are concerned with numbers, data, quantity, structure, space, models, and to solve mathematical problems. change.

One definition of persistence according to Dictionary.com. (n.d.) is "persisting, especially in spite of opposition, obstacles, discouragement, etc." (p.1). This describes how many students identified with a math disability or have math difficulties are willing to work through mathematical problems.

Many times mathematics is considered to be just arithmetic. Arithmetic involves computation and operations of numbers, but it is a small part of mathematics. Yes, getting the correct answer is important, but the emphasis should be on how to solve the problem. Many times students' learning growth is stunted because they struggle to perform operations correctly. Many of these students fall behind trying to learn how to work through longer arithmetic processes, when technology such as a calculator can be an accommodation so students can continue through the problem-solving processes of grade level work.

Garnett (1998) states:

Interestingly, some of the students with these difficulties may be remedial math students during the elementary years when computational accuracy is heavily stressed, but can go on to join honors classes in higher math where their conceptual prowess is called for. Clearly, these students should not be tracked into low level secondary math classes where they will only continue to demonstrate these careless errors and inconsistent computational skills while being denied access to higher-level math of which they are capable. Because there is much more to mathematics than right-answer reliable calculating, it is important to access the broad scope of math abilities and not judge intelligence or understanding by observing only weak lower-level skills. Often a delicate balance must be struck in working with learning disabled math students which include:

- Acknowledging their computational weaknesses;
- Maintaining persistent effort at strengthening inconsistent skills;
- Sharing a partnership with the student to develop self-monitoring systems and ingenious compensations; and at thesame time, providing the full, enriched scope of math teaching (para. 9).

Students who have difficulty in the area of math computation are still able to be progressing mathematicians. Students with disabilities may deal with more problems compared to peers significantly, daily. These students do tend to lack speed in solving problems and coming to the correct conclusions, but many are willing to work to solve difficult problems with persistence. Teachers must allow students to have the proper tools to be able to enter a math problem and solve it with their classmates. High expectations for each and every child to be successful with mathematics must remain the focus as students are supported to learn from high quality instructional materials.

Rationale

The primary target objective of K-12 mathematics education for the state of Nebraska is that all students have access to high quality, equitable, and engaging mathematics programs, and instruction. All students will participate in applicable learning opportunities that establish both theoretical and procedural comprehension. Math teachers create positive classroom communities by promoting student ownership of learning with explicit instruction. This ensures goals are focused for learning through meaningful tasks, manipulatives, and examples that are used to connect mathematical representations, as visual representations aid students' understanding of abstract mathematical concepts and solving problems. Math teachers and students demonstrate resilience and a growth mindset, convinced all students can learn math at high levels with rigorous work.

This document provides an overview of the characteristics of mathematics disabilities and presents evidence-based strategies to support students with mathematics disabilities and/or difficulties at all instructional levels. Importantly, the strategies outlined in this document are targeted at improving learning outcomes for the following student populations:

- Students with a formal school identification of a specific learning disability in mathematics.
- Students with a non-mathematics related disability (e.g., speech and language disorder, specific learning disability in reading) who experience mathematics difficulty.
- At-risk students without a formal disability diagnosis who experience mathematics difficulty.

The Nebraska Department of Education (NDE) and its Office of Special Education is committed to ensuring that the public education system is positioned to advance equitable academic outcomes by providing access to learning environments that meet the needs of its diverse student population. This technical assistance document aligns with those efforts as it outlines effective supports for students with learning disabilities in mathematics; and offers support to school divisions and parents seeking to improve outcomes in mathematics for students with disabilities. It also serves for professional development and technical assistance from NDE.

NeMTSS FRAMEWORK

For those schools who implement Nebraska's Multi-Tiered System of Support (NeMTSS), many of the strategies outlined in this document could support core instruction as well as the intervention and intensive intervention tiers of support.

Least Restrictive Environment (LRE)

The vision is for all Nebraska math students; including students with disabilities, students with a variety of learning needs, and includes a variety of appropriate and least restrictive environment educational settings, while being provided with FAPE (Free Appropriate Public Education)

The education of students with special needs is more effective with maintaining high expectations, inclusivity with their peers, and fair access to their grade level curriculum, according to comprehensive and lengthy research provided by the Individuals with Disabilities Education Act of 2004 (IDEA 2004). Rule 51 (Regulations and Standards for Special Education Programs) explains Least Restrictive Environment (LRE): Nebraska Department of Education Rule 51 see page 44 for least restrictive environment.

The Center for Parent Information and Resources (2021) states:

Placement is not an "either/or" decision, where children are either placed in a regular education classroom or they're not. The intent is for services to follow, or go with, the child, not for the child to follow services. Schools must make provision for supplementary services (such as resource room or itinerant instruction) to be provided in conjunction with regular class placement (p.4).

All children from birth through age 21 (including students who have been suspended, expelled, and/ or residing in a detention facility, jail, or prison) are entitled to FAPE. FAPE encompasses special education and related services to meet individual students' needs. Nebraska Department of Education Rule 51 section 004 and Nebraska Department of Education Rule 52 section 004 explains the responsibilities and protocols in regards to providing Free Appropriate Public Education further.

A review of school divisions Least Restrictive Environment (LRE) data in the Nebraska Department of Education (NDE) Special Education Annual Performance Report in 2019 shows that 79% of students with disabilities in Nebraska are spending at least 80% of their day in the general education classroom. However, according to the 2021-2022 NDE Public Schools State Snapshot, only 18% of students with disabilities met grade level standards in mathematics. Furthermore, statewide data demonstrates that achievement gaps in mathematics continue to exist for students with disabilities across the state.

This discrepancy in data may indicate that educators need more support and guidance in providing specially designed math instruction and appropriate accommodations for students with disabilities in the general classroom. Therefore, to enhance the performance of students with disabilities in the mathematics' classrooms, this document serves as a resource for educators, administrators, and parents to address the educational needs of students with mathematics disabilities and any mathematics difficulty.

Dedicated math teachers will strategically engage students to ensure their students are always learning. A highly qualified teacher with the best math programming, curriculum, and materials can make all the difference for students with special needs to be successful. Math teachers need to reflect "How can I meet the needs of all of my students? Are my students getting what they need? Am I challenging my high ability learners? What modifications, accommodations are needed, or how do I differentiate for my special education learners? Frequent informal and formal assessments need to drive teachers' decisions to determine equitable progress and to ensure students are getting the correct math instructions. Consider visiting the Center for Parent Information & Resources site and check out the document: Considering LRE in Placement Decisions for more detailed information regarding LRE.

Defining & Determining Mathematics Learning Disabilities

Determining Eligibility for Students with a Specific Learning Disability (SLD)

The Nebraska Department of Education updated the Eligibility guidelines in January 2021. All updates can be found on the Eligibility Guidelines page of the Nebraska Department of Education, Office of Special Education's webpage (education.ne.gov/sped).

As stated in the Determining Special Education Eligibility for Specific Learning Disabilities guidance document:

The 2004 Individuals with Disabilities Education Improvement Act (IDEA) and subsequent regulations published in 2006 have significantly changed the identification process for students suspected of having specific learning disabilities. Rather than using a discrepancy model contrasting intellectual and achievement test results, multi-disciplinary teams are now encouraged to consider a variety of methods to identify specific learning disabilities, including response-to-intervention (RTI /Multi-tiered System of Supports (MTSS) that incorporates deeply implemented problem solving, cognitive processing approaches, and the determination of a pattern of strengths and weaknesses.

Please see the full guidance document from the Nebraska Department of Education, Office of Special Education: Determining Special Education Eligibility for Specific Learning Disabilities for general guidance for parents, teachers, special education personnel, administrators, and other professionals with information on the identification and determination of eligibility for special education services for children with specific learning disabilities.

The following table contains links to State and Federal Laws pertaining to Free Appropriate Public Education (FAPE) for students with disabilities. These laws provide clear definitions, regulations, and statutes to determine disabilities and eligibility for Special Education Services.

State and Federal Special Education Law Table

State	Federal
Nebraska Department of Education – Rule 51	Individual with Disabilities Act (IDEA)

See the Following Sections Specifically:
• **Section 003 Definition of Terms**
  ○ 003.24 Definition of Free appropriate public education (FAPE)
  ○ 003.56 Definition of Special Education
  ○ 003.08 Definition of a child with a disability
• **Section 006 Identification of Children with Disabilities, Multidisciplinary Teams and reporting of Diagnostic Data**
  ○ 006.04K for Specific Learning Disability
	See the Following Regulations Specifically:
	• Sec. 300.8 Child with a disability Defines what it means to be a child with a disability as well as definitions of disability terms. See sec 300.8 (c) (10) for the definition of Specific Learning Disability
	• Sec. 300.39 Special education Defines the general definition of Special education and the individual special education terms
	• Sec 300.101 Free Appropriate Education (FAPE) Definition and details regarding FAPE
	• Sec. 300.111 Child Find Regulations regarding locating and evaluating suspected students with disabilities
	• Sec. 300.306 Determination of eligibility Regulations including evaluations to determine eligibility, educational need, and special rule for eligibility.
	• Sec 300.307 Specific learning disabilities Includes the details of the criteria adopted by the state for determining eligibility of students with a Specific Learning Disability
	• Sec 300.309 Determining the existence of a specific learning disability this includes eligibility areas of Specific Learning Disabilities, parental consent, evaluation, and general procedures for determining eligibility as a student with a Specific Learning Disability.
	• Sec 300.311 Specific documentation for the eligibility determination This includes regulations about necessary statements within documentation regarding eligibility
	See the Following Statutes Specifically:
	• Section 1401 (30) Specific learning disability definition
	• Section 1414 Evaluations, eligibility determinations, individualized education programs, and educational placements - see specifically Sections 1414 (a)Evaluations, parental consent, and reevaluations and (b) Evaluation Procedures

Frequently Asked Questions Amongst Different Mathematics Disabilities

What are the causes of a specific learning disabilities in mathematics?

- A mathematics disability is considered a neurodevelopmental disorder that involves dysfunction in specific brain regions that are important for mathematics skills.
    - Brain regions associated with mathematics skills include (but are not limited to): the parietal lobe, prefrontal cortex, dorsal basal ganglia, temporal lobe, and hippocampus (Soares et al., 2018).
- There is a strong genetic influence on the development of mathematical skills.
    - One study found that 50% of siblings of a student with a mathematics disability can be expected to experience similar difficulties (Shalev & Gross-Tur, 2001).
    - Parents of a student with a mathematics disability are 10 times more likely to have a mathematics disability than members of the general population (Hannell, 2013).
- Environmental factors such as motivation, behavior, and attention challenges, and/or poor teaching may contribute to or exacerbate a mathematics disability (Szücs & Goswami, 2013).

What is the incidence of students with a specific learning disability in mathematics in the school-age population?

Despite a wide range of prevalence data, there is general agreement that approximately 3-7% of students are identified as having a specific learning disability in the area of mathematics (Shavel, 2007).

What are the primary criteria for diagnosing a specific learning disability in mathematics?

Although students with mathematics disability experience one or more of the challenges described above, IDEA (2004) states the primary criteria for diagnosis of mathematics disability is difficulty in one or both of the following areas:

- Mathematics calculation
- Mathematical reasoning

What is the difference between mathematics disability, dyscalculia, and mathematics difficulty?

Mathematics disability refers to students who have a formal school identification of a Specific Learning Disability (SLD) in mathematics under the Individuals with Disabilities Education Act (IDEA, 2004) federal mandate. Students with a mathematics disability have Individualized Education Program (IEP) goals in mathematics.

Several terms exist to describe a mathematics disability and often are used interchangeably. The list below highlights terms used to describe conditions equivalent to mathematics disability.

- Dyscalculia
- Developmental dyscalculia
- Arithmetic-Related Learning Disability (AD)
- Arithmetical Disability (ARITHD)
- Mathematical Disability (MD)
- Mathematics Learning Disability (MLD)
- Specific Learning Disability in Mathematics

(Szücs & Goswami, 2013)

The term mathematics difficulty refers to students who:

- Do not have a formal disability identification, but experience mathematics difficulty; or
- Have a non-mathematics related disability (e.g., speech and language disorder, specific learning disability in reading) and experience mathematics difficulty.

Frequently Asked Questions About Characteristics of Disabilities in Mathematics

What are the global characteristics of a specific learning disability in mathematics?

Students with mathematics disability experience difficulty in one or more of the following areas (Hannell, 2013)

- Counting
- Comparison of quantities
- Understanding operations
- Mathematics fact fluency
- Problem solving
- Conceptual understanding
- Procedural efficiency
- Spatial reasoning
- Verbal reasoning

What are the signs that a student has a specific learning disability in mathematics and/or mathematics difficulty?

There are a wide range of signs that indicate that a student has a specific learning disability in mathematics and/or mathematics difficulty. However, it is also important to consider how the student is progressing within the trajectory of mathematics understanding in relationship to these signs.

The following chart outlines several common indicators (Hannell, 2013).

AREA OF DIFFICULTY	WHAT DOES IT LOOK LIKE
Number Sense	• Has difficulty recognizing small quantities without counting • Does not grasp comparison or the relative value of numbers • Has difficulties with mental calculations • Uses fingers to count simple totals - beyond early elementary grades • Uses the 'counts all' method instead of 'counting on' for addition • Finds it difficult to estimate or give approximate answers
Response Time	• Requires extended time to answer mathematics questions • Requires extended time to complete problems • Works very mechanically
Math Language	• Finds it difficult to talk about mathematical processes • Does not ask questions, even when he/she does not understand • Finds it difficult to generalize learning from one situation to another • Makes mistakes in interpreting word problems and instead just 'number crunches' the numbers in the text • Mixes up terms like equal to and greater than
Memory for Facts and Procedures	• Finds it difficult to remember mathematics facts • Has trouble remembering what symbols like + mean • Forgets previously mastered procedures very quickly • Has to recite the entire multiplication table to get an answer such as 4x6=24 • Works multiplication tables out by adding on rather than by automatic recall • Finds mental mathematics difficult • Forgets the questions before the answer can be worked out

Sequencing	• Loses track when counting • Loses track when saying multiplication tables • Has difficulty remembering the steps in a multi-stage process
Spatial Organization	• Is confused about the difference between 21 and 12 and uses them interchangeably • Mixes up the signs (e.g., + and -) • Puts numbers in the wrong places when regrouping or exchanging • Has trouble setting up calculations and work on a page • Scatters tally marks instead of organizing them systematically • Is unaware of the difference between 6 - 2 and 2 - 6 (i.e., thinks the answer for both problems is 4) • Is easily confused with division (e.g., Is it 6 divided by 3 or 3 divided by 6?) • Take the lesser number from the greater, regardless of position • Finds rounding numbers difficult • Finds telling time on an analogue clock difficult • Is easily overloaded by crowded mathematics worksheets • Copies work inaccurately • Relies on imitation and rote learning instead of understanding • Can find the sum when computing, but cannot explain the process • Often uses the wrong working method in exchanging or regrouping (e.g., treating 10 as 1 or vice versa)

Are there other common areas of difficulty for students with a specific learning disability in mathematics?

- Students with mathematics disability and/or difficulty also may present with challenges in the following processing areas (Brodesky et al., 2002; Hannell, 2013):

Language and reading (English and math)	• Understanding the words and vocabulary used in mathematics • Reading and interpreting mathematics texts
Memory and information processing	• Using short-term memory (e.g., when solving a computation problem) • Accessing information from long-term memory • Retaining and using knowledge
Processing speed and response time	• Working problems in a timely manner • Responding to questions • Retrieving math facts
Visual-spatial processing and spatial organization	• Understanding visual information
Fine motor skills	• Making movements with the fingers and hands
Executive functioning skills	• Planning, organizing, and monitoring learning, behavior, and emotions
Attention	• Focusing on tasks without being distracted
Psychosocial skills	• Monitoring emotions and reacting appropriately

High Quality Core Learning Materials

Aligning to Standards

There have been recent updates and changes to Nebraska Mathematics Content Standards due to the Nebraska Revised Statute 79-760.01. The main shifts for K-12 Nebraska mathematics instruction are: focus on fewer concepts, understand mathematics through coherence, and experience rigorous math content. These shifts are necessary for the Nebraska's College and Career Readiness (CCR) Standards for Mathematics to be met.

> *"An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential."*
>
> *(NCTM, 2014, p. 59)*

According to Nebraska Instructional Collaborative (n.d.):

> These instructional shifts are also a part of the Quality Instructional Materials Review Tool for K-8 mathematics from EdReports.org. EdReports.org developed its tool to provide educators, stakeholders, and leaders with independent and useful information about the quality of instructional materials (whether digital, traditional textbook or blended) for those who will be using them in classrooms. Expert educators use the tool to evaluate full sets of instructional materials in mathematics against non-negotiable criteria. (p.1)

The Quality Instructional Materials Review Tool for K-8 mathematics from EdReports.org is a stable origin for Nebraska districts, schools, and educators to utilize when selecting instructional mathematics materials because it shares the instructional shifts (focus, coherence, and rigor) included in the Nebraska's College and Career Readiness (CCR) Standards for Mathematics.

Additionally, Nebraska Instructional Collaborative (n.d.) states, "…the K-8 Mathematics Evidence Guides provide educator reviewers with guidance to identify, collect, calibrate, and report on instructional materials aligned to the standards for mathematical content, the standards for mathematical practice, and the usability of the instructional materials."

Please see the Selection Process – Nebraska Instructional Materials Collaborative for information on specific steps that should be taken into consideration when instructional materials are being selected.

Another helpful tool when considering instructional materials is this Program Comparison Tool from the NeMTSS Framework Website. This tool gives an overview of specific skills programs address, program length, a program description, and cost.

Programs are divided into the areas of reading, writing, math, oral language, and behavior and SEL (Social and Emotional Learning).

Evidence-based Practices

The most effective way for math teachers to teach concepts and procedures is to implement evidence-based practices; strategies that are effective through rigorous research. The use of evidence based practices is mandated by the Every Student Succeeds Act (ESSA) and the Individuals with Disabilities Education Act (IDEA 2004). Mathematics programs must be grounded in scientifically based research; this ensures high quality math instruction. This ensures:

- Effective programs respond better to learners' needs

"Curriculum plays an important role in how students are taught, and there is a strong body of evidence that shows that putting a high- quality curriculum in the hands of teachers can have significant positive impacts on student achievement."

(Boser,et al, 2015, p.1)

- Increased student success in mathematics
- Reliable data which increases accountability to stakeholders
- Better overall math education for students

What is high-quality mathematics instruction and why is it important? (2021)

Universal Design for Learning

What Is Universal Design for Learning?

Universal Design for Learning, or UDL, is an instructional framework that supports flexible ways for educators to teach lessons, as well as multiple ways for students to demonstrate what they know. The goal is to reach all learners, including students with disabilities and English-language learners. For more information on UDL, visit cast.org.

Universally Designing the Mathematics Classroom

How can we redesign our mathematics classrooms so that more students can experience success? UDL is a process best undertaken at the local level by collaborative groups of teachers engaging in the study of their own practices. However, understanding both the shifts in state standards and common barriers for students with a variety of disabilities, there are some important shifts to make mathematics classrooms more accessible to all learners.

Create Safe Classroom Climates

Researchers in UDL have documented how emotions regulate all learning, and thus understanding of the emotional and relationship aspects of learning are critical to providing universal access. With a rigid focus on right and wrong, mathematics classrooms can feel unsafe to students, particularly those who are positioned as less competent. Teachers can develop a safe classroom community in which students are comfortable taking mathematical risks. Students report taking more risks speaking in mathematics class when the teacher values thinking more than accuracy, and explicitly gives students permission to make mistakes. In one inclusive classroom, students and teachers chose their favorite mistake each day, based on which one best helped students learn more about the math. Teachers will need to lead a shift away from valuing mathematical speed towards valuing mathematical thinking and persistence: in addition to eliminating timed tests, teachers can explicitly value thoughtfulness over speed.

Offer Relevance and Choice

A critical component to universal access is student enthusiasm and engagement. UDL tools that serve to increase engagement are flexibility, choice, and relevance. Teachers can focus mathematics class on relevant, engaging, and culturally responsive contexts as well as follow the students' lead into topics for investigation. Rather than insisting on narrow forms of engagement, teachers can provide students with choice in how they engage in mathematical problem- solving (e.g., individually, in pairs, and in groups). These changes can shift math class away from being a disengaging environment towards being an environment in which students see themselves as mathematical thinkers.

"There is no formula or recipe that works for all learners in all times. There is no set of lesson plans or units that can engage the range of learning styles, approaches, and intelligences that are likely to gather in one classroom."

– William Ayres

Focus on Core Ideas

In order to make math class accessible to all learners, teachers need to identify and understand

the core mathematical ideas in each unit. Identifying these will assist in designing a sequence of tasks that engage students in the necessary learning to understand core ideas, which are particularly useful in adapting instruction for students with processing difficulties, intellectual disabilities, and/or limited prior knowledge of mathematics. For example, instruction in fractions should begin with "fair sharing"—the concept that fractional pieces are equal and that different fractions can be equivalent. This core idea can be made accessible to students with a range of prior knowledge through problems in which students must fairly distribute items such as food. Excessive and repetitive assignments are another barrier for students. Instead of worksheets with many problems, classroom and home work should include a smaller number of problems focused on core ideas.

Put Rich, Accessible, and Collaborative Tasks at the Center

While not all work in the math classroom needs to consist of collaborative investigations, there should be some central, class-wide investigations that (a) focus on the core ideas of the mathematics unit; (b) are multi-dimensional (drawing on different strengths); and (c) allow access and sustained learning for students with varying prior knowledge of the topic (low floor–high ceiling). When such central investigations are well designed, they offer opportunities for every child in a class to be part of a collective inquiry.

Represent Concepts in Multiple Modalities

Just as multiple representations are a core feature of UDL, so representation itself is a key aspect of mathematical thinking and learning. For example, the number line is a key mathematical representation across K–12, from early numbers to Cartesian planes. Use of mathematical models can be adapted to include students with visual impairments who can experience such models through touch and sound. However, like most mathematical representations, number lines can be complex for students to learn, particularly when teachers only show these representations, expecting them to make sense immediately to all students. Research has documented how to sequence instruction to develop specific useful mathematical representations, such as the number line. Students should be able to use a variety of representations to model their thinking; multiple representations can support students with memory and processing differences. All of this suggests the central role of representation in mathematical learning and how understanding representation itself as multimodal can make mathematics more accessible to all learners.

Focus on Developing Strategic, Expert Mathematicians

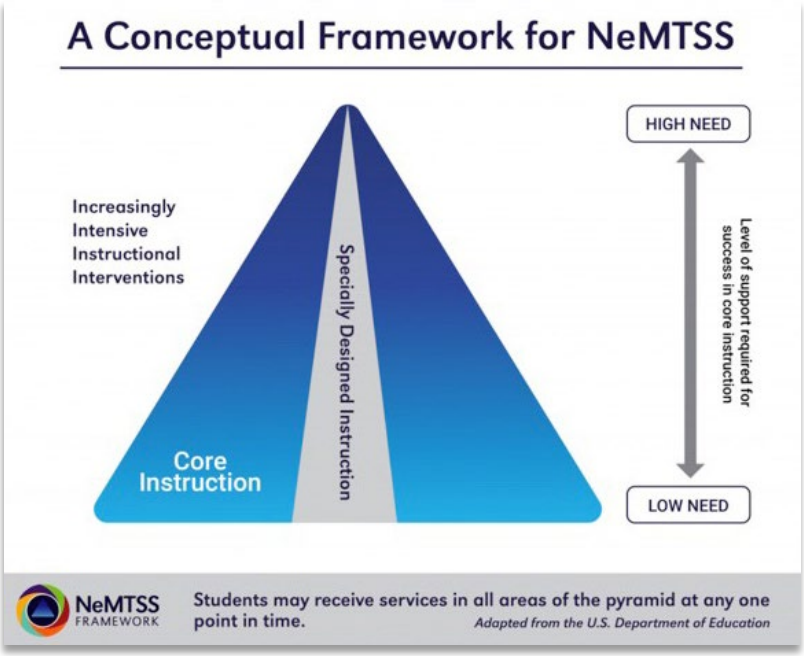
Both UDL and the NeMTSS Framework offer a vision of mathematics education as focused not on memorization and replication of procedures but on engaging in the practices of mathematicians: proof, justification, problem-solving, representation and mathematical modeling. Engaging in these practices can offer long-term benefits for all students, but particularly for students with disabilities who have not historically been given access to sense-making in mathematics. However, all students will need additional scaffolds to develop expertise in these practices. For example, many students have difficulties connecting multiple representations of a particular concept, such as matching an equation with its graph. The instructional routine Connecting Representations provides a sequenced routine that offers guided practice in connecting representations, including language and processing supports to develop strategic expertise.

For additional information, see https://files.eric.ed.gov/fulltext/ED605096.pdf for the full research brief on Increasing Access to Universally Designed Mathematics Classrooms.

Utilizing The NeMTSS Framework within your Universal Design for Learning

NeMTSS is designed to support Nebraska districts and schools in developing sustainable, evidence-based practices that will provide supports for each student within a multi-tiered system. A well designed support model will ensure that ALL students will acquire the foundational skills to be proficient mathematicians who are able to select from multiple problem- solving strategies, justify their solution choices, and describe their results. High quality instruction is the

foundation for a 3- Tiered instructional model. It consists of research-validated classroom instructional practices and a comprehensive core mathematics program that has a strong focus on the eight mathematics practice standards.



An effective 3-Tier model will develop alignment with curriculum and instruction between the tiers and will strive to ensure that 80% of students experience success in Tier I. Early intervention will be provided for students who struggle, and enhanced instruction will be offered for students who need to be challenged. It is important to keep in mind that "you cannot intervene your way out of core instruction that is not effective" (Metcalf, 2009).

Tier I aims to prevent onset of mathematics difficulties through the use of universal screening and should include differentiated instruction and grouping structures. It is essential that valid assessment data be utilized to guide instructional decisions. Extensive research currently exists about the kind of instruction that needs to be provided to children, so that they can gain conceptual understanding AND procedural fluency in mathematics. Finding a balance between direct instruction in basic skills and scaffolded inquiry-based instruction using physical and visual models prior to teaching procedures and standard algorithms, along with specific vocabulary instruction, is essential. High quality instruction involves multiple forms including teacher-directed, peer-mediated and technology-enhanced opportunities for learning, and is differentiated based on learner needs.

Students identified by their performance as being at risk for mathematics problems through the screening process (up to 15% of students) should have ongoing progress monitoring to determine if they require more intensive intervention in Tier II. This first layer of additional support occurs outside of the time dedicated to core instruction, in groups of 5–8 students, and focuses primarily on providing increased opportunities to build conceptual understanding and procedural fluency, and practice and learn skills taught during Tier I Core instruction.

A small percentage of students require even more intensive, specialized services in order to be successful in math. Tier III intervention is more explicit, focuses on remediation of skills, is provided for a longer duration of time (both in overall length of intervention and regularly scheduled minutes of instructional time), and occurs in smaller groups (i.e., groups of 1–3 students).

A key factor that will lead to sustained effective practice is the provision of appropriate support. School administrators must act as instructional leaders to ensure that mathematics instruction is a priority. Teachers and paraeducators must receive adequate training in research-supported programs and understand how the component parts of programs support standards-based learning. Teachers and paraeducators must also understand how to use data to make important instructional decisions.

Resources and Additional Information

For more information on the Nebraska Instructional Materials Collaborative Math Resources visit the NDE, https://nematerialsmatter.org/about/ page for information regarding Nebraska instructional shifts, Nebraska math instructional shifts, practices, and materials; Nebraska's college and career ready standards for mathematics, and the EdReports Bridge Document.

Nebraska's College and Career Ready Standards for Mathematics require key instructional shifts in practice to support student achievement and the promise of the these standards.

- Please visit Nebraska Instructional Materials Collaborative- Reviews to help in the selection process of materials further. This site has reports for instructional materials categorized into the statuses of meets, partially meets, and does not meet.
- Relevant Reading: https://www.brookings.edu/wp-content/uploads/2016/06/0410_curriculum_chingos_whitehurst.pdf

Nebraska Mathematical Processes

Nebraska Mathematics Standards (2015) states: The Nebraska Mathematical Processes reflect overarching processes that students should master as they work towards college and career readiness. The Nebraska Mathematical Processes reflect the interaction of skills necessary for success in math coursework as well as the ability to apply math knowledge and processes within real world contexts. The processes highlight the applied nature of math within the workforce and clarify the expectations held for the use of mathematics in and outside of the classroom.	
Solves mathematical problems	Through the use of appropriate academic and technical tools, students will make sense of mathematical problems and persevere in solving them. Students will draw upon their prior knowledge in order to employ critical thinking skills, reasoning skills, creativity, and innovative ability. Additionally, students will compute accurately and determine the reasonableness of solutions.
Models and represents mathematical problems	Students will analyze relationships in order to create mathematical models given a real-world situation or scenario. Conversely, students will describe situations or scenarios given a mathematical model.
Communicates mathematical ideas effectively	Students will communicate mathematical ideas effectively and precisely. Students will critique the reasoning of others as well as provide mathematical justifications. Students will utilize appropriate communication approaches individually and collectively and through multiple methods, including writing, speaking, and listening.
Makes mathematical connections	Students will connect mathematical knowledge, ideas, and skills beyond the math classroom. This includes the connection of mathematical ideas to other topics within mathematics and other content areas. Additionally, students will be able to describe the connection of mathematical knowledge and skills to their career interest as well as within authentic/real-world contexts (p. 2).

Mathematics Instruction Best Practices

Specially Designed Instruction in Mathematics

What are evidence-based, specially-designed strategies to support students with mathematics difficulty across grade levels during mathematics instruction?

When delivering mathematics instruction to students with mathematics difficulty at any grade level, teachers should incorporate the following practices, all of which have a strong evidence base:

1. Explicit Instruction
2. Formal Mathematical Language
3. Concrete, Representational, and Abstract Connections
4. Fact and Computational Fluency
5. Word-Problem Solving

The following table highlights key points for each of the five evidence-based strategies that are effective in supporting students with mathematics difficulty.

Evidence Based Specially Designed Instructional Strategies	
Strategy	Key Points

Explicit Instruction	Effective explicit instruction involves the teacher providing the following:
	• Modeling steps using concise language
	• Providing guided practice opportunities
	• Providing independent practice opportunities
	• Using supports during modeling and practice:
	  ○ Asking the right questions
	  ○ Eliciting frequent responses
	  ○ Providing feedback
	  ○ Being planned and organized

Formal Mathematics Language	Teacher should promote students' understanding of formal mathematic vocabulary by: • Using formal mathematics vocabulary terms • Using similar or related terms correctly and precisely • Planning for language use prior to instruction • Including explicit vocabulary activities in instruction • Holding students accountable
Concrete, Representational, and Abstract Connections	The Concrete-Representational-Abstract (C-R-A) framework includes forms of mathematics: concrete, representational (pictorial), and abstract. • Concrete: three-dimensional, hands-on materials and objects • Representational (Pictorial): two-dimensional pictures, images, or virtual manipulative • Abstract: numbers, symbols, and words

	• The use of C-R-A supports students in developing a deeper conceptual understanding of mathematics beyond superficial procedural knowledge.
Fact and Computational Fluency	• Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose and are able to explain and produce accurate answers efficiently. • Teachers should use activities and games to promote fact fluency. • Beyond mathematics facts, students should develop fluency with computation (i.e., multi-digit addition , subtraction, multiplication, or division). • Fluency practice should be brief and occur daily.

Word Problem Solving	• The majority of routine word problems that students solve in elementary and middle school fall into one of the six different schemas:
	  ○ Total
	  ○ Difference
	  ○ Change
	  ○ Equal Groups
	  ○ Comparison
	  ○ Ratios or Proportions
	• Teachers should provide students with verbal and gestural cues to review and recall the six schemas.
	• When teaching problem solving strategies, DO NOT tie key words to operations.
	• When teaching problem solving strategies, DO NOT define word problems by the operations.
	• When teaching problem solving strategies, DO teach students an attack strategy to help guide the process of problem solving.
	• When teaching problem solving strategies, DO teach word-problem schemas.

What are examples of specially designed instructional strategies that can be utilized when addressing the needs of students with autism spectrum disorder (ASD) or more significant behavioral needs?

There are important instructional considerations that must be taken into account to promote student learning. Given the difficulty individuals with ASD have with acquiring skills incidentally, it is crucial to provide carefully planned and predictable instruction. Students will benefit from direct teaching of skills and concepts as well as strategies to encourage active engagement. Additionally, some students with autism or more significant behavioral needs may need specially designed instruction in strategies related to emotional regulation and responses. Examples could include role play, feedback, and reinforcement for appropriate responses.

Explicit Instruction

Explicit instruction often is described as the cornerstone of effective mathematics instruction for students with learning difficulties (Hudson et al., 2006; Jitendra et al., 2018; Witzel et al., 2003). Although many helpful models of explicit instruction exist, the model developed by the National Center on Intensive Intervention (NCII) offers a valuable guide to understanding explicit instruction.

There are three main components of explicit instruction:

- Modeling: facilitated by the teacher
- Practice: involves the students and the teacher
- Supports: consist of an ongoing dialogue between the teacher and students. Supports are employed during modeling and during practice. Supports are described within the explanations of modeling and practice.

Modeling

Modeling prepares students to complete a mathematics skill successfully. Modeling includes two main components: clear explanations and planned examples.

- Clear explanations
    - Provide a 2-3 sentence statement about the goals and importance of the lesson.
    - Explicitly model the steps for solving a mathematical problem.
    - Include important vocabulary and concise mathematics language.
    - Choose to model one example or several examples depending on students' familiarity with the mathematics content.
    - Vary modeling based on students' needs and exposure to mathematical content.
- Planned examples
    - Thoughtfully plan examples before the lesson to help students understand the
    - mathematical concept.
    - Ask important questions.

<u>Example for a division lesson:</u>

- How am I going to present the division problems – using partitive division or measurement division?
- Am I going to use a slash, obelus (i.e.,÷), long division bracket, or all three symbols?
- How many examples do I need to include?
- Consider examples that are open-ended, worked examples (i.e., previously-solved problems answered correctly or incorrectly), or non-examples.

MODELING

Clear Explanations	Examples
Goals and importance	*Today we are going to learn about division. Division is important because sometimes you need to share or divide things with your friends, like when you order pizza or want to share candy.* *Let's continue working on our three-dimensional shapes. Today, we will learn about cones. Cones are important because we see examples of cones everyday: ice cream cones, party hats, and orange cones in our school parking lot.*
Explicitly model steps with concise mathematical language *(Note: **bolded words** represent concise mathematical language)*	*To solve 21 **plus** blank equals 73, I first decide about the **operation**. Do I **add, subtract, multiply** or **divide**?* *The **plus sign** tells me to **add**, but I know the **sum** and I am missing one of my **addends**. So, to find out my missing number, I could count from 21 to 73 or subtract 73 minus 21. I'll subtract 73 **minus** 21. I'll use the **partial differences algorithm**. First, I'll **subtract** 70 **minus** 20. What's 70 **minus** 20?* *70 **minus** 20 is 50. I write 50 right here under the **equal line**. Where do I write the 50?* *Then I'll subtract 3 **minus** 1. What's 3 **minus** 1?* *3 **minus** 1 is 2. So, I write 2 here in the ones place.* *Finally, we **add** the **partial differences**: 50 and 2. 50 **plus** 2 is 52. So, 73 **minus** 21 **equals** 52. What's 73 **minus** 21?*
Planned Examples	**For an addition lesson**
Examples	5 + 6 , 9 + 3 , 8 + 8
Worked examples	5 + 6 = 11 ; 12 = 9 + 3 ; 8 + 8 = 16
Non-examples	5 x 6 , 9 ÷ 3 , 8 – 8

Supports During Modeling

Although modeling primarily is teacher-directed, students actively participate through supports. During modeling, teachers should attend to the following four supports.

1. **Ask high- and low-level questions**

   While providing clear explanations and presenting planned examples, teachers should ask students a mix of high- and low- level questions. Teachers should aim to ask students a question at least every 30-60 seconds during modeling.

   By asking a combination of high- and low-level questions, teachers can evaluate students' understanding and monitor that students are paying attention and on-task. Asking a variety of questions also promotes active engagement in the lesson.

   a. High-level questions encourage deeper thinking and reasoning and allow teachers to assess students' conceptual understanding of a mathematics concept.
      i. Example high-level questions:
         1. How does finding common denominators help us in comparing fractions?
         2. When is it necessary to regroup when solving a problem? Explain your thinking.
   b. Low-level questions require simpler answers and are helpful for checking for procedural understanding. The inclusion of low-level questions offers an important way to increase students' participation and minimize frustration.
      i. Example low-level questions:
         1. What is 7 times 9?
         2. What happens when we add 2?
         3. What operations can we use to solve the word problem?
         4. Show me an example of a right angle.

2. **Elicit frequent responses**

   Eliciting frequent responses from students proves essential for maintaining students' attention and determining if lesson components require reteaching or additional planned examples. Teachers should engage students frequently by eliciting responses at least every 30-60 seconds.

   a. Students' responses may include answers to high- or low-level questions.
   b. Students do not need to answer all questions with an oral response.
   c. Students may respond as a group in a choral or partner response or write or draw an answer on paper, a worksheet, or whiteboard.
   d. Students can gesture with a thumbs up or thumbs down, use manipulatives, check the work of a problem, or update a vocabulary term on a word wall to convey a response.

When teachers model a lesson, students must participate. Eliciting frequent responses offers an important way to ensure students participate. The combination of asking questions and eliciting frequent responses often is misunderstood within modeling. Some teachers believe that modeling only consists of teacher demonstration and teacher talk. This is not the case. Effective modeling requires an ongoing dialogue between teachers and students.

3. Provide immediate affirmative and corrective feedback

When students respond to questions during modeling, teachers should provide specific affirmative and corrective feedback. Feedback creates an opportunity to redirect and bolster confidence, both of which are important for students with learning differences, who frequently exhibit low self-esteem and high anxiety related to mathematics. Teachers need to provide feedback to students immediately and as often as possible.

a. Affirmative feedback proves most effective when students receive specific comments about concepts or procedures like:
    i. I noticed you used the counting up strategy to add those two numbers.
    ii. I see you are using the geoboard to demonstrate the fraction two-thirds.
b. Corrective feedback is provided by teachers when students make a mistake or misunderstand a concept or procedure. Corrective feedback often involves asking questions and encouraging students to explain their steps and thinking to ensure teachers understand where and why the students have misconceptions about the task and procedures.
    i. Pose questions, such as:
        1. Can you explain the steps you followed for this problem?
        2. How did you arrive at 4 for the difference?
    ii. Question why students make specific errors to create a learning opportunity for other students by providing confirmation about the steps performed correctly and reviewing the steps needed to correct the mistake.
    iii. Encourage positive peer interaction and feedback, such as:
        1. Tell your partner how you solved the problem.
        2. Turn and talk about how you could create a drawing to represent this word problem.

4. Maintain a brisk pace

During modeling, teachers should maintain a brisk pace. Maintaining a brisk pace requires teachers to plan and organize prior to the lesson.

a. Before the lesson, teachers should consider the following questions:
    i. What materials will I need for the lesson?
    ii. what ways will I incorporate technology?
    iii. Does my seating chart promote optimal student learning and necessary movement during the lesson?
    iv. What planned examples will I use?
    v. Will I use any worked-examples?
    vi. Will I include any non-examples?
b. Teachers should be knowledgeable about the material and ready to provide effective modeling.
c. When teachers maintain a brisk pace, students pay attention and focus on the instruction.
d. Planning for the lesson and organizing needed materials ahead of time allow teachers to maintain a brisk pace, which maximizes student learning.

Practice

While modeling prepares students to complete the mathematics task successfully, practice provides multiple opportunities for students to practice the learned concepts. Practice includes: guided practice and independent practice.

- Guided practice
- Involves the teacher and students working together to solve a mathematics problem. The teacher solves the problem while students solve the same problem.
    - Takes place at a table, with the teacher and students working a problem together, or the teacher can solve the problem on the whiteboard as students complete the problem at their desks.

- Allows students to complete a problem for the first time with supports in place to promote understanding of the lesson concept and to encourage students' success.
- Involves the teacher and students working together using questioning and mathematics tools (e.g., manipulatives, hundreds chart, step-by-step checklist) to guide students through the problem.
- Involves collaboration among the teacher and students or among groups or pairs of students; however, guided practice is most effective when the teacher works with students to complete a few problems.
- Helps students with learning difficulties and provides a gradual release of responsibility from modeling to independent practice.

Many teachers include minimal guided practice opportunities or fail to include guided practice in their lessonsaltogether.

Teachers may model the lesson, then skip directly to independent practice.

Guided practice is essential for supporting students with learning difficulties and should be integral to everymathematics lesson.

- Independent practice
  - Consists of students working independently as the teacher continues to provide feedback and answer questions during completion of the task at hand.
  - Allows the teacher to determine if students understand the concepts and procedures taught during the lesson.
  - Should be implemented under the guidance and supervision of the teacher. o Understand that assigning independent practice as homework does not ensure that students are receiving the level of support necessary to understand or solve problems.

Supports During Practice

Effective practice continues the ongoing dialogue between the teacher and students. Ineffective practice encourages students to work independently without teacher support.

- When students are engaged in guided and independent practice, teachers should continue to attend to the four supports:
  - Ask high- and low-level questions
  - Elicit frequent responses
  - Provide immediate affirmative and corrective feedback
  - Maintain a brisk pace
- Explicit instruction involves modeling and practice, with supports embedded into every lesson.
- During introductory lessons, teachers may model several problems and provide a few practice opportunities while asking the right questions, eliciting responses, providing feedback, and maintaining a brisk pace.
- After teachers have introduced the material, they may choose to model one example and offer several practice opportunities for students while attending to the four supports.
- Explicit instruction is flexible and should vary from day to day and lesson to lesson and is based on the individual needs of the student. It should be used

Here is an action plan checklist to guide teacher to effectively implement explicit instruction. Ask yourself, "Does my explicit instruction include these necessary components?"

- Model steps using concise language
- Provide guided practice opportunities
- Provide independent practice opportunities
- Use supports during modeling and practice
  - Ask the right questions
  - Elicit frequent responses
  - Provide feedback
  - Be planned and organized

in conjunction with other highly effective mathematical practices such as engagement in rich problem solving and mathematical discourse

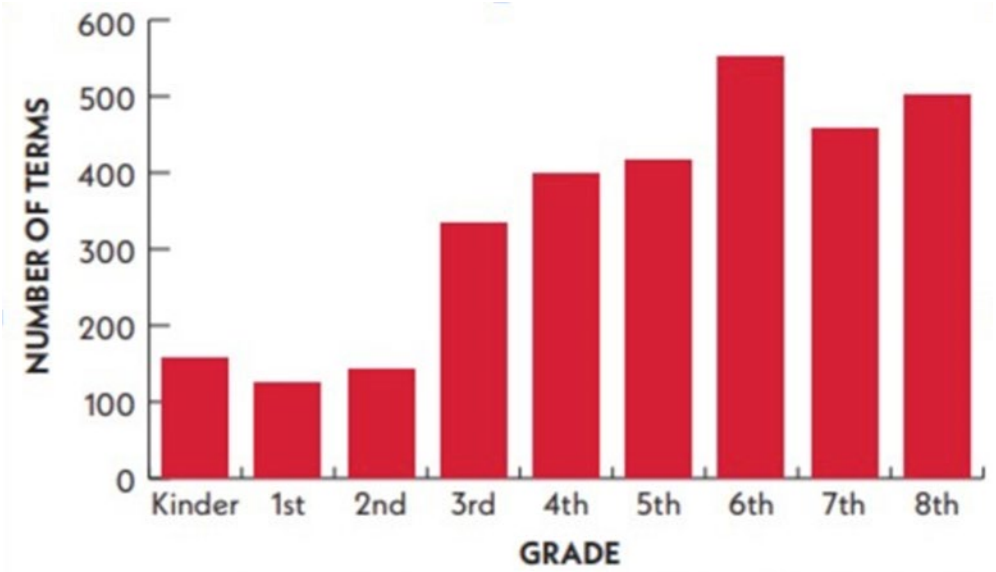
Resources

The National Center for Intensive Intervention has several resources related to explicit instruction.

- This Features of Explicit Instruction Module provides course content focused on enhancing teachers' effective implementation of explicit instruction, including modules on each of the explicit instruction components: modeling, practice, and supports.
- This Instructional Delivery Module provides mathematics-specific examples related to explicit instruction, along with numerous videos of tutors using explicit instruction with students to teach specific mathematics skills, such as eighth-grade algebra and first-grade addition and subtraction.

Formal Mathematical Language

As teachers use explicit instruction to model and provide students with practice, they also need to include formal mathematical language in their instruction. Formal mathematical language refers to the precise mathematical terms used to describe concepts and procedures. Example of formal mathematical vocabulary terms include sum, digit, and numerator. In contract, informal mathematical language consists of words like answer, number, and top number in the fraction.



Mathematics vocabulary across grades (Powell et al., 2019).

The above graph Mathematics Vocabulary across Grades illustrates the approximate number of formal vocabulary terms in commonly used elementary and middle school mathematics textbook glossaries and depicts the average number of formal mathematics vocabulary terms students are expected to learn at each grade level.

Mathematics vocabulary terms prove challenging for students with learning difficulties for the following seven reasons (Riccomini et al., 2015; Rubenstein & Thompson, 2002):

1. Technical terms
    a. Mathematics vocabulary includes technical terms, symbols, and diagrams specific to mathematics.
    b. Most students have never heard or seen the technical terms (e.g., numerator, parallelogram, Pythagorean Theorem).
    c. Many technical terms may be unfamiliar to students with language-related difficulties, including English Learners (ELs).

Examples:

d. The terms sum, value, and product have specific and complex mathematical definitions familiar to most speakers, but new to students with language-related challenges and ELs with limited knowledge of terms (Freeman & Crawford, 2008).

e. Symbolic vocabulary terms such as zero and equal may prove challenging as the symbols used to explain numerals and symbols may be unknown (Powell et al., 2017).

2. Multiple meanings in mathematics and everyday English

   a. Many mathematics vocabulary terms have multiple meanings in mathematics and English.

Examples:

   b. The word volume refers to the amount of space in mathematics but describes a noise level in everyday English.

   c. Cubed means raised to the third power in mathematics but explains a way to cut vegetables in everyday English.

3. Multiple meanings in mathematics

   a. Mathematics vocabulary concepts may be represented in multiple ways.

   b. Mathematics vocabulary concepts may present with multiple mathematical meanings.

   c. The same word may be used to describe more than one situation.

Examples:

   d. Over 10 different terms exist to describe subtraction (Moschkovich, 2002).

   e. A quarter may refer to a coin or a fourth of a whole (Moschkovich, 2002).

4. Multiple meanings across academic content areas and contexts

   a. Many vocabulary terms have multiple meanings across different content areas and various contexts.

Examples:

   b. The term base can refer to the base of an exponent, the base of a three-dimensional shape, a base versus an acid in chemistry, or the bases in baseball. Students also may think of the homonym bass, as in a bass guitarist.

What does the word degree mean?

Possible Responses

Angle *Deodorant* *Level* *Educational Achievement* *Temperature*

Teachers need to explicitly teach mathematics vocabulary terms like degree to help students understand to which definition they are referring.

5. Homonyms

   a. Some mathematics terms sound the same but have different meanings

Examples:

   b. Sum and some have very different meanings yet sound exactly the same.

   c. Additional homonyms like whole/hole, half/have, and symbol/cymbal are very confusing when

teachers only present the terms orally.

6. Vocabulary terms with multiple words
    a. Many vocabulary terms that students need to learn include multiple words (e.g., acute angle, rational number, triangular pyramid).
    b. Students are not only interpreting one word; students are interpreting multiple words and making connections between the words.
    c. Terms that require knowledge of two words for understanding may cause additional difficulty for students.
7. Similarities to or differences from native language words
    a. Students with a native language other than English may experience additional challenges with mathematics vocabulary terms due to their similarities and/or differences to their native language words.

    Example:

    b. The Spanish word for quarter is cuarto, which can mean quarter of an hour, but quarter also refers to a room in a house as in living quarters (Roberts & Truxaw, 2013).

Formal Mathematics Language Strategies

To promote students' understanding of mathematics vocabulary terms, teachers should:

1. Use formal mathematics vocabulary terms
    a. Teachers should consider the importance of using formal mathematical vocabulary terms (i.e., rather than informal phrases).

    Examples:

    b. Teachers may ask students to solve an equation by writing the answer. Even though answer is a commonly used term and can be used when students solve a range of equations, the term does not represent the mathematical operations required to solve each equation.
    c. Using the formal mathematical terms sum, difference, product, or quotient reinforces the calculations that students will perform to solve the problems, thus supporting the conceptual understanding of such terms.
    d. Students will encounter formalized mathematics vocabulary terms in mathematics texts and activities and on high- stakes assessments (e.g., "What is the difference between Teresa's money and Salvador's money?").
    e. Teachers need to expose students to these terms in preparation for such activities and tests.

Examples:

    f. On a test question about "Which three shapes are quadrilaterals?" if a teacher has not used and practiced the term quadrilaterals, it would be difficult for students to answer this question.
    g. When teaching students to tell time, teachers might refer to a clock as having a long hand and short hand. This language may hinder students' understanding of clocks because the informal language does little to convey an understanding of telling time. Instead, teachers should use the terms hour hand and minute hand, and students should use these terms regularly as well.
    h. To ensure students understand the formal mathematical language needed to solve problems and develop solutions, teachers need to explicitly teach mathematics vocabulary terms through the context of the problems being presented.
2. Use similar or related terms correctly and precisely
    a. Teachers need to be correct, precise, and specific when using closely related mathematical terms

(Powell et al., 2018).

Example:

b. Teachers may use the terms factor and multiple interchangeably, but these terms have distinct meanings. Using these terms interchangeably may cause confusion for students. The term factor refers to all of the whole numbers by which you can divide a number with no remainder (e.g., the factors of 6 are 1, 2, 3, and 6), but the term multiple refers to the number after multiplying 6 by another whole number (e.g., 6, 12, and 18 are multiples of 6). Each number has a fixed number of factors (i.e., there are four factors for the number 6), but many possible multiples (e.g., $6 \times 1 = 6$, $6 \times 2 = 12$, $6 \times 3 = 18$, etc.).

c. During lesson planning, teachers should reflect on which formal vocabulary terms to explicitly teach to students through the context of the problems being presented.

d. Teachers should consider any technical, sub-technical, symbolic, and/or general terms related to the lesson content.

    i. Technical terms, or vocabulary words with a specific mathematical meaning, should be explicitly taught to students.

    ii. Sub-technical terms include multiple meanings, at least one of which is related to mathematics. Teachers should think about students' exposure to sub-technical terms to determine which terms need to be explicitly taught in the lesson.

        1. For example, square can describe a shape or a location, such as a town square. Students may also square a number (52).

        2. Teachers may want to engage students in discrimination or sorting activities with a term that has several meanings or with several terms that share similar meanings.

    iii. Teachers should consider symbolic terms, which are the terms used in math to represent symbols.

        1. For example, students might be able to write $, but unable to read the word dollar.

        2. Understanding symbolic terms is essential when students are asked to read and interpret word problems.

    iv. General terms are frequently integrated into mathematics instruction.

Example questions for teachers to consider:

e. Do students know what it means to measure something?

f. Do students understand how to find the longest length or identify the shape located above?

g. For students with mathematics-learning difficulties, language can create an additional challenge.

h. Teachers' selection of terms should directly align with students' language skills, knowledge, and familiarity with the mathematics content.

Why does language present a challenge for students with mathematics difficulty?

- Comorbidity rates, or students identified with at least two disabilities, range between 30-70%.
- In many cases, students with mathematics learning disability or difficulty also experience reading difficulties.
- Let's consider a teacher is working with a small group of 4 students. Researchers estimate at least 2 of the 4students will have comorbid reading and mathematics challenges.

3. Plan for language use prior to instruction
    a. Teachers should consider their language use (as well as students' language use) prior to instruction.
    b. In many cases, teachers limit the mathematical language used within intervention or instruction. Teachers also may try to make the language easier for students (i.e., use informal language instead of formal language). Using limited or informal language does not prepare students for success.
    c. Students are exposed to formal mathematical language in their mathematics textbooks, on assessments, and in online mathematics videos. Teachers should present the same formal mathematical language during instruction to support students' long-term learning and mathematical understanding.
    d. Teachers may need to engage students in activities where they define similar terms (e.g., parallelogram, trapezoid, rhombus, rectangle, square, and kite) and describe how the terms are similar or different. During these explanations, teachers need to be specific and precise.
4. Include explicit vocabulary activities in instruction
    a. Teachers should include explicit vocabulary activities in their instruction to ensure students actively practice using vocabulary terms essential for understanding mathematics concepts.
    b. Instead of informally exposing students to mathematics vocabulary, teachers should directly teach vocabulary and provide meaningful practice opportunities for students (Riccomini et al., 2015).
        i. During explicit instruction of vocabulary terms, teachers and students can co-create concept maps, develop word walls, and encourage students to maintain individual dictionaries of mathematical terms.
        ii. Teachers are encouraged to use Virginia's Mathematics Vocabulary Word Wall Cards for reference.
    c. Teachers also may consider limited utilization of mnemonic devices to improve students' understanding of terms and access prior knowledge.
    d. Multiple exposure to mathematics terms that build fluency are essential for developing students' mathematical vocabulary competency.
        i. The use of flashcards (with the word on one side of the index card and the definition and a picture on the other side) and game-like activities offer useful ways to reinforce previously introduced mathematics vocabulary during instruction.
    e. Teachers can use flashcards as fluency builders during instruction, as a transition game as students wait in a line, or as an exit slip that requires students to describe a vocabulary word before exiting the classroom.
    f. Students may add their vocabulary cards to a ring and practice throughout the school day.
    g. Vocabulary game-like activities also increase students' motivation and learning.
        i. Word-O is an adapted form of Bingo and Word Sorts allow students to categorize, compare, and contrast words.
        ii. For a more extensive list and explanations of popular vocabulary games, visit the Flocabulary website. (Riccomini et al., 2015).
5. Hold students accountable
    a. Teachers should hold students accountable for using formal mathematical language correctly.
    b. In addition to listening to the language of mathematics, teachers need to create opportunities for students to speak about mathematics and write about mathematics using formal mathematical language.
        i. Some students go through an entire mathematics lesson without speaking the formal language of mathematics.
        ii. Without practice listening, speaking, writing, and reading in mathematics, students will not develop a strong lexicon of mathematical language.
        iii. Most mathematics content is disseminated through oral listening (i.e., listening to teachers or peers talking about mathematics) and reading (i.e., reading text online or in a textbook), whereas most learning occurs through speaking and writing.
        iv. To maximize students' learning, teachers must focus on the mathematics concepts and procedures; language is an essential component of this learning.

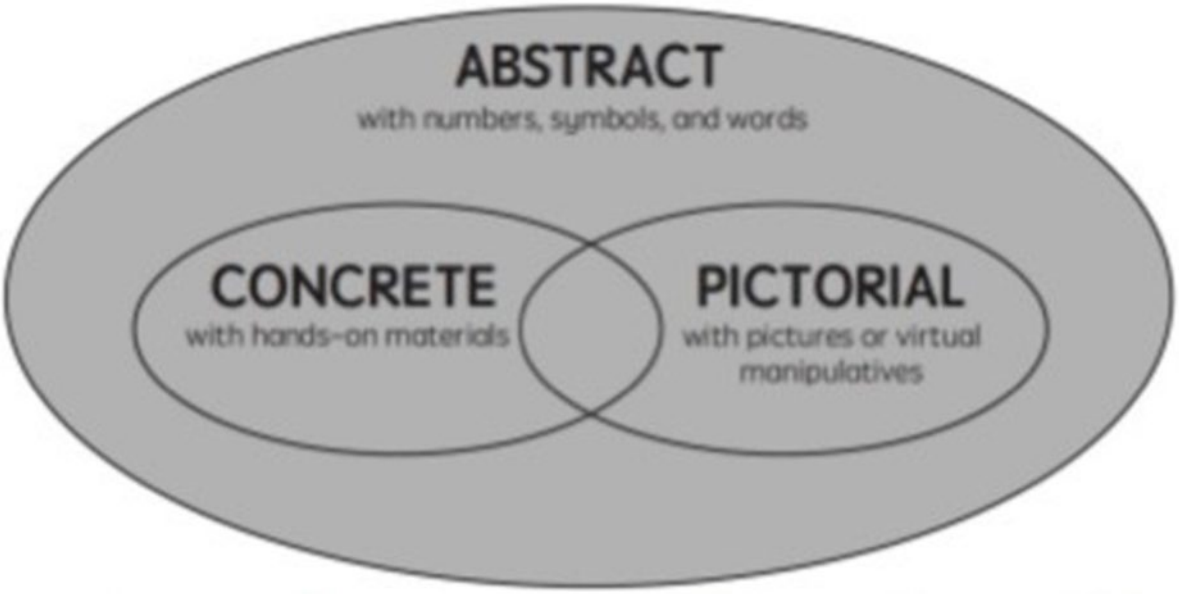
Resources

National Center for Intensive Intervention provides best practice for teaching mathematical language during K-12 mathematics instruction.

Concrete, Representational, and Abstract Connections

The concrete-representational-abstract (C-R-A) sequence is an evidence-based practice supported by research (e.g., Flores et al., 2014; Witzel et al., 2008). In this Resource Guide, C-R-A is presented as a framework in which the concrete, representational (pictorial), and abstract forms of mathematics work collaboratively to facilitate students' deeperunderstanding of mathematics concepts.

The C-R-A framework, displayed below, considers the diverse needs of students with learning difficulties by utilizing supports when necessary. That is, students with learning difficulties require varied levels of support; some students may require more practice with concrete forms while other students may benefit from a combination of concrete and pictorial supports to access the abstract. Viewed as a framework, students are better able to generalize their conceptual knowledge from the representational and concrete understandings to the abstract (Peltier & Vannest, 2018). Additionally, it is important for teachers to make explicit connections between all three levels of the understanding (Strickland & Maccini, 2013).



Concrete-Representational-Abstract framework

This framework includes three forms of mathematics: concrete, representational (pictorial), and abstract.

1 Concrete form
   a. Refers to the three-dimensional, hands-on materials and objects that students can touch and move to promote understanding of different concepts and procedures.
   b. Includes hands-on formal manipulatives such as fraction bars, algebra tiles, tangrams, geoboards, and two-color counters
   c. Includes hands-on manipulatives that are less formal (e.g., straws for measurement, paper clips for place value, or shoeboxes for three-dimensional figures).
2 Representational (Pictorial) form
   a. Includes two-dimensional pictures, images, or virtual manipulatives.
   b. In many cases, the pictorial is referred to as the semi-concrete or representational. As the abstract, concrete, and pictorial are considered representations of mathematics, the term pictorial is used to describe the third component of the multiple representations framework.
   c. Pictorial images may be presented within textbooks or workbooks, in teacher and student drawings, and on high- stakes standardized assessments.
   d. The pictorial form includes graphic organizers that help students understand mathematics concepts (e.g., Jitendra & Star, 2012).
   e. The pictorial may be presented in the form of technology through the use of virtual manipulatives.

i. For every hands-on manipulative, a corresponding virtual manipulative exists. Many of these virtual manipulatives are free or provided at little cost.

3 Abstract form

a. Consists of numbers, symbols, and words, and reflects the typical view of mathematics (e.g., 42 + 102 = 144).

b. In many cases, the abstract form of mathematics is students' destination, but teachers use the concrete and pictorial representations to support students' understanding of abstract concepts and procedures.

Benefits of C-R-A Connections

- Presenting mathematics content in many ways promotes understanding of abstract concepts for all students, especially those with mathematics difficulty.
- Together, the concrete and pictorial forms support the explanations of abstract concepts and procedures.
- When using the concrete or the pictorial, teachers should display and discuss the abstract form of a problem.

Example:

- If teachers use two-color counters to show the fraction two-thirds, they also need to write the fraction on the board.
- When learning about the relationship between fractions and decimals, students may use a digital or concrete geoboard to understand the connection.
- During this lesson, teachers also should display the abstract form of the fraction and decimal (e.g., is the same as 0.60).
- The use of C-R-A supports students developing a deeper conceptual understanding of mathematics beyond superficial procedural knowledge.
- Evidence shows that some students may need more practice with concrete or pictorial materials, whereas other students may benefit from more practice with abstract forms.
- Teachers need to remain flexible with their use of C-R-A to address the unique and diverse needs of students with disabilities.
- As students develop a conceptual understanding of the content, they often transition from concrete and pictorial representations to more abstract representations.
- Teachers must provide C-R-A with explicit instruction to explain and practice with the representations (van Garderen et al, 2012).

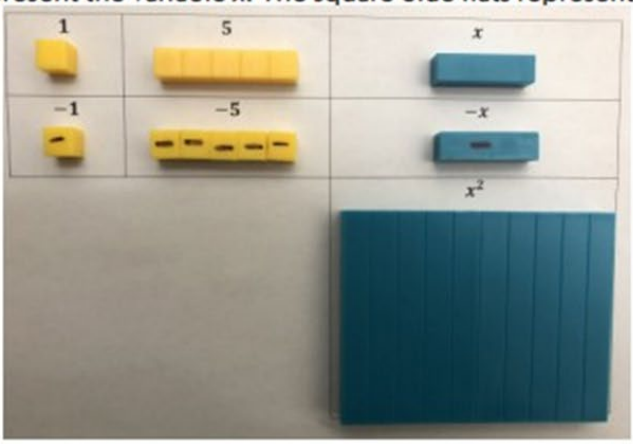
The charts on the next page further illustrate the benefit of using C-R-A. The charts compare teaching a mathematics concept with manipulatives versus without manipulatives. Specifically, the first example focuses on teaching middle and high school students to understand quadratic expressions, and the second example focuses on teaching elementary students to understand addition using the partial sums strategy.

The charts demonstrate that the simultaneous presentation of abstract, concrete, and pictorial forms of mathematics supports students with disabilities above and beyond the abstract presentation alone. At any grade level, the use of manipulatives benefits students in their development of conceptual understanding and procedural knowledge.

WITH MANIPULATIVES	WITHOUT MANIPULATIVES
Lesson Objective: Using x-rods, students will represent and sketch rectangle area problems involving linear expressions containing only positive terms to produce a quadratic expression.	*Lesson Objective:* Students will learn how to multiply binomials to create a quadratic expression.

WITH MANIPULATIVES

1. Students participate in an introductory lesson to multiply two binomial expressions using manipulatives as a visual aid/tool to represent quantity. The yellow units are used to represent integer constants; the long blue rods represent the variable x. The square blue flats represent x^2.



2. Students will utilize the manipulatives to demonstrate how they are manipulating the blocks to multiply the binomial expressions.
3. The teacher demonstrates how to use the manipulatives when multiplying linear expressions. The teacher emphasizes the distributive property using the corner piece and manipulatives.

4. Students multiply the linear expressions with manipulatives to complete the table below. They also sketch the manipulatives and write an equation in the form of the area formula for rectangles (length • width = area). The students use concrete manipulatives, sketched representations of the manipulatives, and abstract symbols such as variables and constants.

We all have a square shaped rec room where we like to hang out with our friends. We would like to buy a ping pong table but will need to expand all of our basements so that the length is increased by 4 feet and the width is increased by 2 feet. Fill in the table below.

	Side of original rec room in feet	*New length in Feet*	*New width in feet*	*New Area in square feet*
Amy	*9*	*9+4=13*	*9+2=11*	*13•11=143*
Shania	*10*	*10+4=14*	*10+2=12*	*14•12=168*
Delsa	*11*	*11+4=15*	*11+2=13*	*15•13=195*
Any square rec room	*x*	*X+4*	*X+2*	*(x+4)(x=2)*

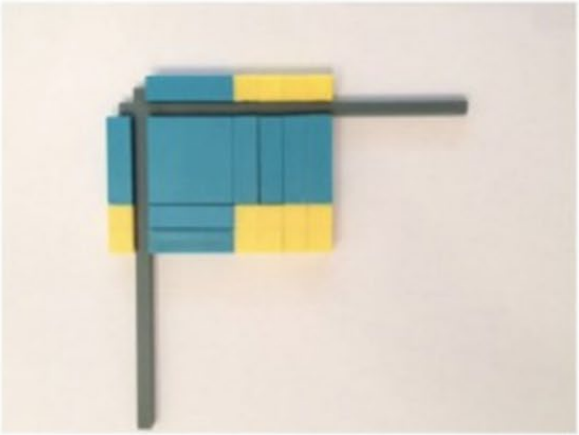
WITHOUT MANIPULATIVES

1. Students learn about quadratic expressions.
•
2. The teacher demonstrates how to multiply two binomials, such as $(x + 4)(x + 2)$. The teacher discusses multiplication using the distributive property and shows how to multiply $x \cdot x$, then $x \cdot 2$, then $x \cdot 4$, then $4 \cdot 2$.

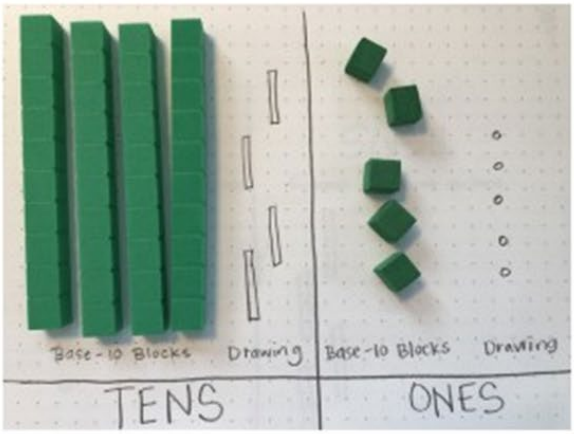
3. The teacher discusses simplifying $x^2 + 4x + 2x + 8$ which simplifies to $x^2 + 6x + 8$.

4. The students work a problem on their own.

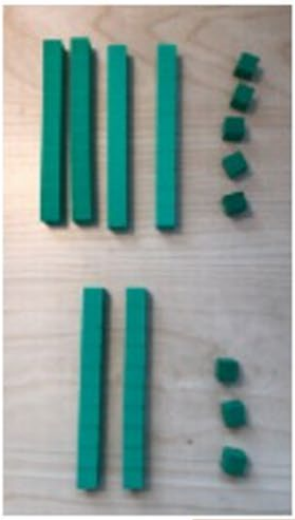
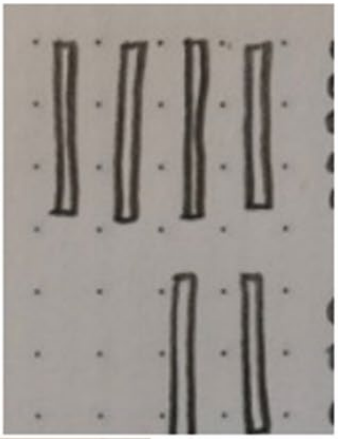
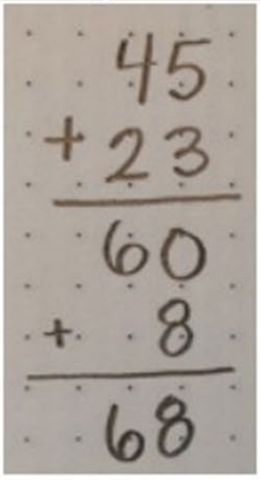
Teaching multiple linear quadratic expressions with and without manipulatives (Strickland, 2017)

WITH MANIPULATIVES	WITHOUT MANIPULATIVES
 *Write area equation: length • width = area* *$(x+4)(x+2) = x^2 + 6x + 8$* *What happened to the area of the bedroom?* *It became bigger* *What about the shape?* *Changed from a square to a rectangle* *How does the shape of the tiles compare to your drawings of Our renovated rec rooms?* *same* In the problem above $(x + 4)(x + 2)$, the first x-rod on the top of the corner piece is multiplied by the x-rod on the side of the corner piece to equal x^2, which is inside the corner piece. Second, the x-rod on the side of the corner piece is multiplied by the four yellow units on the top. This equals four x-rods that are placed inside the corner piece to the right of the x^2 flat and directly under the four yellow units on the top of the corner piece. Third, the two yellow units on the side of the corner piece are multiplied by the x-rods on the top of the corner piece. This equals two x-rods, which are placed inside the corner piece under the x^2 flat. Last, the four yellow units on the top of the corner piece are multiplied by the two yellow units on the side to equal 8 yellow units inside the corner piece to complete the rectangle. This translates to the quadratic expression of $x^2 + 4x + 2x + 8$ which simplifies to $x^2 + 6x + 8$.	

Teaching multiple linear quadratic expressions with and without manipulatives (Strickland, 2017)

WITH MANIPULATIVES	WITHOUT MANIPULATIVES
Lesson Objective: Using Base-10 blocks, students will learn how to solve two-digit problems using the partial sums strategy.	*Lesson Objective:* Students will learn how to solve two-digit problems using the partial sums strategy.
1. Students participate in an introductory lesson to solve two-digit problems using manipulatives as a visual aid/tool to represent quantity. The green rods represent tens; the green units represent ones. The teacher asks students what they recall about addition or what it means to add. Responses will likely include ideas such as addition is putting together or adding on. In an introduction to the Base-10 blocks, students recognize that the green rod represents ten of the unit cubes.  2. Students will utilize the manipulatives to demonstrate how they are solving the problem using the partial sums strategy. The students will add the tens for a partial sum by moving the rods together. The students will add the ones for a partial sum by moving the ones together. 3. The teacher demonstrates how to use the manipulatives when adding using the partial sums strategy. The teacher emphasizes the adding the tens then adding the ones, and the shows how to add the partial sums by counting all the rods and all the units. 4. Students add with manipulatives to complete the problem. Students also sketch the manipulatives. The students use concrete manipulatives, sketched representations of the manipulatives, and abstract symbols.	1. Students learn about the partial sums strategy in which students will add the tens then add the ones. 2. The teacher demonstrates how to solve a problem, such as 45 plus 23. The teacher describes adding the tens: 40 plus 20 to equal 60. The teacher writes 60. 3. The teacher describes adding the ones: 5 plus 3 to equal 8. The teacher writes 8. 4. The teacher shows adding 60 plus 8 for a sum of 68. The teacher writes 68. 5. The students work a problem on their own.

Teaching Addition Using Partial Sums with and without Manipulatives

WITH MANIPULATIVES	WITHOUT MANIPULATIVES
   In the problem above (45 + 23), the student shows or draws 45 as 4 rods and 5 units. The student shows or draws 23 as 2 rods and 3 units. The student sets up the manipulatives or drawing similar to the abstract form of the problem – with one addend placed on top of the other addend. The student adds the tens (green rods) together to equal 60. Then, the student adds the ones (green units) together to equal 8. The student adds 60 plus 8 for a sum of 68. 45 plus 23 equals 68.	

Teaching Addition Using Partial Sums with and without Manipulatives

Resources

Virtual Manipulatives

- National Library of Virtual Manipulatives
- Toy Theater Virtual Manipulatives
- Math Playground Virtual Manipulatives
- Math Learning Center Free Apps
- Geogebra

Project STAIR (Supporting Teaching of Algebra: Individual Readiness) YouTube channel

The links below provide examples for how to multiply linear expressions using a variety of manipulatives and graphic organizers within contextualized problems.

- Multiplying Linear Expressions – Part 1 – Using Algebra Blocks
- Multiplying Linear Expressions – Part 2 – Multiplying Positive Terms
- Multiplying Linear Expressions – Part 3 – Multiplying Positive and Negative Terms
- Multiplying Linear Expressions – Part 4 – Graphic Organizers and Contextualized Problems

National Center for Intensive Intervention

- Teach Counting Using Manipulatives Video
- Manipulatives to Illustrate Basic Facts: Addition Problem Structures Video
- Represent Place Value Concepts with Base-10 blocks Video
- Manipulatives to Illustrate Place Value Computation Video
- Manipulatives to Convert Improper Fractions to Mixed Numbers Video

Fact and Computational Fluency

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A student exhibits computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, are able to explain, and produce accurate answers efficiently.

The computational methods used by a student should directly align with the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties.

Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades and builds the foundation for later mathematics concepts. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of Grade 2 and those for multiplication and division by the end of Grade 4. Teachers should encourage students to use computational methods and tools that are appropriate for the context and purpose.

Strategies for Building Computational Fluency

Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. Flexible thinking strategies can be used to develop computational fluency with basic facts and can then be expanded to use with larger numbers.

Beyond basic facts, students should develop fluency with computation (i.e., multi-digit addition, subtraction, multiplication, or division).

- Teachers can use explicit instruction to model and practice different computational algorithms.

For Example:

- When teaching multi-digit addition, teachers should demonstrate two or more of the following algorithms across a number of weeks:
    - Traditional (work right to left)
    - Partial sums (work left to right)
    - Opposite change (round one number to the nearest ten; amount added is subtracted from the other number)
    - Column addition (work left to right).
- Students should share their ideas collaboratively with how to solve problems in different ways and then choose a strategy that they understand, can explain, and produces accurate answers efficiently.

To develop fluency with whole-number computation and rational-number computation, students must have ongoing, spaced practice opportunities. Teachers should introduce different algorithms for solving multi-digit problems and allow students to choose the method that works best for them. With computational fluency, memorization is not the goal but rather the development of efficiency and flexibility with computational procedures. Ultimately, teachers should consider embedding fluency activities into instruction when teaching any mathematics topic or concept.

Strategies for Building Fact Fluency

Fluency practice is an evidence-based strategy for supporting students with mathematics difficulty. Students with learning difficulties should be provided brief daily opportunities for fact practice as needed and include numeracy routines. Facts can and should be practiced in a variety of ways (i.e., using a variety of games, activities, songs, and worksheets). Students need practice in the use and selection of efficient strategies.

STRATEGIES FOR DEVELOPING ADDITION AND SUBTRACTION BASIC FACTS

STRATEGY	EXAMPLE
Use Counting on/Counting Back	5 + 3 = 8 (Students counts on from 5 to reach 8) 6 – 4 = 2 (Students count back from 6 to reach 2)
Use One More Than/Two More Than; One Less Than/Two Less Than	1 + 1 ; 2 + 1 ; 3 + 1 or 1 + 2 ; 2 + 2 ; 3 + 2 9 – 1 ; 8 – 1 ; 7 – 1 or 9 – 2 ; 8 – 2 ; 7 – 2
Use Doubles/Near Doubles	2 + 2 ; 3 + 3 ; or 3 + 4 = (3 + 3) + 1; 6 + 7 = (6 + 6) +1
Make 10 Facts	7 + 4 = (7 + 3) + 1 = 11; 5 + 8 = (5 + 5) + 3 = 13
Think Addition for Subtraction	For 9 – 5, think "5 and what number makes 9?"
Use of the Commutative Property	4 + 3 has the same value as 3 + 4
Use of Related Facts	4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4
Use of the Additive Identity Property	4 + 0 = 4; 9 + 0 = 9
Use Patterns to Determine Sums	0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.

The development of computational fluency, as it relates to working with larger whole numbers, relies on quick access to number facts. The patterns and relationships that exist within the multiplication and division facts are helpful in learning and retaining fact fluency. Studying the patterns and relationships provides students the opportunity to build a foundation for fluency with multiplication and division facts.

STRATEGIES FOR DEVELOPING MULTIPLICATION AND DIVISION BASIC FACTS

STRATEGY	EXAMPLE
Use Skip Counting	Students count by multiples of a number; 5, 10, 15, 20, 25 = 5 x 5
Use Doubles	Students know that doubling a number is the same as multiplying by 2; 7 x 2 = 7 + 7; 6 x 2 = 6 + 6
Use Foundational Facts to Derive Unknown Facts	Decomposing one of the factors in 7 x 6, allows for the use of the foundational facts of 5s and 2s. This knowledge can be combined to learn the facts for 7 (e.g., 7 x 6 can be thought of as (5 x 6) + (2 x 6))
Derive Unknown Facts	Deriving unknown facts from known facts may include: doubles (2s facts), doubling twice (4s facts), five facts (half of ten), decomposing into known facts (e.g., 7 x 8 can be thought of as (5 x 8) + (2 x 8)).
Use Properties of the Operations	commutative property (5 x 8 = 8 x 5); distributive property of multiplication allows students to find the answer to a problem such as 6 x 7 by decomposing 7 into 3 and 4 (e.g., 6 x 7= 6 x (3 + 4)) allowing them to think about (6 x 3) + (6 x 4) = 18 + 24 = 42
Think Multiplication for Division	For 30 ÷ 5, think "5 times what number equals 30?"
Use of Related Facts	6 x 3 = 18 , 3 x 6 = 18, 18 ÷3 = 6, and 18 ÷6 = 3
Use of the Multiplicative Identity Property	4 x 1 = 4; 10 x 1 =10; 1 x 22 = 22

Strategies for solving problems that involve multiplication or division may include mental strategies, the use of place value (i.e., partial products, the standard algorithm) and the properties of the operations (i.e., commutative, associative, multiplicative identity, and distributive, etc.).

When practicing facts, students may benefit from opportunities to individually graph their scores, as age appropriate, related to fact knowledge in order to track how their fluency is progressing.

Technology can serve as a valuable tool for developing students' fluency skills. Prior to choosing a fluency game or activity, teachers should select a game or activity that will:

- Provide practice with small sets of facts;
- Track and allow students to monitor their progress; and
- Provide feedback to students (particularly when students make errors).

Problem Solving

Teaching problem solving strategies is an essential evidence-based strategy for supporting students with mathematics difficulty. The focus of this section is effective instruction related to problem solving. Problem solving provides an opportunity for students to demonstrate mathematics competency, yet often proves challenging for students with learning difficulties.

To become a mathematical problem solver, students must apply mathematical concepts and skills and understand the relationships among them to solve problems of varying complexities.

Students also must recognize and create problems from real-world data and situations within and outside mathematics and apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students need to develop a set of skills and strategies for solving a range of problems.

To assess a student's problem-solving skills, word problems are often presented. A word problem (see below for a sample) is a scenario presented with words and numbers that requires students to interpret the prompt or question and provide a response. Word problems are often challenging for students with learning difficulties.

- Because reading, language, and the concepts and procedures of mathematics serve as prerequisite skills for understanding word problems, many students, especially those with learning difficulties, experience challenges when solving word problems (Jitendra & Star, 2011; Krawec et al., 2012; Xin et al., 2005).
- For students with learning difficulties, there are seven areas where students experience difficulty with word-problem solving: reading the problems, understanding vocabulary, identifying relevant information, interpreting charts and graphs, identifying the appropriate operation(s), and performing the computation(s).

Students solve word problems that typically fall into three different categories:

1. Directive word problems
    a. Students are provided directions to complete a task or find missing information.
    b. Directive problems are usually not contextualized.
2. Routine word problems
    a. Present numbers within the problem (or within a chart or graph).
    b. Include a word-problem prompt or question that encourages students to manipulate the numbers to find the answer.
    c. May involve one or two steps.
3. Non-routine word problems.
    a. Usually include multiple solutions or multiple ways to solve the problem.

The Following chart provides examples of each type of word problem.

DIRECTIVE	ROUTINE	NON-ROUTINE
Which expressions are equivalent to 3(2x-2y)?	A brownie recipe requires cup of sugar. How many batches can be made with cups of sugar?	The Pencil Company sells pencils in the following quantities: • Singles (1 pencil) • Bundles (10 pencils) • Boxes (100 pencils) • Cases (1,000 pencils) The Pencil Company just received an order for 2,342 pencils. However, they currently have only one case of pencils in stock, but they have a large quantity of the other packing sizes. Show at least three different ways that the pencils could be packed for this order. Explain your thinking using pictures, numbers, and words.
Rotate the shape 90 degrees counter-clockwise around the origin.	On July 1, the value of a stock was $37.40. On July 31, the value of the stock changed by a gain of $12.75. What was the value of the stock at the end of July?	

Examples of directive, routine, and non-routine word problem

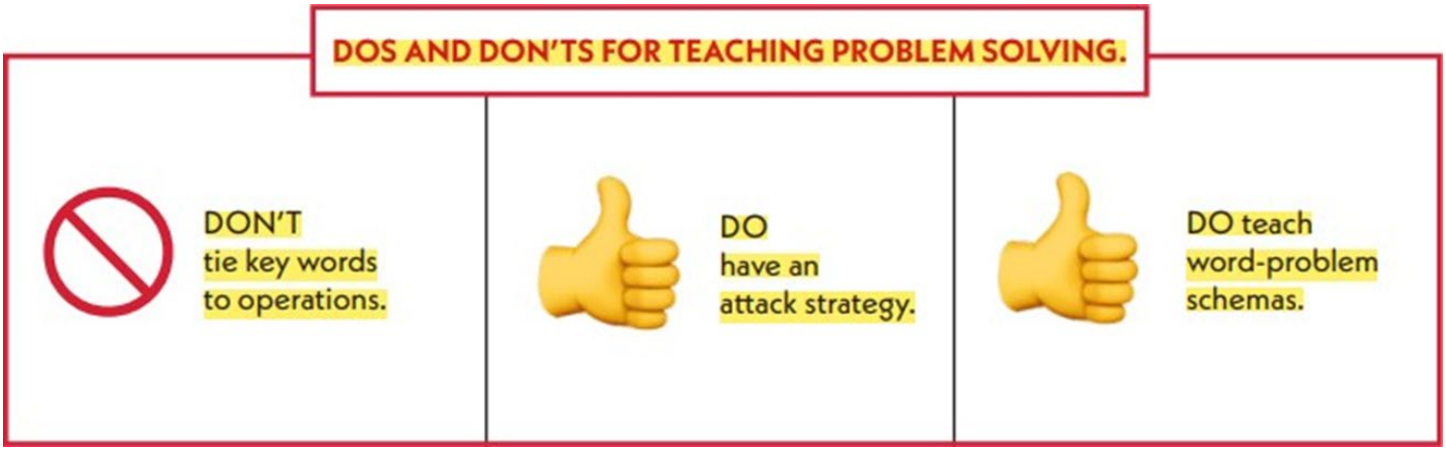
Strategies for Teaching Word Problem Solving

Because of the importance placed on problem solving within elementary, middle, and high school, a strong research base exists to help teachers understand how to provide effective word-problem instruction.

- First, students need to learn an attack strategy to help guide the process of problem solving (Jitendra & Star, 2012; Montague, 2008; Xin & Zhang, 2009).
- Second, students need to recognize and solve word problems according to the schema of the word problem

(Jitendra et al., 2002; Jitendra & Star, 2012; Van de Walle et al., 2013; Xin & Zhang, 2009).

- As part of appropriate schema instruction, teachers need to use appropriate mathematical language to help students understand the meaning of each word problem.
- Teachers should consider the following three points displayed in the figure below when teaching students with learning difficulties to solve problems.



1 DON'T tie key words to operations.

a. Teaching key words as a strategy for solving problems is an ineffective practice.
b. When keywords are tied to operations, students play seek and find with word problems. That is, students look for a keyword and then use the operation signaled by the key word without thinking conceptually about what the problem is asking.
c. In problem solving, emphasis should be placed on thinking and reasoning rather than on keywords. Focusing on key words such as in all, altogether, difference, etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses.
d. From a teaching perspective, it does not make sense to teach students a strategy (e.g., key words tied to operations) that leads to incorrect responses.

Examples:

e. Students often interpret share as a term that means division.
f. Share works as a division term in: Nevaeh wants to share 36 brownies with 6 friends. How many cookies will each friend receive?
g. Share does NOT work as a division term in: Nevaeh baked 36 brownies and shared 16 brownies with her friends. How many brownies does Cynthia have now?
h. Teachers should avoid defining word problems by the operation. Activities that label the operation (e.g., Division Problems) prove especially problematic because students may solve problems in different ways.
i. Neither tying key words to operations nor defining word problems by the operation have an evidence base to support their use.

2 DO make sure students have an attack strategy.

a. An attack strategy serves as a tool that students can use to structure their thinking before and during the solving of a word problem.
b. An attack strategy is grounded in metacognitive strategies and commonly uses a mnemonic and/or acronym to prompt students' recall.
c. An attack strategy often includes self-regulation components, whereby students ask themselves questions and monitor their performance as they solve a word problem.
d. An attack strategy generally follows the outline of problem solving from George Pólya (1945):
    i. Understand
    ii. Plan
    iii. Carry out
    iv. Look back

e. Teachers should confirm that the first step in the attack strategy is to read the problem.
f. Teachers can use their preferred attack strategy with students (and students only need one).
g. Regardless of the selected attack strategy, students should use the same attack strategy when solving every word problem.
h. A frequently used attack strategy is called UPS Check. Students follow a set of explicit steps for setting up the word problem by being asked to:
    i. Understand: involves a careful reading of the problem and asking, "What is the problem mostly talking about?"
    ii. Plan: includes a focus on the underlying schema of the word problem (see next section below).
    iii. Solve: involves solving the problem using a computational strategy of the student's choosing.
    iv. Check the answer: includes using self-regulation skills to ask, "Does this answer make sense and why?"
i. With any attack strategy, students need explicit instruction (i.e., teaching with modeling and practice opportunities) to learn how to apply the attack strategy to different word problems.
j. When problem solving is the focus of instruction, attack strategies must be modeled regularly and practiced by the students.
k. An attack strategy poster displayed on the wall of the classroom only proves helpful when teachers refer to the poster and students utilize the attack strategy.
l. At some point, some students rely less on the attack strategy as the problem-solving process becomes ingrained. However, teachers should model the use of an attack strategy until students understand and routinely use the attack strategy independently when solving word problems.

3 **DO Teach word-problem schemas:**
Once students learn an attack strategy, the next step is to teach students to understand word problems according to their schema, or problem structure.

a. A schema is a framework for solving a word problem (Powell, 2011). In schema instruction, students are taught to recognize a given word problem as belonging to a schema (i.e., problem type) and employ strategies to solve that word-problem type.
b. Almost all routine word problems that students see and solve in elementary and middle school fall into one of six different schemas:
    i. Total
    ii. Difference
    iii. Change
    iv. Equal Groups
    v. Comparison
    vi. Ratios or Proportions

Schema Instruction for Solving Word Problems

Research on schema instruction has highlighted the benefit of teaching this strategy to students (Fuchs, et al., 2004; Fuchs et al., 2008; Jitendra et al., 2007). Many students with learning difficulties have trouble setting up and solving word problems. Students experience challenges when identifying the relevant information necessary for solving the problem (Krawec, 2014) and when selecting the operation to use for computation or performing the computation (Kingsdorf & Krawec, 2014). To alleviate these difficulties, teachers can help students focus on the underlying schema of a word problem and provide practice identifying relevant information, using graphic organizers to organize important information, and solving the problems.

ADDITIVE SCHEMAS

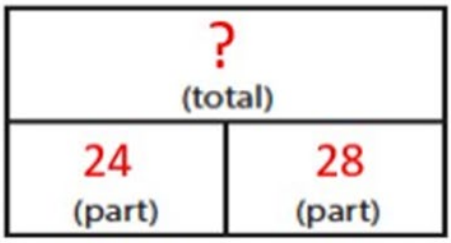
- The three additive word-problem schemas include Total, Difference, and Change problems.
- Additive schemas involve word problems in which addition or subtraction may be used for solving the problem. The operation, however, does not define the word problem – the schema defines the word problem.

1 **Total problems**

a. In Total problems, two or more separate parts combined for a sum or total (Kintsch & Greeno, 1985; Fuchs et al., 2014).
b. Total problems may be referred to as Combine or Part-Part-Whole problems.
c. Total problems require an understanding of part-part-whole relationships (i.e., the whole is equal to the sum of the parts; Jitendra et al., 2007).
d. In Total problems, the unknown may be the total or one of the parts.
e. After determining that a word problem adheres to the Total schema, students can use a graphic organizer to organize the word-problem information.
f. Determining how to apply the numbers from the word problem and use the numbers appropriately often proves difficult for students with learning difficulties; therefore, a graphic organizer makes this task easier.
g. Total problems also may include more than two parts, which requires an adjustment to the graphic organizer.
h. The figure below shows a worked example of a Total problem.
    i. The student used the UPS Check attack strategy to set up and solve the word problem.
    ii. The student underlined the focus of the problem (i.e., cookies).
    iii. The student determined that there was no irrelevant information; all of the numbers referenced cookies.
    iv. The student decided this problem was a Total problem because the statement asked about the total number of cookies Meghan baked in all. The student used the graphic organizer to organize the numbers from the word problem.
    v. The student identified one part as 28 and one part as 24.
    vi. The student used a question mark to represent the unknown. In this example, the unknown was the total.
    vii. The student added to determine the total was 52.
    viii. The student wrote a label, cookies, for the number answer and checked to make sure the answer made sense (i.e., the sum was greater than both addends).

Sample Total problem:

Megan baked 28 sugar cookies and 24 chocolate chip cookies. Enter the total number of cookies Megan baked in all.



Total = 52 cookies

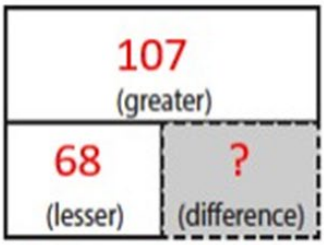
2 **Difference problems**

a. In Difference problems, students compare an amount that is greater and an amount that is less to find the difference.
b. The unknown may be the amount that is greater, the amount that is less, or the difference.
c. Difference problems also may be referred to as Compare problems.
d. Students may use a graphic organizer to solve difference problems.
e. The figure below shows a worked example of a Difference problem.
    i. The student used the UPS Check attack strategy to set up and solve the word problem.
    ii. The student underlined the focus of the problem (i.e., wooden beads and glass beads).
    iii. The student determined that there was no irrelevant information; all of the numbers referenced beads.
    iv. The student determined the problem was a Difference problem because the problem was comparing Jana's wooden beads to Jana's glass beads to find the difference.

v. The student used a graphic organizer to organize the numbers from the word problem.
vi. The student identified Greater as 107 and Less as 68.
vii. The student used a question mark to mark the unknown. In this example, the unknown was the difference.
viii. The student subtracted to determine the difference was 39.
ix. The student wrote a label, (more) beads, for the number answer and checked to make sure the answer made sense (i.e., the minuend was greater than the subtrahend and difference).

Sample Difference problem

Jana has 107 wooden beans and 68 glass beads. How many more wooden beads than glass beads does Jana have?



Difference = 39 Beads

3 Change problems

a. Change problems usually begin with an initial quantity and something happens to increase or decrease that quantity.
b. In Change problems, the starting amount, the change amount, or the end amount can be unknown.
c. Change problems also may be referred to as Join or Separate problems.
d. A graphic organizer can be used to represent the starting amount, change, and ending amount.
e. There are a few unique features in Change problems.
   i. First, students must determine if the change amount increases or decreases the end quantity to accurately solve the word problem.

   Examples:

   1. Sarah had $55. Then, her friend gave her more money to go shopping. Now, Sarah has $78. How much money did her friend give her? The change amount increases the overall end quantity and students should use the graphic organizer to show change increase.
   2. Sarah had $55. Then, she gave her friend some money. Now, Sarah has $32. How much money did Sarah give her friend? Students should use the graphic organizer to show change decrease because the change amount decreases the overall end quantity.
   3. Change problems may include several change amounts within the problem that either increase or decrease the end quantity.

   Example:

   4. Ebony had $12. Then, her dad gave her $23. Eboni then spent $5 on candy. How much money does Eboni have now? In these cases, the graphic organizer can be altered to show a change increase and then a change decrease.

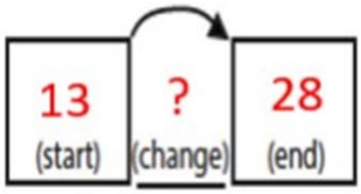
   The figure on the following page shows a worked example of a Change problem.

   1. The student used the UPS Check attack strategy to set up and solve the word problem.
   2. The student underlined the focus of the problem (i.e., passengers).
   3. The student determined that there was no irrelevant information; all of the numbers referenced passengers.
   4. The student determined the problem was a Change problem because the problem described a bus that started with some passengers, then more passengers got on the bus.

5. Because the change amount increased the overall end amount (i.e., start amount was 13; end amount was 23), the student used the graphic organizer to organize the numbers from the word problem and show a change increase.
6. The student identified start as 13, change as missing, and end as 28.
7. The student used a question mark to represent the unknown. In this example, the unknown was the change amount.
8. The student subtracted to determine the change amount was 15.
9. The student wrote a label, passengers, for the number answer and checked to make sure the answer made sense (i.e., the sum was greater than both addends).

Sample Change problem

A bus had 13 passengers. At the next stop, more passengers got on the bus. Now, there are 28 passengers. How many passengers got on the bus?

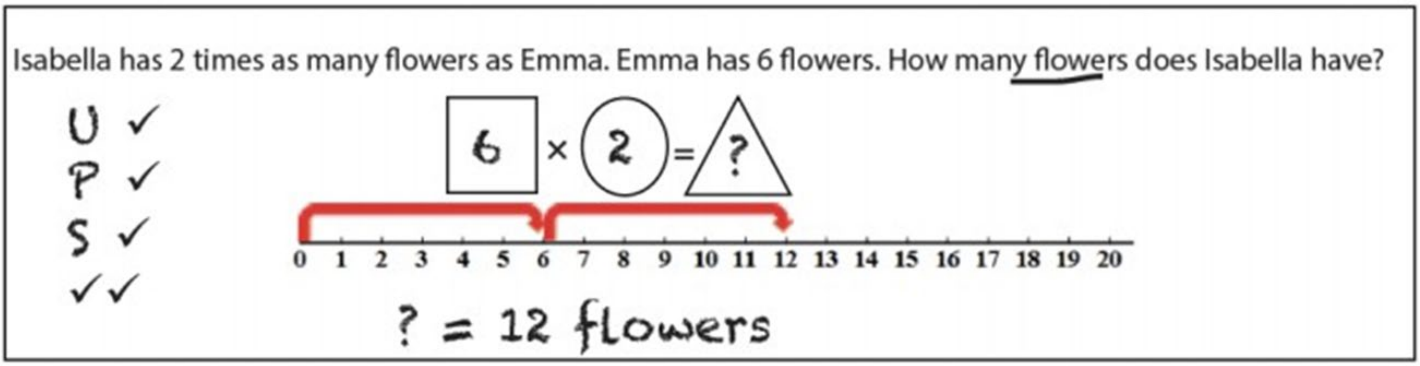


Change = 15 passengers

4 Comparison problems

a. In Comparison problems, a set is multiplied a number of times for a product.
b. Most comparison problems require students to determine the product; however, the unknown may be the set, the multiplier, or the product.
c. Students can use a graphic organizer or students can visualize the comparison of the original set on a number line.
d. Typically, Comparison problems are introduced in the elementary grades after the Equal Groups schema, and students continue to solve Comparison problems throughout late elementary school and into middle school.
e. The figure on the following page shows a worked example of a Comparison problem.
    i. The student used an open number line and the graphic organizer to focus on the set that was multiplied two times (i.e., twice).
    ii. The student used the UPS Check attack strategy to set up and solve the word problem.
    iii. The student underlined the focus of the problem (i.e., flowers).
    iv. The student also determined that there was no irrelevant information; all of the numbers
    v. referenced flowers.
    vi. The student determined the problem was a Comparison problem because the problem had a set, Emma's flowers, multiplied two times for a product.
    vii. The student wrote the 6 and 2 in the corresponding spaces for the set and multiplier in the graphic organizer.
    viii. The student used a question mark in the corresponding area for the product in the graphic organizer to mark the unknown.
    ix. The student used the number line to count 6 two times and determined the product was 12 (see red arrows in the figure).
    x. The student wrote a label, flowers, for the number answer and checked to make sure the answer made sense (i.e., the product was greater than both factors).

Sample Comparison Problem



5 Ratios or Proportions problems

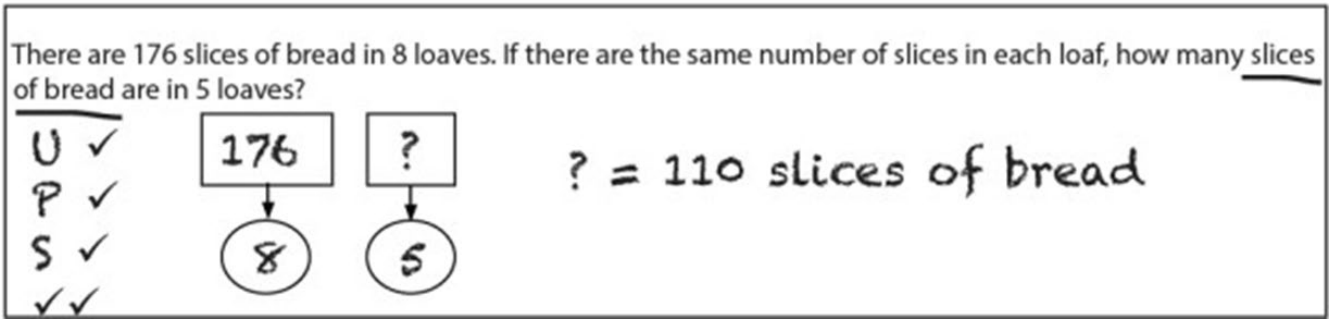
a. In Ratios or Proportions problems, students identify the relationships among quantities. This schema can be used to solve word problems about ratios, proportions, percentages, or unit rate, and the unknown may be any part of the relationship.

b. In middle school, particularly for Grade 7 and Grade 8, one of the most widely used schemas that could be used is Ratios or Proportions.

The figure on the following page presents a worked example of a typical Ratios or Proportions word problem using a graphic organizer from Jitendra and Star (2011).

i. The student used the UPS Check attack strategy to set up and solve the word problem.
ii. The student underlined the focus of the problem (i.e., slices of bread).
iii. The student determined that there was no irrelevant information; all of the numbers referenced bread.
iv. The student determined the problem was a Ratios or Proportions problem because the problem described a relationship among quantities: the number of slices of bread in loaves.
v. The student wrote the 176, 8, and 5 in the corresponding spaces in graphic organizer.
vi. The student used a question mark to reference the unknown, or the number of slices in 5 loaves.
vii. After the student completed the graphic organizer and filled in the important information, the student could have solved the problem using several methods:
    1. The student could have used cross multiplication and division.
    2. The student could have identified the relationship between 8 and 176 (i.e., 8 times 22 equals 176) and applied this relationship to the 5 by multiplying 5 by 22.
ii. Finally, the student wrote a label, slices, for the number answer of 110 and checked to make sure the answer made sense (i.e., both improper fractions yielded the same number, 22).

Sample Ratios Problem



The sample proportions problem below shows how the Ratios or Proportions schema may be presented in another way in middle school.

- The student was presented with a ratio of 3 to 5.
- The student learned to interpret this ratio as a fraction (i.e., 3 of every 8 students are boys).
- By translating the ratio to a fraction, the student was able to use a similar set up to the Proportions problem solved in the sample Ratios problem. In this way, the Ratios or Proportions schema offers flexibility that helps students to solve complex word problems in middle school.

Sample Proportions Problem

Dale Middle School has 440 students. The ratio of boys to girls is 3:5. How many boys are in the school?

$$\frac{3}{8} = \frac{?}{440} \qquad ? = 165 \text{ boys}$$

Finally, providing students with verbal and gestural cues to review and recall the six schemas has benefitted students with learning difficulties. As students work to determine the specific schema for a word problem, teachers should include language prompts by asking the following questions:

As students question, it is recommended that teachers pair the question with an accompanying gesture for each of the six schemas. The Resource Project STAIR (Supporting Teaching of Algebra: Individual Readiness) Youtube channel provides demonstrations of the gestures for Total, Difference, Change, Equal Groups, Comparison, and Ratios or Proportions. Listed below are some of the additional demonstration videos.

RESOURCES

Project STAIR (Supporting Teaching of Algebra: Individual Readiness) Youtube channel

- Schema Gestures provides a demonstration of the gestures for the three additive schemas (Total, Difference, and Change) and the three multiplicative gestures (Equal Groups, Comparison, Ratios or Proportions).
- Total Schema provides a demonstration of how to solve Total word problems.
- Difference Schema provides a demonstration of how to solve Difference word problems.
- Change Schema provides a demonstration of how to solve Change word problems.
- Equal Groups Schema provides a demonstration of how to solve Equal Groups word problems.
- Comparison Schema provides a demonstration of how to solve Comparison word problems.
- Ratios or Proportions Schema provides a demonstration of how to solve Ratios and Proportions word problems.

Problem Solving Attack Strategies

TOTAL
Are parts put together for a total?

DIFFERENCE
Are two amounts compared for a difference?

CHANGE
Is there a starting amount that increases or decreases to a new amount?

EQUAL GROUPS
Are there groups with an equal number in each group?

COMPARISON
Is there a set compared a number of times?

RATIOS OR PROPORTIONS
Are there relationships among quantities – if this, then this?

Understand **Plan** **Solve** **Check** *(adapted from Pólya, 2009)*	**Find the problem** **Organize information using a diagram** **Plan to solve the problem** **Solve the Problem** *(Jitendra and Star, 2012)*
Search the word problem **Translate the words into an equation or picture** **Answer the Problem** **Review the solution** *(Gagnon and Maccini, 2001)*	**Read (for understanding)** **Paraphrase (your own words)** **Visualize (a picture or a diagram)** **Hypothesize (a plan to solve the problem)** **Estimate (predict the answer)** **Compute (do the arithmetic)** *(Montague, 2008)*

High Leverage Practices

High leverage practices were created by a team of special education researchers and published by the CEC and the

Collaboration for Effective Educator, Development, Accountability, and Reform (CEEDAR) in a book titled High Leverage Practices in Special Education (Riccomini et al., 2017 as cited in McLesky et al., 2017). The 22 HLPs are beneficial to Special Education Teachers, Prospective Teachers, and all K-12 Student learning as collaboration, assessment, social-emotional-behavior supports, and instruction are the four categories High Leverage Practices are organized within.

Riccomini et al. (2017) as cited in McLesky et al. (2017) states:

> HLPs address many aspects related to the delivery of special education—collaboration, assessment, social-emotional-behavior supports, and instruction. Criteria for selecting the HLPs specify that each must (a) focus directly on instructional practice, (b) occur with high frequency in teaching in any setting, (c) be research based and known to foster student engagement and learning, (d) be broadly applicable and usable in any content area or approach to teaching, and (e) be fundamental to effective teaching when executed skillfully.

The HLPs are organized as:

Collaboration

1 Collaborate with professionals to increase student success.
2 Organize and facilitate effective meetings with professionals and families.
3 Collaborate with families to support student learning and secure needed services.

Assessment

4 Use multiple sources of information to develop a comprehensive understanding of a student's strengths and needs.
5 Interpret and communicate assessment information with stakeholders to collaboratively design and implement educational programs.
6 Use student assessment data, analyze instructional practices, and make necessary adjustments that improve student outcomes.

Social/ Emotional/ Behavioral

7 Establish a consistent, organized, and respectful learning environment.
8 Provide positive and constructive feedback to guide students' learning and behavior.
9 Teach social behaviors.
10 Conduct functional behavioral assessments to develop individual student behavior support plans.

Instruction

11 Identify and prioritize long- and short-term learning goals.
12 Systematically design instruction toward a specific learning goal.
13 Adapt curriculum tasks and materials for specific learning goals.
14 Teach cognitive and metacognitive strategies to support learning and independence.
15 Provide Scaffolded Supports
16 Use Explicit Instruction
17 Use Flexible Grouping
18 Use strategies to promote active student engagement.
19 Use assistive and Instructional Technologies.
20 Provide Intensive Instruction.
21 Teach Students to maintain and generalize new learning across time and settings.
22 Provide positive and constructive feedback to guide students' learning and behavior.

Please see the High-Leverage Practices from TeachingWorks for examples of how High Leverage Practices can be implemented in the classroom for all areas of instruction.

Assistive Technology

Definition

IDEA defines an Assistive technology as "any item, piece of equipment, or product system, whether acquired commercially off the shelf, modified, or customized, that is used to increase, maintain, or improve the functional capabilities of a child with a disability. The term does not include a medical device that is surgically implanted, or the replacement of such a device. (34 C.F.R. § 300.5; 92 NAC 51 § 003.02)"

> The Illinois Assistive Technology Guidance breaks down the IDEA definition as:
>
> "Any item" can be interpreted broadly. AT ranges from more complex items such as computer-based technology and software to everyday items like small balls that can be used to modify pencils for alternative grasps. "Product system" refers to the idea that an AT solution often requires multiple technologies working together to benefit a student with a disability. The concept of a product system is analogous to a computer and software. Software alone cannot run without a computer, and a computer is unable to provide much benefit without the software. An example of this concept in application is a student who requires an augmentative or alternative communication (AAC) device mounted to his or her wheelchair, as well as a switch to activate the device. All the technologies must work in concert for the student to benefit from the AT system.
>
> "Whether acquired commercially off the shelf, modified, or customized" means that commonly available technology may be used as AT tools or AT systems purchased and used as AT to increase functional capabilities. Often, however, they need to be adapted to a student's individual needs. This idea is similar to buying a car. Before driving it, the buyer will most likely adjust the seat positions, mirrors, tilt of the steering wheel and so forth. The buyer may even add a wrap to keep the steering wheel from getting hot in the summer. All those changes make the car better for the driver who bought it. The same is true of AT. Once out of the box, AT may need to be modified or customized for the individual student. Support personnel may need to adjust the device or system programming or alter the way the student physically interacts with it.
>
> "That is used to increase, maintain, or improve the functional capabilities of a child with a disability" relates to the reason the AT tool or system is provided to the student. Functional capabilities are the skills and activities students must perform effectively to succeed in school. Among them are eating, drinking, toileting, seeing, hearing, communicating, reading, writing, paying attention, and getting to and around school. (p.1-2)."

Furthermore, it can be said that AT provides a compensatory benefit to a student with a disability, according to Cook, Polgar, & Encaracao, (2020), Edyburn (2000), Lewis (1993), Parette, Peterson-Karlan, Wojcik and Bardi (2007), and Wojcik (2005). All proposed that AT is any tool (or system of tools) allowing a person to complete a task at an expected performance level when that would not otherwise be possible. In short, AT helps students show what they know and compensate for a barrier posed by their disability.

IDEA also states under Part B that Assistive Technology should be made available to a child if it's needed as a part of the child's Special Education services, related services, or supplementary aids and services. For further information about this specific regulation visit Sec. 300.105 Assistive Technology.

Assistive Technology Resources

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- Educationtechpoints.org has some useful tools when considering students' needs for classroom materials, accommodations, modifications, and assistive technology.
- The following table provides examples of possible assistive technology resources that can support students in mathematics. This list is not endorsed by the NDE, nor is it exhaustive.

NAME OF RESOURCE	DESCRIPTION
Accessible Math Tools for the Classroom	National Center on Accessible Educational Materials (AEM)- This page provides an overview of classroom tools for making math content more accessible with supports such as text to speech for reading math expressions aloud, handwriting recognition, sonification of graphs and more.
Mathshare	Mathshare helps students show and organize their math work. Mathshare is a free, open source tool developed by Benetech, a nonprofit that empowers communities with software for social good, whose mission is to make math accessible for all students. Mathshare is free for schools to use and for learning management systems to integrate into their platforms. This Mathshare Accessibility Features document describes the accessibility features that are offered in Mathshare.
Desmos	The online calculator, Desmos, offers several accessibility features such as keyboard shortcuts and features for visually impaired students.
Equatio	Equatio software allows users to create mathematical equations, formulas and more directly on their computer. A user can type, handwrite, or dictate any expression, and Equatio will convert it to accurate digital math which can be added into a Microsoft Word doc or G Suite apps. This software is offered free for teachers.

Nebraska Practices

Individual IEP and IFSP teams have the primary responsibility for determining the assistive technology needs of students and children in Nebraska.

Assistive technology and technology devices are to be considered when developing, reviewing, and revising students' IEPs. This document provides multiple resources to further educate IEP team members of potential assistive technology and services. Districts should use a well defined decision process when discussing assistive technology needs of students. Assistive technology, accommodations, and modifications should only be implemented in a child's IEP if the child truly needs the support. Students should be utilizing these supports in their daily assignments and learning; and should never be utilizing these supports only on a district or standardized test. Always reference back to the IEP created as a team when considering what is best for the student's success and revise when needed.

A student is not simply given AT. AT must be embedded in a process that ensures that it is successful (Bowser et al., 2015). Four main components are included in this process: AT consideration, Provision of AT, AT implementation, and the AT Progress Monitoring. Each component is explained in the following table.

Explanations of the Four Main Components within an AT Process

Component	Explanation
AT Consideration	An educational/IEP team determines whether or not the student needs AT through reviewing data related to a student's current knowledge/skills, areas of difficulty, and curricular/IEP goals.
Provision of AT	If AT is determined to be necessary for a student to make progress, the team determines how the selected AT will be acquired, provided, and funded.
AT Implementation	The team ensures that the student can use the AT through determining what training needs to be provided, who needs the training and what supports need to be in place for the AT to be used effectively. Back-up plans are developed for when the AT malfunctions or is not accessible.
AT Progress Monitoring	The team engages in collecting data to determine if the AT a student is using is working, if a new AT tool/system is needed, or a student no longer needs the AT.

Nebraska AT Contact Information

Nebraska ATP Education Program	Mailing Address: 3901 N. 27th Street, Suite 5 Lincoln, NE 68521
Toll Free: 877-713-4002 Phone: 402-471-0734	Website: https://education.at4all.com
Email: atp.education@nebraska.gov	

For additional information regarding Assistive Technology please view this document: Illinois Assistive Technology Guidance Manual

Math Modifications and Accommodations

Modifications

What are general recommendations for modifications?

A modification is a change in the course of study, standards, test preparation, location, timing, scheduling, expectations, student response and/or other attributes, which provide access for a student with a disability to participate in a course, standard, or test. It does fundamentally alter or lower the standard or expectation of the course, standard, or test.

Modifications are generally made in content for students with significant cognitive disabilities and/or physical disabilities. Modifications involve lowering the level of the materials presented and/or reducing the depth and complexity of the content.

Examples of modifications include:

- Materials are adapted; texts are simplified by modifying the content areas—simplifying vocabulary, concepts, and principles.
- Grading is subject to different standards than general education, such as based on IEP goals.
- Assignments are changed using lower level reading materials, worksheets, and simplified vocabulary.
- Testing adaptations are used, such as lowering the reading level of the test.

Modifications to the curriculum for students with more significant learning and behavior needs can be made in the general education setting if the IEP team determines appropriate. The student will have an IEP which specifies which specific learning objectives will need modifications and will identify how the student will be held accountable for making progress. In some cases, the student will receive educational benefit by being exposed to more of the general education curriculum than the student is being held accountable for or required to master. The student may be working on objectives below grade level, but the subject should be the same as the rest of the class. In addition, depending upon the nature and severity of the student's disability, it may be that the student requires in-class support to be successful within the LRE.

Below is a general list of modifications for teachers to consider as they plan their mathematics instruction for students with disabilities.

General recommendations for modifications in mathematics include changes to the following:

- Instructional level of the concept or task
    - If a fourth-grade class is solving word problems but a student is functioning on a second-grade level, modify the content of the word problems to align with the second-grade standards.
    - If a student is working on developing knowledge about addition and the class is practicing addition and subtraction facts, present the student with fact fluency practice activities that focus solely on addition.
    - If a student has not mastered a skill (e.g., multiplication), continue to practice the skill with the student while the class moves onto a different skill or standard.
    - Use alternative books or materials on the topic being studied.
- Performance criteria
    - Assess students using a different standard than other students,
    - Offer a pass/no pass option, and
    - Modify grades based on the IEP.

- Assignments
    - Answer different homework problems than peers,
    - Answer different test questions than peers,
    - Create alternative projects or assignments,
    - Rewrite questions using simpler language, and
    - Allow outlining instead of writing for a mathematics writing prompt.

Accommodations and Instructional Strategies

What are accommodations?

Accommodations refer to the supports that allow students to access their grade-level curriculum and demonstrate learning and mastery of grade-level content. IEP teams determine both accommodations and modifications based on individual needs. Some of the most common accommodations include calculators, multiplication charts, and manipulatives. Accommodations provided to students with disabilities as part of the instructional and assessment process should allow equal opportunity to access the assessments. Accommodations used on the state assessments must be documented in the student's Individualized Education Program (IEP) or 504 Plan and used in daily instruction. The Nebraska Student-Centered Assessment System (NSCAS) Manual (pages 35- 39) includes allowable accommodations for students taking the NSCAS State Assessment.

Resources

- For some examples of accommodations and instructional strategies that can be used to support students with mathematics difficulty please visit the document- Students with Disabilities in Mathematics from the Virginia Department of Education and view the charts on pages 17 through 24.
- As referenced in the Assistive Technology Resources section of this document, the Assistive Technology Consideration Resource Guide is a useful tool when considering students' needs for classroom materials, accommodations, modifications, and assistive technology. This guide provides examples of each of these categorized into instructional areas and tasks.

Additional Resources: Zearn & TNTP

Zearn is an online math program that was created for teachers, by teachers. Zearn has evidence of effectiveness and a proven impact on student learning (Zearn, n.d.). The Nebraska Department of Education is offering all Nebraska schools an opportunity to access Zearn Math for Summer 2022 and the 2022-2023 school year. Visit the website https://about.zearn.org/partners/nebraska-zearn-math-support for more information including digital lessons, overviews, webinars, and many other learning opportunities for utilizing Zearn.

TNTP has utilized data from Zearn, which is used by one in four elementary students in the nation.

Due to the Covid-19 pandemic, research suggests more students have fallen short academically over the last year than ever before. Now school districts are planning carefully on how to best respond. Should remediation be the key? Or should "just-in-time" supports be implemented by starting with students' grade level content and providing "just-in-time" supports when it is necessary- this is known as acceleration (TNTP, 2021).

TNTP 2021 found the following information by utilizing data from Zearn:

Findings include:

- Students who experienced learning acceleration struggled less and learned more than students who started at the same level but experienced remediation instead.
- Students of color and those from low-income backgrounds were more likely than their white, wealthier peers to experience remediation—even when they had already demonstrated success on grade-level content.
- Learning acceleration was particularly effective for students of color and those from low-income families.

This is strong evidence that learning acceleration works, and that it could be key to unwinding generations-old academic inequities the COVID-19 pandemic has only exacerbated. System leaders have an important opportunity in the months ahead to start providing teachers with the resources and support they need—and to start building the skill and belief that's necessary—to help every student engage in grade-level work right away (para.2-3).

Visit the website https://tntp.org/resources for more essential resources.

Writing Math Goals

IEP teams must write math goals that are specific, brief, and with clarity to ensure comprehension by all members of the IEP team.

- Math goals should provide clarity to the following questions:
- What does the student need?
- How will the student learn it?
- How will data be collected?

This child's strengths and needs need to be measured by informal testing, formal testing, teacher observations, and reviews of curriculum.

Math goals in the IEP are individualized; goals do not come from a math curriculum textbook, instead they come from evaluative measures. Annual, measurable goals describe what the student is expected to achieve during the academic year, for the following 12 months. The Nebraska Department of Education recommends the SMART acronym.

S – Specific- clearing defining the conditions of academic or function skill, the timeframe in which the skill is to be complete by

M – Measurable – clearly defining how the standard is being observed, how the skill is being evaluated

A – Actively phrased- specific action statements in the IEP that are clearly defined

R – Realistic- practical challenging academic progress based on student's current Present Levels of Academic Performance

T – Time- specific timeframe in which the goals are expected to be accomplished by

Developing meaningful math goals assumes the learner's current levels of academic performance or Present Levels of Educational Performance are up to date. This information is determined by a variety of sources that are accurate and comprehensive. This may include: assessment results, school work, observations, and other reliable math data (Vanderbilt University, 2021)

Professional Learning Opportunities

The following table contains websites with a plethora of information pertaining to math across the state of Nebraska, as well as nationally. Many of these websites have information for professional learning opportunities such as webinars, workshops, and many other resources for professional development.

Nebraska Sites	National Sites	Nebraska Universities &State College Sites
Nebraska Instructional Materials Collaborative	NCTM (National Council of Teachers of Mathematics)	University of Nebraska- Lincoln Center for Science, Mathematics & Computer Education
NATM (Nebraska Association of Teachers of Mathematics)	NCSM (National Council of Supervisors of Mathematics)	University of Nebraska- Lincoln Department of Mathematics
Nebraska Department of Education- Mathematics Education	Mathematical Association of America	University of Nebraska- Kearney Mathematics and Statistics
		University of Nebraska- Omaha Department of Mathematics
		Chadron State College Mathematics Webpage
		Peru State College Mathematics
		Wayne State College Department of Physical Sciences and Mathematics

Additional Math Resources

Resources for students with Autism

Name of Resource	Website/ Link
Equals Math	Equals_Overview_Manual_Organization.pdf (ablenetinc.com)
Morningside Model Math	https://www.morningsidepress.org/about

Resources for students who are Blind or Visually Impaired

The following resources listed are taken from the American Printing House for the Blind (APH) Instructional Products Catalog, 2020-2021 edition.

For the most updated resources please visit the American Printing House for the Blind (APH) website at www.aph.org and view the annual Instructional Products catalog.

Name of Resource	Description
Hundred Board and Manipulatives	This can be used to teach basic math concepts including patterning, counting, ordering numbers, comparing numbers, addition, and subtraction, and base ten number system. Available in Nemeth or UEB
Math Drill Cards	The cards include a math fact on one side of the card, and the fact with the answer featured on the opposite side. Available in Braille or Large Print.
Place Value Setter	With its refreshable and concrete display, this gives students a prompt way to represent numbers using written digits, and is useful for setting up place value problems quickly
Tactile Algebra Tiles	This set includes magnetic tiles that students can manipulate on a steel board and uses tactile symbols to help students differentiate between tiles. Colors were chosen with consideration for the students who have low vision.
Tactile Tangrams	This puzzle encourages the development and reinforcement of skills including spatial reasoning, shape recognition, size comparison, pattern replication, and independent problem solving.
Number Lines	With a variety of number lines available, these help to reinforce number concepts. Available in Braille/Tactile and Large Print.
Tactile Compass	This tool includes two different spur wheels to draw both single-line and double-line circles. They are used in combination with braille paper or plastic film and a rubber mat.

Hundreds Chart	The chart is printed/embossed on white paper. Alternating rows are highlighted to help students with low vision easily track numbers.
Braille/Large Print Protractor	Bold, large print numbers and raised dots mark the 180 degrees of this protractor, with markings every 5 degrees. The lines are bold and the type is 18 point.
Abacus	Each abacus consists of a black frame with white beads that slide against a red felt backing. The background provides a good visual contrast and prevents beads from slipping.
Analog Clock Model	This tool has braille and raised large print markings, and hands that are easily rotated.
Braille/Large Print Yardstick	This yardstick has raised-line markings along one edge, with braille markings every inch and black large print markings along the opposite edge.
Hundred Board and Manipulatives	This can be used to teach basic math concepts including patterning, counting, ordering numbers, comparing numbers, addition, and subtraction, and base ten number system. Available in Nemeth or UEB.

Resource for Students who are Deaf or Hard of Hearing

Name of Resource	Website/ Link
TTAC Online- Mathematics Differentiated Instructional Strategies - Deaf and Hard of Hearing	https://ttaconline.org/differentiated-instructional-strategies-deaf

Resources for Parents

Name of Resource	Website/ Link
Teaching Math to Young Children for Families and Caregivers	https://ies.ed.gov/ncee/edlabs/regions/central/resources/teachingearlymath/index.asp
10 Nonfiction Children's Books That Humanize Mathematics	https://www.kqed.org/mindshift/54983/10-nonfiction-childrens-books-that-humanize-mathematics
Family Activities for Nebraska Mathematics Standards: Grades 1 and 4 February 2002	https://www.education.ne.gov/wp-content/uploads/2017/07/NE_math_standards_parent_activity.pdf

One Last Word: “When Will I Ever Use this?”

Students frequently ask, “When will I ever use this?”

Mathematics explains the world around us, and most jobs require at least basic mathematics and reasoning skills. In fact, in an increasingly technological world, almost all jobs need math, even jobs that students might not think need math or technology skills! For example, to get into the military, one has to take the Armed Services Vocational Aptitude Battery (ASVAB) assessment. The ASVAB includes a math section, and the higher one scores on the ASVAB, the better the position in the military and the larger the monetary bonuses. Police officers also take an exam that includes math prior to being accepted into police academies. Firefighters are required to take an exam that includes math, too.

Many adolescents want to program video games—one of the most frequent career aspirations of middle school boys. Fortunately, coding classes and other programming opportunities are increasingly offered in K-12 settings. However, software development for video games requires incredible amounts of mathematics, particularly for first person shooters! Arcs of objects, distances to impact, velocity of objects, and many more mathematical concepts embedded in video games require advanced algebra, trigonometry, and calculus skills.

Most jobs obtained through workforce services require either a minimum ACT score for young workers or completion of several assessments including math, reading and information location. Many states use the WorkKeys Assessment created by ACT. Some states’ workforce or job placement services use the Tests of Adult Basic Education (TABE) work-related foundational and problem-solving skills. If we don’t help our students become proficient in mathematics, even entry level jobs become unavailable to them, much less jobs that will help them gain economic independence and security!

Related Definitions

FAPE – Free Appropriate Public Education. IDEA defines FAPE as "free appropriate public education must be available to all children residing in the State between the ages of 3 and 21, inclusive, including children with disabilities who have been suspended or expelled from school, as provided in §300.530(d)" (Individual with Disabilities Education Act, 2004).

High Leverage Practices (HLPs) – High Leverage Practices are 22 practices organized within four elements of practice: Collaboration, Assessment, Social/emotional/behavioral, and Instruction. These practices were assembled around the critical key components that all K-12 special education teachers should practice and have shown positive results for students if implemented correctly (Riccomini et al., 2017).

Individuals with Disabilities Education Act (IDEA) – Is the law that makes a free appropriate public education (FAPE) available to children eligible with disabilities throughout the United States, including special education and related services. IDEA governs how states and public agencies provide special education, related services, and early intervention services to eligible infants, toddlers, children, and youth with disabilities (Individuals With Disabilities Education Act, 2004).

Multi-Tiered System of Supports (MTSS) – "…is a framework that promotes an integrated system connecting general education and special education, along with all components of teaching and learning, into a high quality, standards-based instruction and intervention system that is matched to a student's academic, social-emotional and behavior needs." (https://www.education.ne.gov/nemtss/)

SEL – "Social and emotional learning (SEL) is an integral part of education and human development. SEL is the process through which all young people and adults acquire and apply the knowledge, skills, and attitudes to develop healthy identities, manage emotions and achieve personal and collective goals, feel and show empathy for others, establish and maintain supportive relationships, and make responsible and caring decisions" https://casel.org/what-is-sel/

Specially Designed Instruction – Means adapting as appropriate to the needs of an eligible child under this Chapter the content, methodology, or delivery of instruction to address the unique needs of the child that result from the child's disability and to ensure access to the general curriculum so that the child can meet the educational standards within the jurisdiction of the public school district or approved cooperative that apply to all children (Nebraska Department of Education, Nebraska Department of Education Rule 51: Regulations and Standards for Special Education Programs, Title 92, Nebraska Administrative Code, Chapter 51 , 2017).

Supplementary Aids and Services – Means, aids, services, and other supports that are provided in regular education classes or other education-related settings and in extracurricular and non-academic settings to enable children with disabilities to be educated with nondisabled children to the maximum extent appropriate in accordance with Section 008 of this Chapter (Nebraska Department of Education, Nebraska Department of Education Rule 51: Regulations and Standards for Special Education Programs, Title 92, Nebraska Administrative Code, Chapter 51 , 2017).

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