NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 6

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

NEBRASKA DEPARTMENT OF EDUCATION

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Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of "I do." The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with "Ask students to . . ." and are examples of moving from "I do" activities to "we do" and "you do" activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 6
MA 6.1 Number

MA 6.1.1 Numeric Relationships

MA 6.1.1.a

Determine common factors and common multiples using prime factorization of numbers with and without exponents.

Extended: Identify the common factors of 4 and 6, 6 and 9, 8 and 10, given the factors of both numbers.

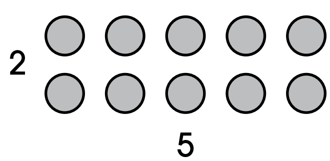
Scaffolding Activities for the Extended Indicator

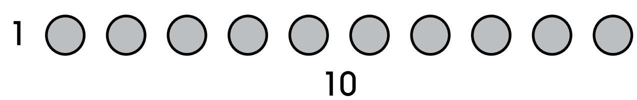
❑ **Identify the factors of 4, 6, 8, 9, and 10 when given an array or a multiplication sentence.**

- Use a multiplication sentence to demonstrate how to identify the factors of a number. For example, present the multiplication sentence 3 × 2 = 6. Explain that factors are numbers that are multiplied to get a product. Therefore, 3 and 2 are factors of 6. Another multiplication sentence with a product of 6 is 1 × 6 = 6. Explain that 1 and 6 are also factors of 6. The factors of 6 are 1, 2, 3, and 6.

  Repeat the process of using multiplication sentences to identify all the factors of 4, 8, 9, and 10.

- Use an array to demonstrate finding the factors of 10. Present an array of 2 rows of 5. Explain that the number of rows and the number of columns represent factors of 10. Therefore, 2 and 5 are factors of 10. Present an array of 1 row of 10 to demonstrate that 1 and 10 are also factors of 10. The factors of 10 are 1, 2, 5, and 10.

  Repeat the process of using arrays to identify all the factors of 4, 6, 8, and 9.

- Ask students to identify the factors when given a multiplication sentence with a product of 4, 6, 8, 9, or 10.

- Ask students to identify the factors when given an array for 4, 6, 8, 9, or 10.

- ❑ **Identify the common factors of 4 and 6, 6 and 9, and 8 and 10 when given the factors of both numbers.**

  - Use multiplication sentences to demonstrate how to identify common factors of 4 and 6. Present the multiplication sentences with a product of 4 and a product of 6, as shown. Model making a list of the factors for 4 and a list of the factors for 6. Explain that the common factors of 4 and 6 are the numbers that are in both lists of factors. Identify 1 and 2 as the common factors of 4 and 6.

    $1 \times 4 = 4$ $1 \times 6 = 6$

    $2 \times 2 = 4$ $2 \times 3 = 6$

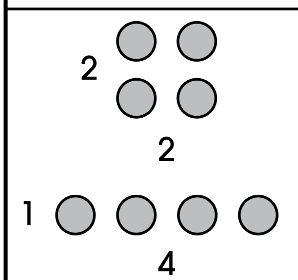
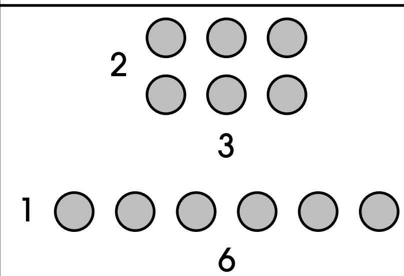
    factors of 4: 1, 2, 4

    factors of 6: 1, 2, 3, 6

    common factors: 1 and 2

    Repeat the process to identify the common factors of 6 and 9 and the common factors of 8 and 10.

  - Use arrays to demonstrate how to identify the common factors of 4 and 6. Present the arrays for 4 and 6 as shown. Model making a list of factors for each number and then identifying the common factors as the numbers that appear in both lists.

Arrays of 4	Arrays of 6
 	
Factors	**Factors**
1, 2, 4	1, 2, 3, 6

    Repeat the process to identify the common factors of 6 and 9 and the common factors of 8 and 10.

  - Ask students to identify the common factors of 4 and 6, 6 and 9, or 8 and 10 when given a list of factors for each number.

  - Ask students to identify the common factors of 4 and 6, 6 and 9, or 8 and 10 when given arrays or multiplication sentences for each number.

Prerequisite Extended Indicator

MAE 4.1.1.e—Identify the factors of 4, 6, 10, 15, and 20.

Key Terms

array, column, common, factor, product, row

Additional Resources or Links

https://www.map.mathshell.org/download.php?fileid=1590

http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html?from=search.html?qt=factor

(Note: Java required for website. Most recent version recommended, but not needed.)

MA 6.1.1.b

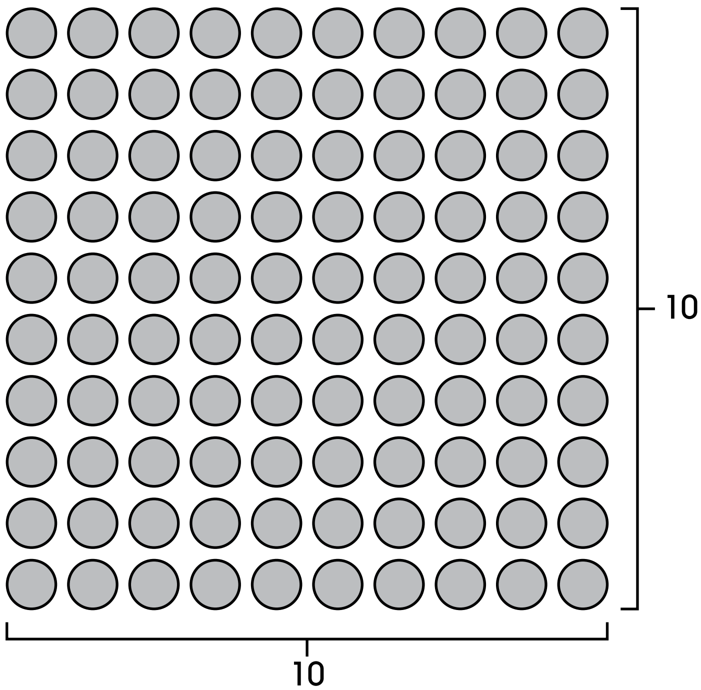
Represent non-negative whole numbers using exponential notation.

Extended: Represent 10, 100, 1,000, or 10,000 as a power of 10.

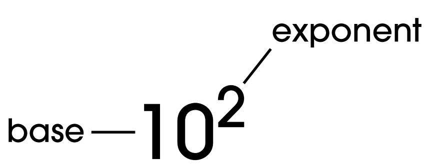
Scaffolding Activities for the Extended Indicator

- ❑ **Use a model to show powers of 10 as multiplication.**

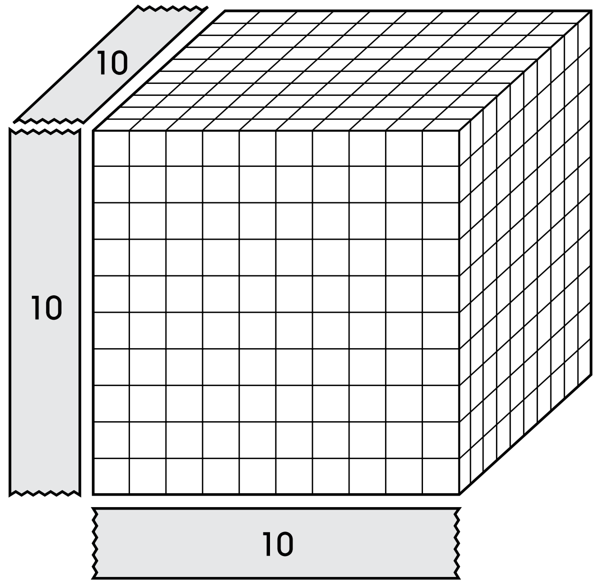
  - Show an array that represents 10 rows of 10 dots, which may also be written as 10 × 10 (10 groups of 10) and is equal to 100.



Explain that 10 × 10 may be written another way. An exponent can be used. An exponent is a small number written to the right and above the big number (base number). The exponent indicates how many times the base is a factor. For example, 10^2 means 10 is a factor two times, which is written as 10 × 10.



- Show a base ten block that represents 1,000. Use tape to label all of the dimensions. Explain that the number of cubes can be expressed as 10 × 10 × 10. Notice that 10 is a factor three times. Using exponents, 10 × 10 × 10 is written as 10^3 and is equal to 1,000.



- When shown a picture of a 10 × 10 array, ask students to represent the number of dots in the array as a power of 10.
- Ask students to match equivalent values when shown 10^2, 10^3, a 10 × 10 array, and a 10 × 10 × 10 cube.

❑ **Represent a multiplication expression as a power of 10.**

- Model determining the correct exponent and writing a power of 10.

  10 × 10 × 10 = 10^3
  10 = ______
  10 × 10 × 10 × 10 = ______
  10 × 10 = ______

- Ask students to determine the correct exponent given an expression of factors of 10.

- ❑ **Represent 10, 100, 1,000, and 10,000 as a power of ten.**

  - Use a table to explain the connection between the number of zeros in 10, 100, 1,000, and 10,000 and the exponent that is used for the power of ten. For example, in 100 there are two zeros and the power of ten for 100 is 10^2. Refer to the models shown above for 10^2 and 10^3.

Power	Expression	Standard Form
10^1	10	10
10^2	10 × 10	100
10^3	10 × 10 × 10	1,000
10^4	10 × 10 × 10 × 10	10,000

  - Demonstrate sorting cutout cards. For example, 10 × 10, 100, and 10^2 all belong in the same pile because they all represent the same value.

10	10 × 10 × 10	10^1	1,000
10^3	10^2	100	10 × 10
10,000	10^4	10 × 10 × 10 × 10	

  - Ask students to sort the cutout cards.
  - Model representing 10, 100, 1,000, and 10,000 as powers of ten.

Number	Power of Ten
10,000	
10	
1,000	
100	

  - Ask students to represent 10, 100, 1,000, and 10,000 as powers of ten.

Prerequisite Extended Indicator

MAE 4.1.1.d—Count by twos and fives, and tens with numbers, models, or objects up to 40.

Key Terms

base, exponent, factor, power of ten, standard form

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-1-topic-lesson-3

https://www.engageny.org/resource/grade-5-mathematics-module-1-topic-lesson-4

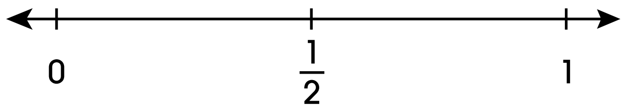
MA 6.1.1.c

Compare and order rational numbers both on the number line and not on the number line.

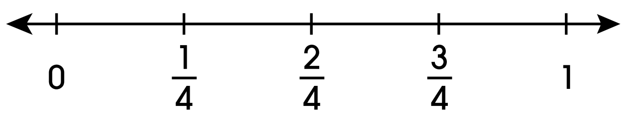
Extended: Compare and order halves, quarters, and tenths of whole numbers 0–1 on a number line.

Scaffolding Activities for the Extended Indicator

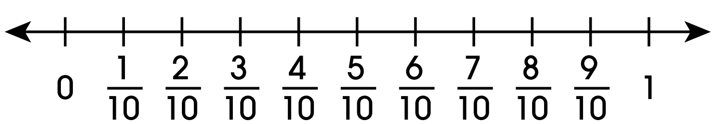
- ❑ **Identify halves, quarters, and tenths on a number line.**
  - Use a number line to show the location of $\frac{1}{2}$. Explain that $\frac{1}{2}$ is between 0 and 1. Emphasize that the whole number 1 is equal to the fraction $\frac{2}{2}$.



  - Ask students to identify $\frac{1}{2}$ and $\frac{2}{2}$ on a number line.
  - Ask students to place the fractions $\frac{1}{2}$ and $\frac{2}{2}$ on a number line from 0 to 1 with tick marks at 0, $\frac{1}{2}$, and 1 and the 0 and 1 tick marks labeled.
  - Use a number line to show the location of quarters. Another term for quarters is fourths. Fourths divide the distance from 0 to 1 on a number line into 4 equal-size parts, each part representing $\frac{1}{4}$ of the whole. As the number line goes from left to right, the number of fourths increases: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$. Emphasize that $\frac{4}{4}$ is equal to 1 whole.



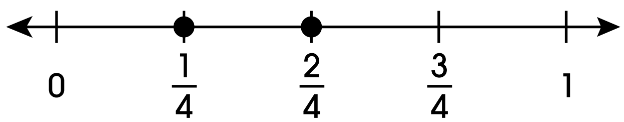
  - Ask students to identify $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ on a number line.
  - Ask students to place the fractions $\frac{1}{4}$ and $\frac{4}{4}$ on a number line from 0 to 1 with tick marks at each quarter interval and the 0, $\frac{2}{4}$, $\frac{3}{4}$, and 1 tick marks labeled.
  - Use a number line to show the location of tenths. Tenths divide the distance from 0 to 1 on a number line into 10 equal parts, each part representing $\frac{1}{10}$ of the whole. As the number line goes from left to right, the number of tenths increases: $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, $\frac{10}{10}$. Emphasize that $\frac{10}{10}$ is equal to 1 whole.



- Ask students to identify $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, and $\frac{10}{10}$ on a number line.
- Ask students to place the fractions $\frac{3}{10}$ and $\frac{8}{10}$ on a number line from 0 to 1 in which the other tenths are labeled.

❑ **Compare halves, quarters, and tenths on a number line.**

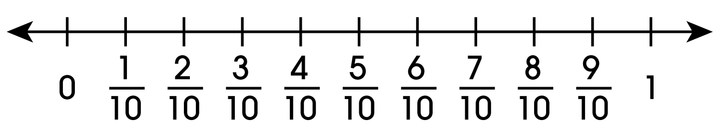
- Use symbols to show comparisons of numbers. For example, the symbol > means "greater than" and can be used in the number sentence 5 > 1. And the symbol < means "less than" and can be used in the number sentence 2 < 7. It may be helpful to demonstrate that the opening of the symbols always goes toward the greater number, like a wide-open mouth that is hungry for the greater amount.
- Ask students to use the symbols > and < to compare whole numbers 1 through 9.
- Use a number line to demonstrate how to compare quarters, or fourths. For example, the points on the number line are at $\frac{2}{4}$ and $\frac{1}{4}$. Explain that numbers on a number line go from left to right and that the farther to the right on a number line, the greater the number is. So, in this example, $\frac{2}{4}$ is greater than $\frac{1}{4}$, which can be written as $\frac{2}{4} > \frac{1}{4}$.



- Repeat the process to demonstrate comparing halves and tenths on the respective number lines and using the symbols > and < to write expressions.
- Ask students to select the correct symbol to compare halves, quarters, and tenths on the respective number lines. For example, ask students to select the symbol > or < to fill in the box. Students should select the < symbol to place in the box.

$$\frac{3}{10} \square \frac{7}{10}$$

- Ask students to compare halves, quarters, and tenths on a number line. For example, give students the following figure and ask, "Which statement is true?"



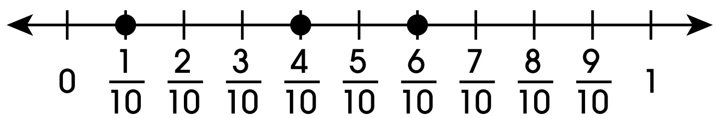
$\frac{7}{10} < \frac{9}{10}$ $\frac{7}{10} = \frac{9}{10}$ $\frac{7}{10} > \frac{9}{10}$

Students should choose $\frac{7}{10} < \frac{9}{10}$.

MA 6.1.1 Numeric Relationships

- ❑ **Order halves, quarters, and tenths on a number line.**

  - Use a number line to demonstrate ordering halves, quarters, and tenths from least to greatest. For example, present the number line shown. Explain that to put the fractions in order from least to greatest the points should be read from left to right. The correct order from least to greatest is $\frac{1}{10}$, $\frac{4}{10}$, $\frac{6}{10}$.



  - Repeat the process to demonstrate putting halves and tenths in order from least to greatest using the respective number lines.
  - Ask students to order halves, quarters, and tenths when given the respective number line and three points plotted.
  - Ask students to order halves, quarters, and tenths by selecting the correct order of three fractions when presented three choices with one correct answer showing the three fractions in order from least to greatest.

Prerequisite Extended Indicators

MAE 4.1.1.k—Compare and order mixed numbers with fourths and halves less than 3.

MAE 3.1.1.i—Use a model to compare unit fractions one-half, one-third, and one-fourth.

MAE 3.1.1.e—Compare and order whole numbers, 1–20. Compare the lesser/least or greater/greatest value without using symbols (>, <).

MAE 3.1.1.d—Represent halves and wholes on a number line.

Key Terms

fourth, greater than, half, less than, number line, order, quarter, tenth

Additional Resources or Links

https://www.mathlearningcenter.org/apps/number-line

https://www.engageny.org/resource/grade-3-mathematics-module-5-topic-d-overview/file/63071

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB3SUP-A5_NumFractions-201304.pdf

MA 6.1.1.d

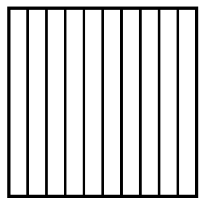
Convert among fractions, decimals, and percents using multiple representations.

Extended: Convert halves, fourths, and tenths to decimals using a model.

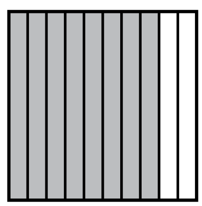
Scaffolding Activities for the Extended Indicator

❑ **Convert tenths into decimal numbers using a model.**

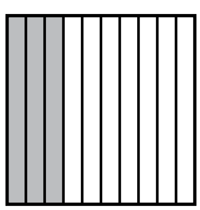
- Use a model to show how to convert fractions with tenths into decimal numbers. Present a figure as shown. Explain that the square represents the whole, and each part is $\frac{1}{10}$ of the whole.



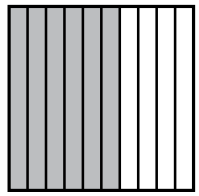
Explain that $\frac{1}{10}$ is equal to 0.1 and shading in 8 of the $\frac{1}{10}$ parts represents $\frac{8}{10}$. Eight-tenths is equal to the decimal number 0.8 since there are 8 of the 0.1 parts shaded. Continue to demonstrate by shading in other fractional parts of the whole and equating the number of tenths shaded as a fraction and a decimal.



- Ask students to convert tenths into decimal numbers when given a model and the equivalent fraction as shown.

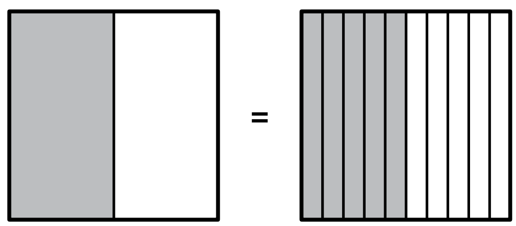
Model	Fraction	Decimal Number
	$\frac{3}{10}$	

- Ask students to convert tenths to decimals when given a model as shown.

Model	Decimal Number
	

- [ ] **Convert halves into decimal numbers using a model.**

- Use a model to demonstrate converting halves into decimal numbers. Present a figure as shown. Explain that the square on the left represents $\frac{1}{2}$ and the square on the right has the same amount shaded, representing $\frac{5}{10}$.

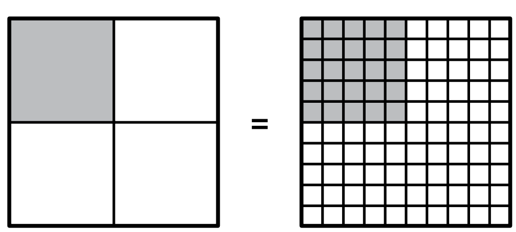


One-half, or $\frac{1}{2}$, of the square is the same as five-tenths, or $\frac{5}{10}$. Since $\frac{1}{10} = 0.1$, that means that $\frac{5}{10} = 0.5$. So one-half is equal to 0.5 as a decimal number. Continue to demonstrate using a variety of models representing one-half and converting $\frac{1}{2}$ to 0.5.

- Ask students to identify a model representing $\frac{5}{10}$.
- Ask students to convert halves to decimals using a model.

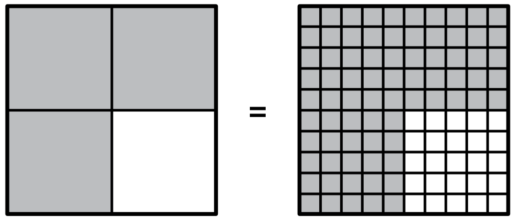
- [ ] **Convert fourths into decimal numbers using a model.**

- Use a model to demonstrate converting one-fourth into a decimal number. Present a figure as shown. Explain that the square represents one whole. The square on the left has 1 part shaded out of 4 equal parts, representing $\frac{1}{4}$. The square on the right has 25 out of 100 equal parts shaded, representing $\frac{25}{100}$.

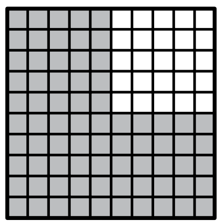
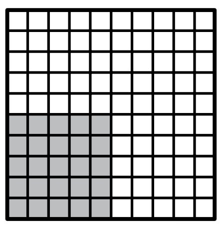


The two large squares each have the same amount shaded, which means that $\frac{1}{4} = \frac{25}{100}$. Each of the small squares on the model for $\frac{25}{100}$ is $\frac{1}{100}$ of the whole. The fraction $\frac{1}{100}$ can be written as the decimal number 0.01. So 25 hundredths is written as 0.25. Therefore, $\frac{1}{4} = 0.25$.

- Repeat the process to demonstrate that $\frac{3}{4} = \frac{75}{100} = 0.75$, using the figure shown.



- Ask students to convert fourths to decimals when given a model and the equivalent fractions as shown.

Model	Fraction	Decimal Number
	$\frac{75}{100}$	
	$\frac{25}{100}$	

- Ask students to convert halves, fourths, and tenths to decimals using a model.

Prerequisite Extended Indicators

MAE 4.1.1.h—Identify decimals on a number line from 0 to 1 (tenths only).

MAE 3.1.1.i—Use a model to compare unit fractions one-half, one-third, and one-fourth.

Key Terms

decimal number, fourth, fraction, half, tenth

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/4/NF/C/5/tasks/154

http://tasks.illustrativemathematics.org/content-standards/4/NF/C/5/tasks/103

https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Models/

http://nlvm.usu.edu/en/nav/frames_asid_264_g_2_t_1.html?from=category_g_2_t_1.html
(Note: Java required for website. Most recent version recommended, but not needed.)

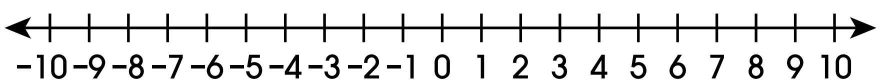
MA 6.1.1.g

Model integers using drawings, words, manipulatives, number lines, and symbols.

Extended: Identify models of integers –10 to 10 using drawings, words, manipulatives, number lines and symbols.

Scaffolding Activities for the Extended Indicator

- ❑ **Understand the meaning of the word "integer."**
  - Use a number line to define whole numbers. Start by presenting a number line from 0 to 10, and indicate the whole numbers shown on the number line. Next, extend the number line to the right. For example, show the number line from 0 to 20. Explain that when the number line is extended to the right, greater whole numbers are shown.
  - Demonstrate that a number line can be extended in either direction to introduce integers. Present a number line from –10 to 10, and emphasize that now the number line has been extended to the left. When the number line is extended to the left from 0, negative numbers are shown.

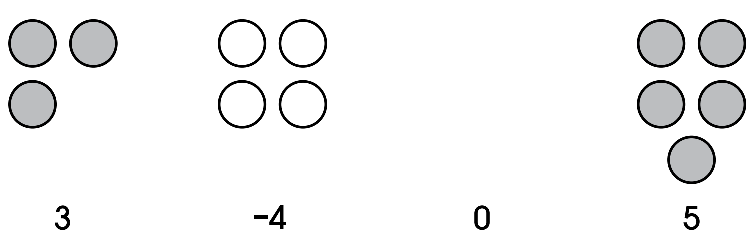


Negative numbers are the opposite of positive numbers, and each whole number starting with 1 has an opposite that is the same distance from 0 on the left side of the number line. So the opposite of 1 is –1, the opposite of 2 is –2, and so on. This category of numbers that includes whole numbers and their opposites are called "integers."

  - Ask students to identify opposite integers from a number line or list of positive and negative numbers.

- ❑ **Use manipulatives to represent integers.**
  - Use tokens of two different colors (e.g., blue and red) to represent positive integers and negative integers. For this example, gray tokens represent positive integers and white tokens represent negative integers.

Represent a variety of integers using the tokens as shown.

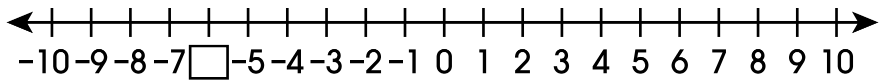


Show how to represent the integers from –10 to 10 using the tokens.

- Ask students to use manipulatives to represent integers from –10 to 10.

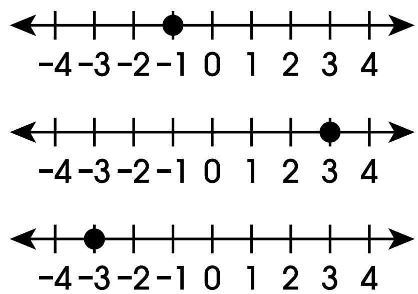
❑ **Identify an integer on a number line.**

- Use a number line to model identifying missing integers.



In the number line shown, there is a missing integer. Demonstrate how to identify the missing integer. Some strategies include finding the opposite (i.e., 6) or counting down from 0.

- Ask students to identify an integer on a number line. For example, present the following three number lines.



Then ask students to identify which number line has a point at the integer –3. Students should identify the last number line as the correct choice.

Prerequisite Extended Indicators

MAE 3.1.1.c—Identify a number closer to a given number on a number line, 1–20.

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

MAE 3.1.1.a—Read, write, and demonstrate whole numbers up to 20 that are equivalent representations including visual models, standard form, and word form.

Key Terms

integer, negative, number line, opposite, positive

Additional Resources or Links

https://www.mathlearningcenter.org/apps/number-line

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB4SUP-A11_NumOpNegNum-201304.pdf

https://nysed-prod.engageny.org/resource/grade-6-mathematics-module-3-topic-overview

MA 6.1.1.h

Compare and order integers and absolute value both on the number line and not on the number line.

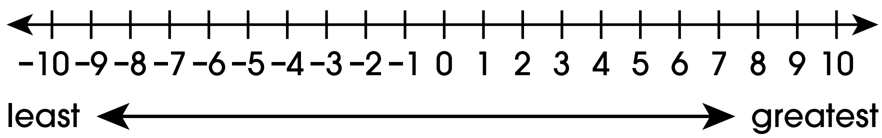
Extended: Compare and order integers (–10 to 10) on a number line.

Scaffolding Activities for the Extended Indicator

❑ **Compare integers on a number line.**

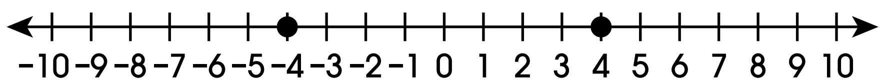
- Discuss relevant real-world examples of values less than zero to introduce the concept of negative integers (e.g., very cold temperatures, the score of a game that involves negative numbers, a debt).

- Use a number line from –10 to 10 to show how to compare integers. Indicate that numbers on the right are greater than numbers on the left.



  For example, 7 is greater than 6 and 0 is greater than –1. Comparing negative integers can be confusing. For example, –2 is greater than –10. It might be helpful to use created or purchased number line slider boards to reinforce the concept that numbers on the right are greater than numbers on the left.

- Demonstrate comparing two integers on a number line. Present the number line as shown and explain that 4 is greater than –4 because it is farther to the right on the number line.



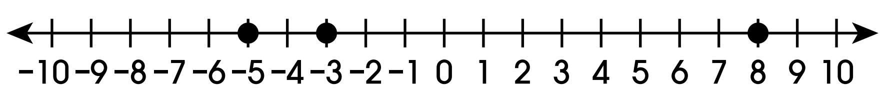
  Be sure to provide examples of comparing two positive integers, two negative integers, and a positive and a negative integer, as well as examples comparing other integers to zero.

- Ask students to compare two positive integers on a number line.

- Ask students to compare a positive and negative integer on a number line.

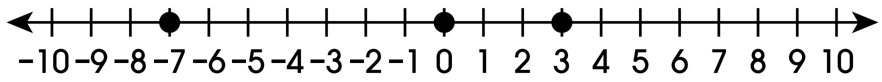
❑ **Order integers on a number line.**

- Use a number line from –10 to 10 to show how to order integers from least to greatest. Present a number line with points plotted at –5, 8, and –3.



  Move from left to right to list the integers from least to greatest: –5, –3, 8. Demonstrate with a variety of examples including positive numbers, negative numbers, and zero.

- Ask students to identify the correct order of integers. For example, present the number line as shown and three possible choices for the order from least to greatest.



0, 3, –7

–7, 0, 3

3, –7, 0

Students should choose –7, 0, 3 as the correct order for the integers given.

Prerequisite Extended Indicators

MAE 6.1.1.g—Identify models of integers –10 to 10 using drawings, words, manipulatives, number lines and symbols.

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

Key Terms

compare, greatest, integer, least, negative, number line, order, positive

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-lesson-1

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-7

MA 6.1.1.i

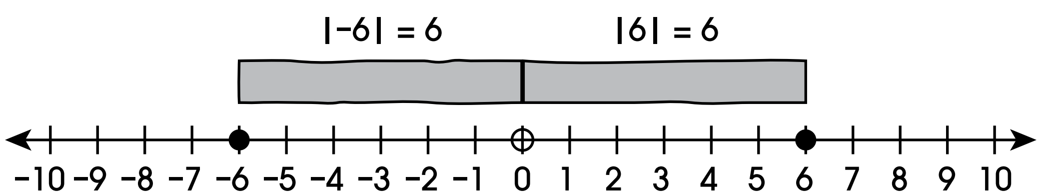
Determine absolute value of rational numbers.

Extended: Identify the absolute value of an integer –10 to 10.

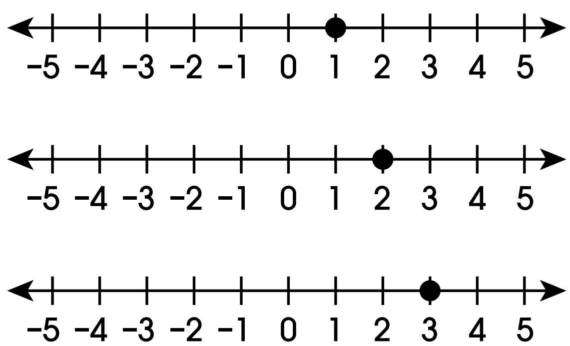
Scaffolding Activities for the Extended Indicator

❑ **Use a horizontal number line to recognize the absolute value of an integer from –10 to 10.**

- Explain that the absolute value of a number is its distance from 0. Using the number line, explain that the absolute value of –6 equals 6 because –6 is 6 units away from 0. Similarly, explain that the absolute value of 6 equals 6 because 6 is 6 units away from 0. Cut a strip of paper that matches the distance from 0 to –6 on the number line. Place one end of the strip of paper on the number line at –6 and the other end at 0. To demonstrate that both 6 and –6 are the same distance from 0, move the strip of paper on the number line so that one end is at 6 and the other end remains at 0. Show that absolute value is indicated with two vertical lines; |–6| = 6 can be read aloud as "the absolute value of negative six equals six."

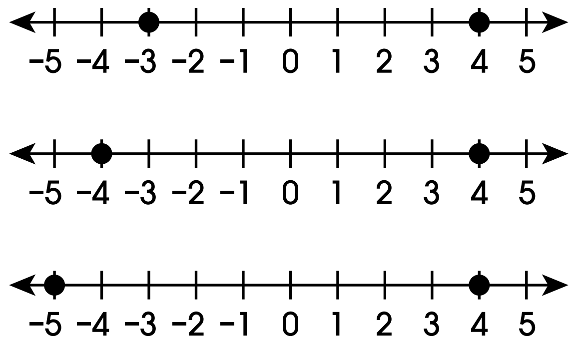


- Demonstrate recognizing number lines with a point plotted at a given absolute value. For example, present the three number lines shown and three questions. Which number line has a point that represents the absolute value of 1? Which number line has a point that represents the absolute value of 2? Which number line has a point that represents the absolute value of 3? Model answering each question by indicating the correct number line.



- Repeat the process with number lines that show points plotted at negative integers. Then, repeat the process with number lines that show points plotted at both positive and negative integers from –10 to 10.

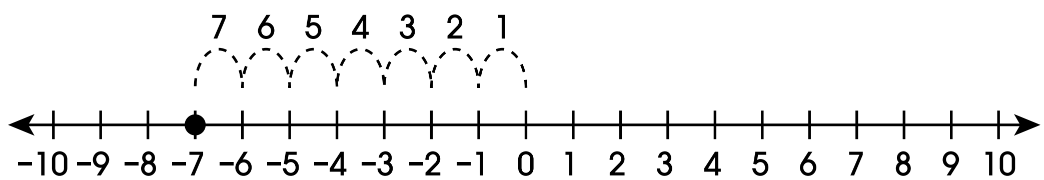
- Demonstrate recognizing pairs of numbers that have the same absolute value. For example, present three number lines as shown and indicate the number line showing two points that represent the same absolute value.



- Ask students to recognize number lines with points plotted that represent a given absolute value.

❑ **Use a horizontal number line to identify the absolute value of an integer from –10 to 10.**

- Present a number line with a point plotted at –7. Demonstrate identifying the absolute value of –7 by counting the intervals from 0 to –7. The absolute value of –7 is 7.



As appropriate, progress to identifying the absolute value of an integer from –10 to 10 without using a number line.

- Ask students to identify the absolute value of integers from –10 to 10 when plotted on a number line and without a number line.

Prerequisite Extended Indicators

MAE 6.1.1.h—Compare and order integers (–10 to 10) on a number line.

MAE 6.1.1.g—Identify models of integers –10 to 10 using drawings, words, manipulatives, number lines and symbols.

MAE 3.1.1.c—Identify a number closer to a given number on a number line, 1–20.

Key Terms

absolute value, distance, integer, number line

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-11/file/41656

https://curriculum.illustrativemathematics.org/MS/students/1/7/6/index.html

MA 6.1.2 Operations

MA 6.1.2.a

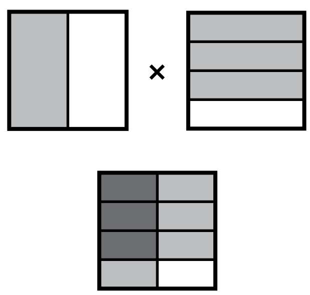
Multiply and divide non-negative fractions and mixed numbers.

Extended: Multiply and divide positive fractions, halves, fourths, thirds, and tenths using models.

Scaffolding Activities for the Extended Indicator

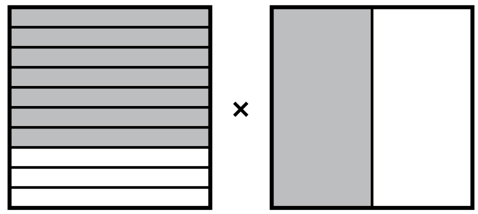
- ❑ **Use area models to represent multiplication problems with positive fractions (halves, fourths, thirds, and tenths).**
  - Recognize area models that represent multiplication problems with positive fractions and identify multiplication problems represented by area models. Present a fraction problem and an area model that represents the problem as shown.

$$\frac{1}{2} \times \frac{3}{4}$$



Explain that the shaded part of the square on the left represents the fraction $\frac{1}{2}$ because it is partitioned into two equal parts with one part shaded. The shaded part of the square on the right represents the fraction $\frac{3}{4}$ because it is partitioned into four equal parts with three parts shaded. Emphasize that for both fractions the whole is the same size. The problem $\frac{1}{2} \times \frac{3}{4}$ can be thought of as "$\frac{1}{2}$ of a set of $\frac{3}{4}$," and the product can be found by showing $\frac{3}{4}$ and then taking $\frac{1}{2}$ of it. This is modeled by the bottom square. Explain that the darkest shaded parts of the model represent the product, which is $\frac{1}{2}$ of $\frac{3}{4}$ of the whole.

- Ask students to recognize a multiplication problem with positive fractions, such as $\frac{7}{10} \times \frac{1}{2}$. Present an area model and three multiplication problems as shown, and ask students to select the multiplication problem that is represented by the area model.



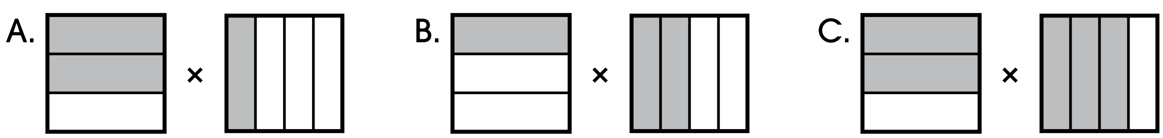
A. $\frac{1}{10} \times \frac{1}{2}$

B. $\frac{2}{10} \times \frac{1}{3}$

C. $\frac{7}{10} \times \frac{1}{2}$

- Ask students to identify the area model that represents a multiplication problem with positive fractions, such as $\frac{2}{3} \times \frac{1}{4}$. Present a multiplication problem and three area models as shown, and ask students to select the area model that represents the multiplication problem.

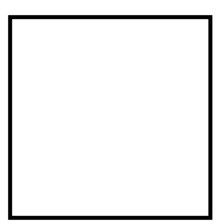
$$\frac{2}{3} \times \frac{1}{4}$$



❑ **Use area models to solve multiplication and division problems with positive fractions (halves, fourths, thirds, and tenths).**

- Use an area model to demonstrate solving multiplication problems with positive fractions. Present a fraction problem and a square to represent the whole as shown.

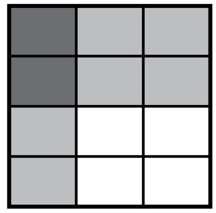
$$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$$



Explain that the multiplication problem can be thought of as $\frac{2}{4}$ of $\frac{1}{3}$. Begin demonstrating how to find the product of $\frac{2}{4}$ and $\frac{1}{3}$ by dividing the whole into four rows to represent fourths. Indicate the denominator of four on the first multiplicand. Demonstrate shading two of the four rows to represent $\frac{2}{4}$.

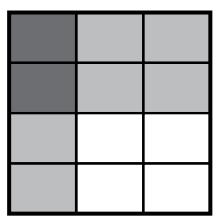
Next, divide the whole into three columns and indicate the denominator of three on the second multiplicand. Demonstrate shading in one of the three rows to represent $\frac{1}{3}$.

$$\frac{2}{4} \times \frac{1}{3}$$



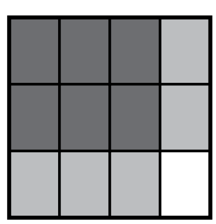
Count the total number of equal parts the whole is now divided into, which is twelve, and indicate that this is the denominator of the product. Count the number of parts that were shaded twice (or the darkest shaded parts), which is two, and indicate that this is the numerator of the product. Explain that the product is represented by the darkest shaded parts of the model. In this model, the whole is partitioned into 12 equal parts. Two of the parts have the darkest shading and the darkest shading represents the product, which is $\frac{2}{12}$. So $\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$.

$$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$$



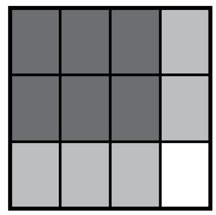
- Ask students to identify the product or solution to a multiplication problem with positive fractions such as $\frac{2}{3} \times \frac{3}{4}$ when given a completed area model and the multiplication equation as shown.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$



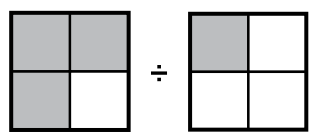
- Ask students to solve a multiplication problem with positive fractions such as $\frac{2}{3} \times \frac{3}{4}$ when given a completed area model as shown.

$$\frac{2}{3} \times \frac{3}{4} =$$

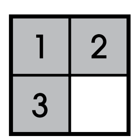


- Use area models to solve division problems with positive fractions. Start with division problems in which the divisor is a unit fraction, such as $\frac{1}{4}$. Present a fraction problem and an area model that represents the problem as shown.

$$\frac{3}{4} \div \frac{1}{4} = 3$$



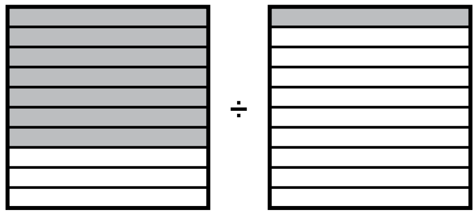
Explain that the shaded part of the square on the left represents the fraction $\frac{3}{4}$ because it is partitioned into four equal parts with three parts shaded. The shaded part of the square on the right represents the fraction $\frac{1}{4}$ because it is partitioned into four equal parts with one part shaded. The problem $\frac{3}{4} \div \frac{1}{4}$ can be thought of as "How many parts of size $\frac{1}{4}$ are in a part of size $\frac{3}{4}$?"



Explain that there are 3 parts of size $\frac{1}{4}$ in the part of size $\frac{3}{4}$. Therefore, $\frac{3}{4} \div \frac{1}{4} = 3$.

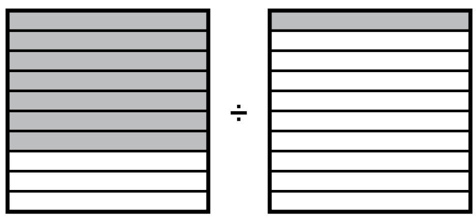
- Ask students to identify the quotient or solution to a division problem with positive fractions such as $\frac{7}{10} \div \frac{1}{10}$ when given a completed area model and the division equation as shown.

$$\frac{7}{10} \div \frac{1}{10} = 7$$



- Ask students to solve a division problem with positive fractions such as $\frac{7}{10} \div \frac{1}{10}$ when given a completed area model as shown.

$$\frac{7}{10} \div \frac{1}{10} =$$



When appropriate, progress to modeling division problems with non-unit fractions as the divisor, the same denominators, and one of the numerators a multiple of the other numerator, such as $\frac{9}{10} \div \frac{3}{10} = 3$.

Prerequisite Extended Indicators

MAE 5.1.2.d—Divide a whole number by $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ using a visual model (e.g., 3 divided by one-half).

MAE 5.1.2.c—Multiply $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ by 2, 3, and 4.

MAE 4.1.2.f—Add and subtract halves to halves, thirds to thirds, fourths to fourths, and fifths to fifths . . . to a whole.

Key Terms

denominator, divide, fourth, fraction, half, multiply, numerator, tenth, third

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-4-topic-e-overview

https://www.engageny.org/resource/grade-5-mathematics-module-4-topic-g-overview

MA 6.1.2.c

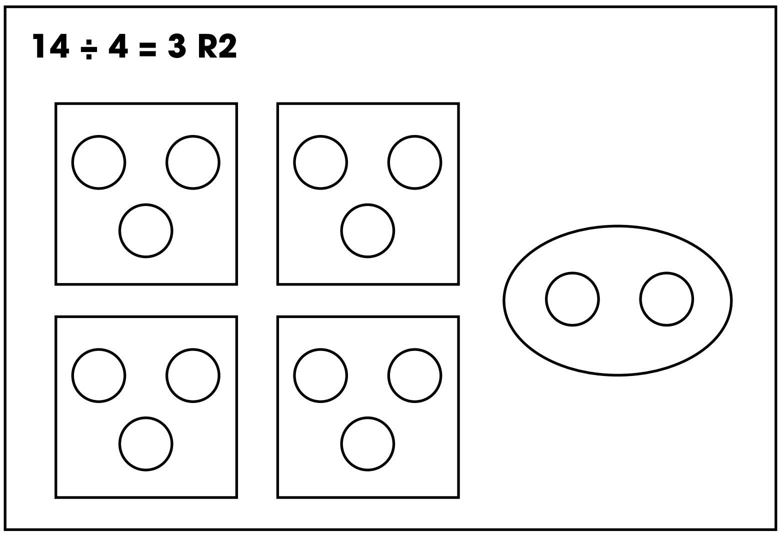
Divide multi-digit whole numbers using the standard algorithm.

Extended: Divide a two-digit number by a one-digit number with a remainder.

Scaffolding Activities for the Extended Indicator

- ❑ **Use manipulatives (chips, popsicle sticks, blocks, dried beans, stickers, etc.) and a counting mat to divide a two-digit number by a one-digit number with a remainder.**

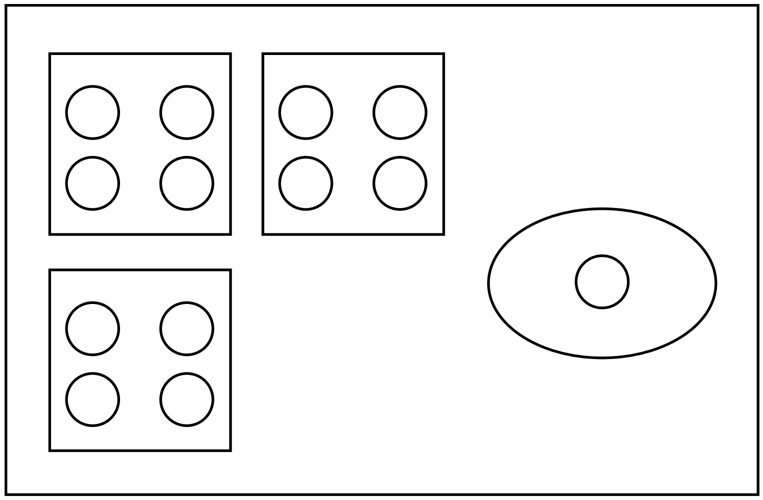
  - Use manipulatives to demonstrate that when a set of objects is divided into smaller groups of an equal amount, sometimes there are objects left over that are called remainders. For example, use 14 counters and a divide-by-4 counting mat, as shown.

    Demonstrate how counters can be placed one at a time in each of the four squares on the mat. After the first four are placed, point out that there are enough counters to place another one in each box. Repeat again. After the third placement, point out that there aren't enough counters left to put one in each box, so those two counters go into the remainder oval. Write the number sentence 14 ÷ 4 = 3 R2. Indicate that 14 is the total number of counters, 4 is the number of squares, 3 is the number of counters in each square, and 2 is the number of counters that remain. Therefore, 14 divided into 4 groups equals 3 in each group with 2 left over.

  - Ask students to follow the process modeled with the counting mat using a division problem with a two-digit number divided by a one-digit number with a remainder. Present the student with the appropriate counting mat for the divisor and the appropriate number of counters or manipulatives. Ask students to determine the number of counters in each group and the number remaining to indicate the solution to the division problem.

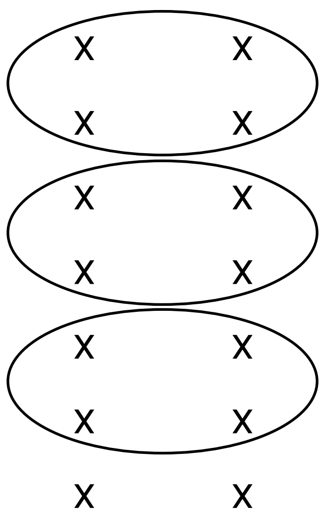
- Ask students to write or select the division problem or the answer to the division problem when given a counting mat with counters already placed on the mat to represent a division problem. For example, provide the following division mat for a student.



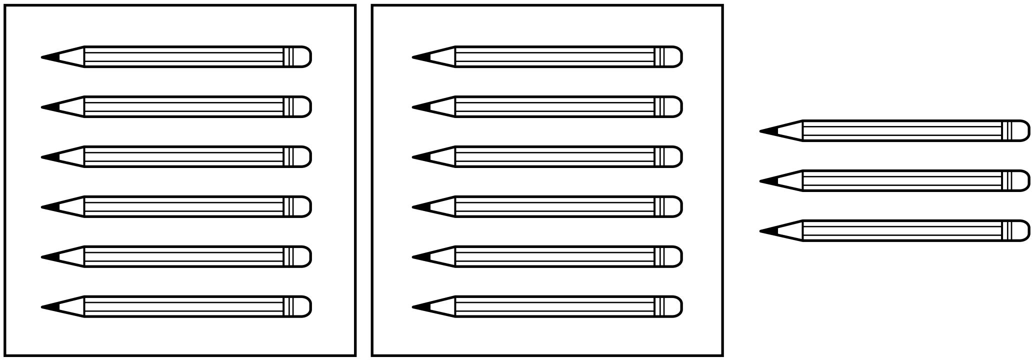
The division problem shown is 13 ÷ 3 = 4 R1.

❑ **Use an array to divide a two-digit number by a one-digit number with a remainder.**

- Use the array shown to demonstrate 14 ÷ 4. Explain that the array shows the original whole of size 14. Indicate that the division by 4 is represented by circling groups of size 4.



Circle all the groups of size 4 and indicate that there are 3 groups of 4 with 2 remaining. Write the number sentence 14 ÷ 4 = 3 R2 and indicate that 14 divided into groups of 4 equals 3 groups with 2 left over. Demonstrate this same activity using manipulatives. For example, use a box of 15 pencils to demonstrate 15 divided by 6.



Write the number sentence 15 ÷ 6 = 2 R3.

- Ask students to use an array to divide a two-digit number by a one-digit number.
- Ask students to use real-world objects to divide a two-digit number by a one-digit number.

Prerequisite Extended Indicators

MAE 5.1.2.b—Divide a two-digit whole number by a single-digit number with no remainder.

MAE 4.1.2.d—Identify numbers 2–20 in equal-size groups.

Key Terms

divide, equal, groups, left over, remainder

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-e-lesson-15/file/34136

https://www.insidemathematics.org/sites/default/files/materials/diminishing%20return.pdf

MA 6.1.2.d

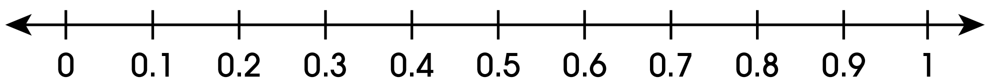
Add, subtract, multiply, and divide decimals using the standard algorithms.

Extended: Add and subtract numbers 0–10 with one decimal place without regrouping.

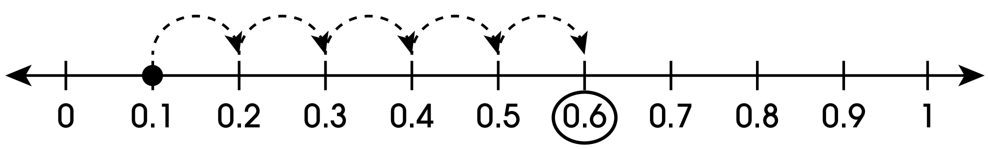
Scaffolding Activities for the Extended Indicator

❑ **Add numbers 0–1 with one decimal place without regrouping.**

- Use a number line to demonstrate adding tenths. Explain that a number line from 0 to 1 can be divided into ten equally sized intervals (or sections) called tenths, as shown.



  The number line can be used to add numbers by starting at the first addend and using arrows to represent the second addend until the sum is found, just like is done with whole numbers. Demonstrate adding 0.1 + 0.5 by placing a point at 0.1 and then moving five intervals to the right. Each arrow represents adding one-tenth, so there are 5 of them for 0.5. They end at 0.6, so the sum of 0.1 and 0.5 is 0.6.



$$0.1 + 0.5 = 0.6$$

- Demonstrate using the standard algorithm for adding decimal numbers, such as 0.2 + 0.7. First, emphasize lining up the decimal points. It might be helpful to present the problem on grid paper.

$$\begin{array}{r} 0.2 \\ +\ 0.7 \\ \hline \end{array}$$

  With the decimal points properly aligned, each column of numbers can be added together to find the sum. Since 2 + 7 = 9, the number after the decimal point will be 9 in the sum. And for the ones place, 0 + 0 = 0. Make note that the decimal point in the answer also aligns with the other decimal points.

$$\begin{array}{r} 0.2 \\ +\ 0.7 \\ \hline 0.9 \end{array}$$

  It may be helpful to demonstrate solving the same addition problem on a number line and comparing the answers.

- Ask students to add numbers from 0–1 with one decimal place using a number line.
- Ask students to add numbers from 0–1 with one decimal place using the standard algorithm.

- ❑ **Add numbers 0–10 with one decimal place without regrouping.**

  - Demonstrate using the standard algorithm to add numbers 0–10 with one decimal place, such as 8.4 + 1.3. Explain the process of first adding the tenths together (the digits 4 and 3) to get 7 in the tenths place and then adding the ones together (the digits 8 and 1) to get 9 in the ones place. The final sum is 9.7.

$$\begin{array}{r} 8.4 \\ +\ 1.3 \\ \hline 9.7 \end{array}$$

    It might be helpful to use visual supports, including but not limited to writing the addition problem on grid paper, writing the addition problem on a place value mat, or using base ten blocks to represent the problem on a place value mat. Demonstrate solving addition problems involving tenths using a variety of numbers from 0–10 with one decimal place.

  - Ask students to add numbers 0–10 with one decimal place when given visual supports.
  - Ask students to add numbers 0–10 with one decimal place using the standard algorithm.

- ❑ **Subtract numbers 0–10 with one decimal place without regrouping.**

  - Demonstrate using the standard algorithm to subtract numbers 0–10 with one decimal place, such as 9.4 – 5.1.

$$\begin{array}{r} 9.4 \\ -\ 5.1 \\ \hline 4.3 \end{array}$$

    Use visual supports (e.g., grid paper, place value mats, base ten blocks) as needed to demonstrate lining up the decimal points and subtracting one place value column at a time. Demonstrate solving subtraction problems involving tenths using a variety of numbers from 0–10 with one decimal place.

  - Ask students to subtract numbers 0–10 with one decimal place when given visual supports.
  - Ask students to subtract numbers 0–10 with one decimal place using the standard algorithm.

Prerequisite Extended Indicators

MAE 4.1.1.h—Identify decimals on a number line from 0 to 1 (tenths only).

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

add, decimal number, difference, place value, subtract, sum, tenth

Additional Resources or Links

https://www.mathlearningcenter.org/apps/number-pieces

https://www.insidemathematics.org/sites/default/files/materials/courtney%27s%20collection.pdf

MA 6.1.2.e

Estimate and check reasonableness of answers using appropriate strategies and tools.

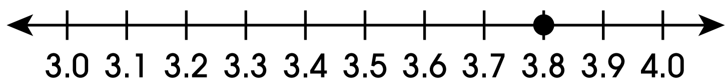
Extended: Estimate the sum of two decimal numbers with tenths (e.g., 5.2 + 3.7 is about 9).

Scaffolding Activities for the Extended Indicator

❑ **Round a decimal number with tenths to the nearest whole number.**

- Introduce "estimate" as another word for rounding. Rounding numbers to the nearest whole number can help estimate the value of the number. Provide relevant examples of using the estimate of a decimal number in a real-world situation.

- Use a number line to demonstrate the location of a decimal number. For example, the number line shown has a point at 3.8.



  Just like a whole number can be rounded to the nearest multiple of 10, a decimal number can be rounded to the nearest whole by finding which whole number it is closest to on the number line. For the number 3.8, the two closest whole numbers are 3 and 4. Looking at the number line, the point is closer to 4 than it is to 3, so 3.8 rounds to 4. Show students a variety of decimal numbers with tenths and demonstrate rounding them on the number line. Be sure to include a number with five-tenths to demonstrate that the digit 5 indicates the number will be rounded to the greater whole number. For example, 8.5 will round to 9, not 8.

- Ask students to round decimal numbers with tenths to the nearest whole number. Focus on the numbers 1.0 through 10.0.

❑ **Estimate the sum of two decimal numbers with tenths.**

- Use rounding of the addends to estimate a sum. Demonstrate that an addition problem can be estimated by first finding the whole numbers closest to the decimal numbers given. Present the problem 2.3 + 4.6.

2.3	2.3 is closest to 2
+ 4.6	4.6 is closest to 5

  Explain that rounding the decimal numbers helps estimate the sum (or the answer), and 2 + 5 = 7. So, 7 is the estimate of the sum. Show students a variety of addition problems.

- Ask students to estimate the sum of two decimal numbers with tenths. For example, show the following problem.

7.9	7.9 is about ______
+ 1.1	1.1 is about ______

  Then give students three equations for the estimate of the sum: 7 + 1 = 8, 8 + 1 = 9, and 8 + 2 = 10. Students should identify 8 + 1 = 9 as the best estimate.

Prerequisite Extended Indicators

MAE 4.1.1.h—Identify decimals on a number line from 0 to 1 (tenths only).

MAE 4.1.1.g—Round a 2-digit number, 1–100, to the nearest ten using a number line.

MAE 3.1.1.c—Identify a number closer to a given number on a number line, 1–20.

Key Terms

add, closest, decimal number, estimate, nearest, round, sum, tenth

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-6-topic-d-overview

https://www.insidemathematics.org/common-core-resources/5th-grade

http://tasks.illustrativemathematics.org/content-standards/5/NBT/A/4/tasks/1804

https://nysed-prod.engageny.org/resource/grade-5-mathematics-module-1-topic-d-lesson-9/file/39496

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Mathematics—Grade 6
MA 6.2 Algebra

MA 6.2.1 Algebraic Relationships

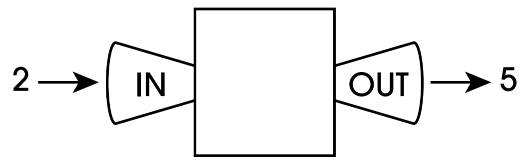
MA 6.2.1.a

Create algebraic expressions (e.g., one operation, one variable as well as multiple operations, one variable) from word phrases.

Extended: Match a simple word phrase with an input/output box.

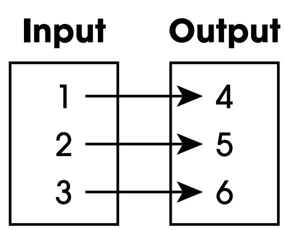
Scaffolding Activities for the Extended Indicator

- ❑ **Match an addition word phrase with an input/output box.**
  - Use a number machine (also called a function machine) to demonstrate an addition input/output pattern or rule. If a 2 goes in the machine, a 5 comes out.



Since the number 2 gets larger when it comes out of the machine, the machine must add something to the 2. To get from 2 to 5, the machine must add 3. So, for this machine, if you put a 4 in, a 7 would come out because 4 + 3 = 7. The word phrase for this machine is “add 3,” which can also be called the rule. Show a variety of number machines that have rules to add a value to the input number, such as “add 5” and “add 1.”

  - Ask students to match an addition word phrase with an input/output box. For example, present the following figure.



This input/output box shows that when a 1 goes in, a 4 comes out; when a 2 goes in, a 5 comes out; and when a 3 goes in, a 6 comes out. Ask students to choose the word phrase that matches the input/output box.

“add 2”
“add 3”
“add 4”

Students should choose “add 3” as the correct word phrase because each input has 3 added to it to find the corresponding output. The rule in this input/output pattern is “add 3.”

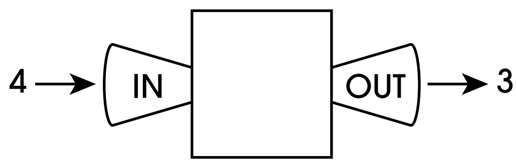
❑ **Match a subtraction word phrase with an input/output box.**

- Use a number machine, an input/output box, or an input/output table to demonstrate a subtraction input/output pattern or rule. For example, present the following table.

Input	Output
7	5
8	6
9	7

Since each number in the input column corresponds to a smaller number in the output column, this table involves subtraction. Each number in the output column is 2 less than the corresponding number in the input column, so the word phrase (or rule) for this pattern is “subtract 2.” Show a variety of subtraction examples.

- Ask students to match a subtraction word phrase with a number machine, an input/output box, or an input/output table. For example, give students the following number machine.



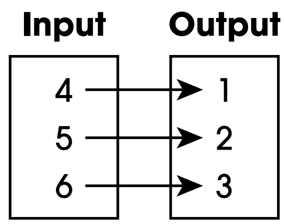
Have students choose one of the following word phrases to match the rule of the number machine. It may help to ask the question “What does the machine do to the number 4 to make it 3?”

“subtract 1”
“subtract 2”
“subtract 3”

Students should choose “subtract 1” as the rule for this input/output number machine.

- ❑ **Match a simple word phrase with an input/output box.**

    - Use a number machine, an input/output box, or an input/output table to model how to decide whether a number pattern shows an addition rule or a subtraction rule. For example, present the following input/output box.



First show how to decide whether the phrase starts with “add” or “subtract,” then demonstrate finding the number value to add to the rule. Since each output number here is less than the corresponding input number, the word phrase will start with the word “subtract.” To find the number value needed, ask the question “What number is subtracted from each number to get the new number?” In this case, that number is 2, so the rule for this example is “subtract 2.” Continue to demonstrate the process of identifying the pattern or rule with a variety of input/output boxes using both addition and subtraction.

- Ask students to match a simple word phrase with a number machine, an input/output box, or an input/output table. For example, present the following table.

Input	Output
1	6
2	7
3	8

Give students three word phrases to choose from: “add 3,” “add 4,” and “add 5.” Students should choose the word phrase “add 5” as the rule for this example.

Prerequisite Extended Indicator

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

add, input, output, pattern, rule, subtract, table

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/growing%20staircases.pdf

http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

http://tasks.illustrativemathematics.org/content-standards/5/OA/B/3/tasks/1895

MA 6.2.2 Algebraic Processes

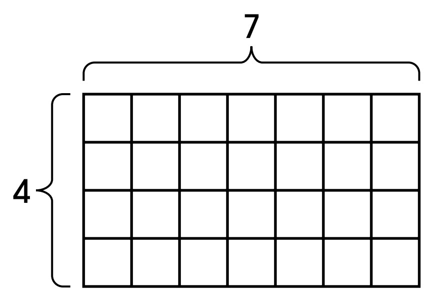
MA 6.2.2.a

Simplify expressions using the distributive property and combining like terms.

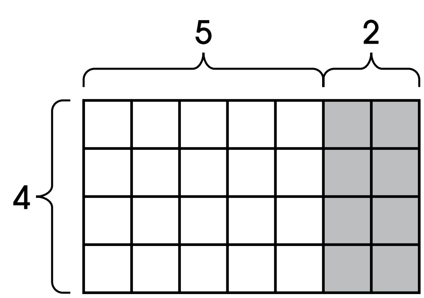
Extended: Identify whole number expressions using the distributive property (e.g., 2(3 + 4)).

Scaffolding Activities for the Extended Indicator

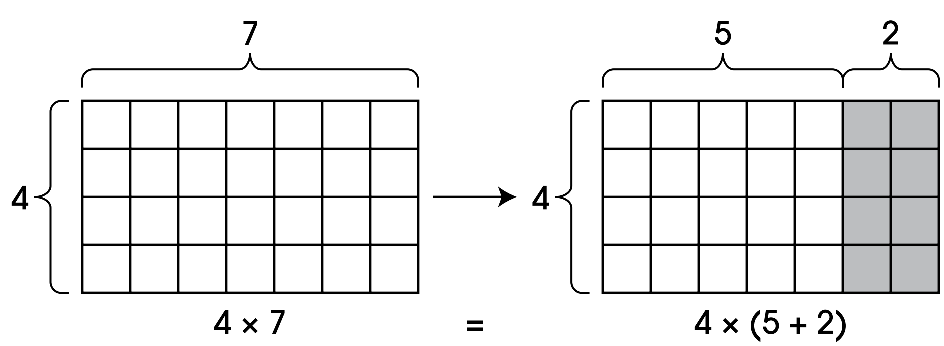
- ❑ **Determine an equivalent expression without distributing.**
  - Use models to show that some multiplication problems can be made easier to solve by decomposing one of the terms into smaller numbers. Explain that the model shown represents the expression 4 × 7. There are 4 rows and 7 columns.



The number 7 in the model may be decomposed into smaller numbers to make it easier to solve the multiplication problem. For example, the value 7 may be “broken apart” into 5 and 2.

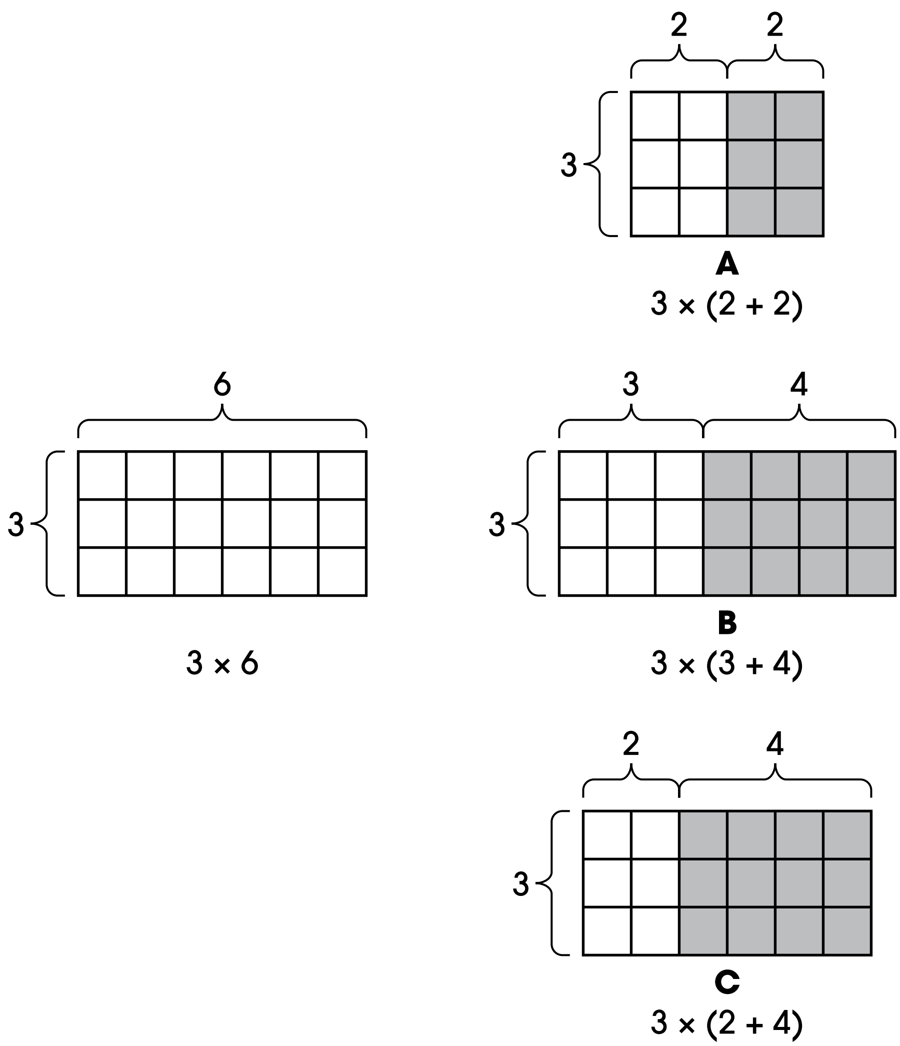


Explain that the 5 and 2 are multiplied by 4, the number of rows: 4 × (5 + 2).



Continue to demonstrate decomposing a variety of multiplication expressions into smaller numbers using rectangle models with unit squares.

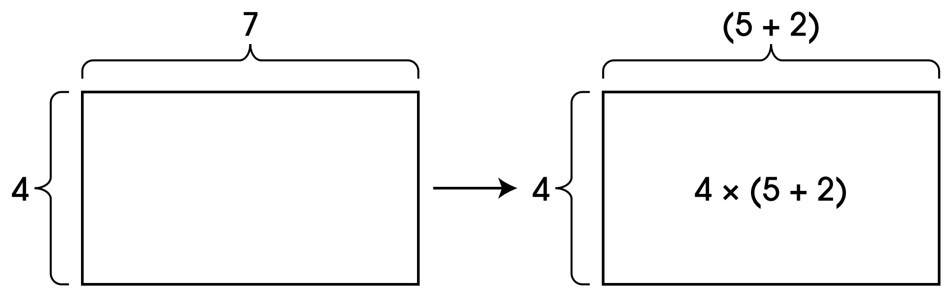
- Ask students to select the model that shows a correctly decomposed representation of the given rectangle when given two or more choices. For example, students select the model on the right that shows an equivalent decomposition to the model on the left.



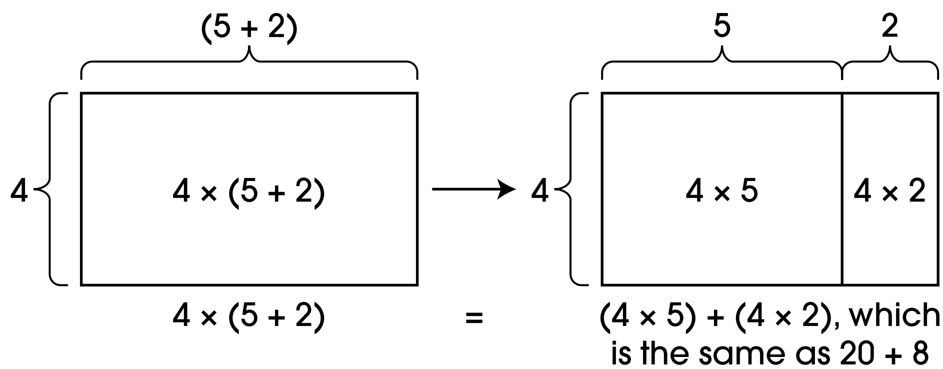
- Ask students to decompose an expression such as 6 × 8 into expressions such as 6 × (1 + 7), 6 × (2 + 6), etc.

- **Identify whole number expressions using the distributive property.**

  - Use models to show how to use the distributive property to create equivalent expressions. For example, using the same rectangle as above, represent the area of the rectangle using the decomposed expression of 5 + 2 for the side length of 7.

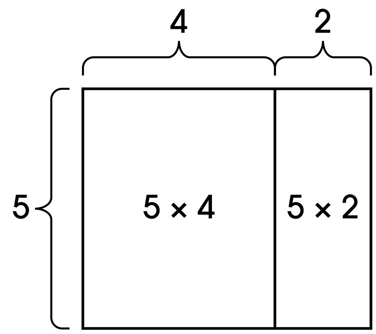


Explain that the expression for the area of the rectangle can also be decomposed by "distributing" the width, 4, to each value from the decomposed length.



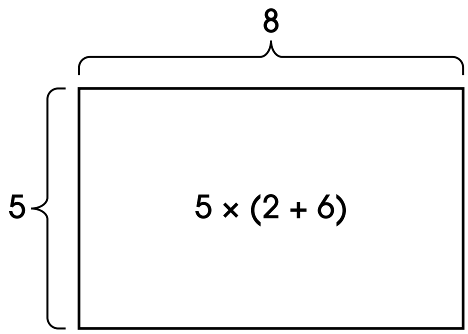
Continue to use a variety of rectangle models to demonstrate using the distributive property to create equivalent expressions.

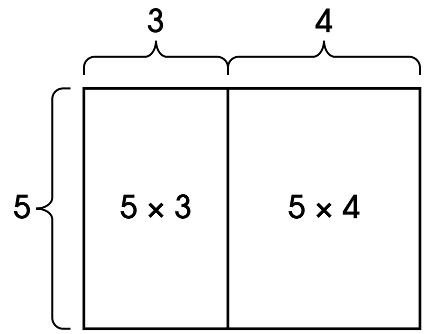
- Ask students to identify visual models using the distributive property when given two or more choices. For example, students select the model on the right that shows an equivalent decomposition to the model on the left.



A

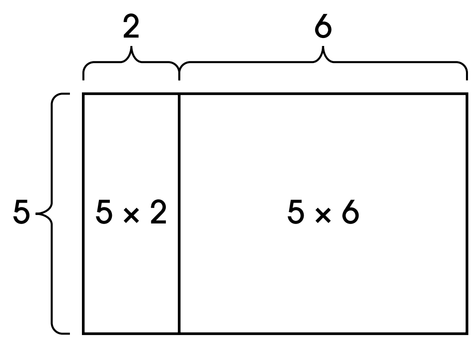
(5 × 4) + (5 × 2),
which is the same as 20 + 10





B

(5 × 3) + (5 × 4),
which is the same as 15 + 20



C

(5 × 2) + (5 × 6),
which is the same as 10 + 30

- Ask students to identify equivalent whole number expressions using the distributive property. For example, ask students to identify an equivalent expression to 3 × (5 + 2) as (3 × 5) + (3 × 2), which is the same as 15 + 6.

Prerequisite Extended Indicators

MAE 5.2.2.a—Evaluate a numerical expression with addition or subtraction and multiplication, 1–5.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

distribute, distributive property, expression, multiplication

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-e-lesson-16

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-d-lesson-12

MA 6.2.2.c

Evaluate numerical expressions, including absolute value and exponents, with respect to order of operations.

Extended: Demonstrate understanding of order of operations involving addition, subtraction, and multiplication.

Scaffolding Activities for the Extended Indicator

❑ **Identify the order in which operations are evaluated.**

- Show 4 + 7 – 2 as an example of a numerical expression. Explain that there are special rules to follow when evaluating numerical expressions. When a numerical expression has addition and subtraction, start evaluating on the left. For example, in 3 + 6 – 4, 3 + 6 is evaluated first. In 8 – 3 + 4, 8 – 3 is evaluated first.

- Ask students to circle, underline, highlight, etc. the part of the expression that needs to be evaluated first. For example, in the expression 6 + 3 – 5, students should identify 6 + 3 as the part of the expression to evaluate first.

6 + 3 – 5	9 – 3 + 4	4 – 2 + 7	5 + 5 – 3

- Explain that when multiplication is included in a numerical expression, multiplication is evaluated before addition and subtraction. In 5 × 4 + 3, 5 × 4 is evaluated first. In 6 + 3 × 2, 3 × 2 is evaluated first.

  Also, sometimes numerical expressions are written using parentheses, like 4(2 + 1). Emphasize that the operation inside the parentheses is always done first and that when a number is directly in front of a set of parentheses, it means to multiply. So in the case of 4(2 + 1), the parentheses containing 2 + 1 need to be evaluated first, and then the answer is multiplied by 4.

  Continue to model identifying the operation that needs to be evaluated first in examples such as the ones below by circling, underlining, highlighting, etc. Be sure to include the number on each side of the operator when identifying the expression that needs to be evaluated first.

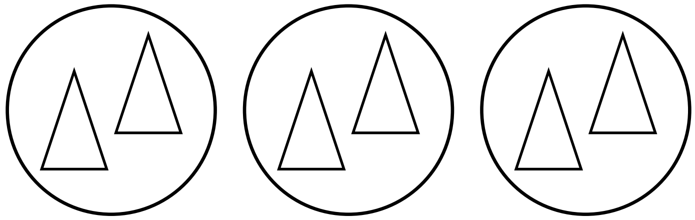
5(3 + 2)	6 – 2 + 5	3 + 4 × 6	5 × 2 – 6

- Ask students to circle, underline, highlight, etc. the expression that needs to be evaluated first in examples such as the ones below.

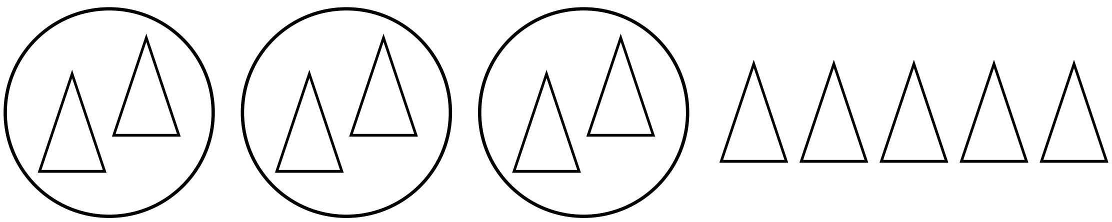
5 × 3 + 4	9 – 2 × 4	6(8 – 3)	4(2 + 4)

- **Evaluate two-step numerical expressions involving multiplication with addition or subtraction.**

  - Explain that 5 + 3 × 2 is an example of a numerical expression with two steps. There are special rules to follow when multiplication is in the numerical expression. Multiplication comes before addition and subtraction. Model solving 3 × 2 with manipulatives or a drawing. Explain that 3 × 2 can be thought of as 3 groups of 2.

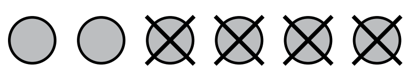
    Explain that the next step in the expression is to add the groups to 5. This gives a total of 11. Therefore, 5 + 3 × 2 = 11.

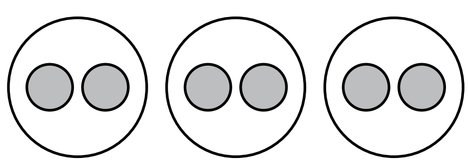
    Continue to model evaluating a variety of two-step numerical expressions involving multiplication with addition or subtraction.

  - Ask students to evaluate numerical expressions involving multiplication and addition or subtraction using manipulatives or drawings. For example, ask students to evaluate expressions such as 3 × 4 + 5 and 7 + 5 × 4.

  - Explain that when parentheses are included in a numerical expression, the operation inside the parentheses is always done first, and if a number is directly in front of a set of parentheses, as in 3(6 – 4), it means to multiply. Model evaluating 3(6 – 4) for students using manipulatives. Explain that 6 – 4 is inside parentheses, so it is evaluated first, which results in 2.

    Since 3 is directly in front of the parentheses, the next step is to multiply 3 × 2. Three times two can be reworded as three groups of two. This gives a total of 6. Therefore, 3(6 – 4) = 6.

    Continue to model evaluating a variety of two-step numerical expressions involving multiplication with addition or subtraction.

MA 6.2.2 Algebraic Processes

- Ask students to evaluate numerical expressions such as the following with parentheses using manipulatives or drawings.

6(2 + 3)	4(8 – 4)	9(9 – 7)	2(6 – 3)

Prerequisite Extended Indicators

MAE 5.2.2.a—Evaluate a numerical expression with addition or subtraction and multiplication, 1–5.

MAE 4.2.2.a—Evaluate numerical expressions using order of operations using numbers 1 through 5.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

addition, multiplication, numerical expression, parenthesis, subtraction

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-2-topic-b-lesson-3

http://nlvm.usu.edu/en/nav/frames_asid_189_g_2_t_2.html?open=activities&from=category_g_2_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

MA 6.2.2.e

Solve one-step equations with non-negative rational numbers using addition, subtraction, multiplication, and division.

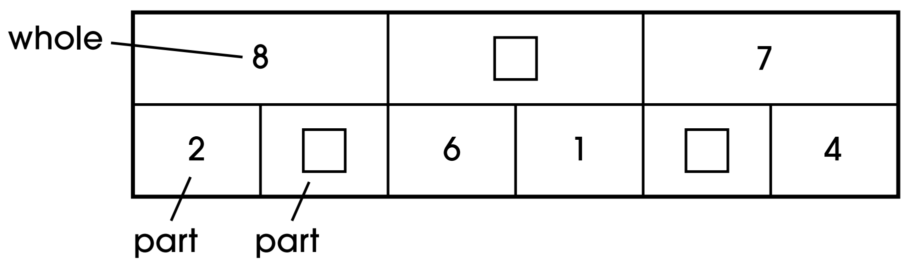
Extended: Solve a one-step equation using addition and subtraction.

Scaffolding Activities for the Extended Indicator

❑ **Identify a variable as an unknown number in an addition or subtraction sentence.**

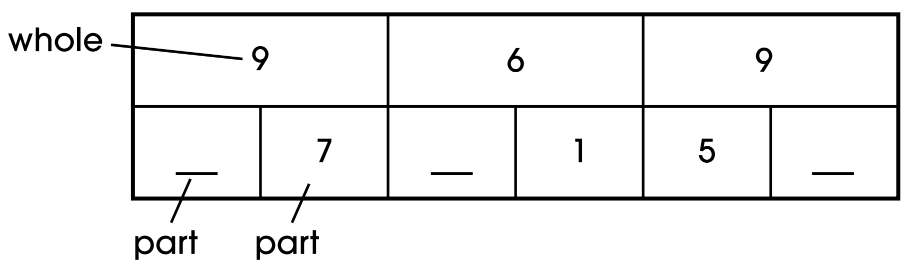
- Show a variety of number sentences where the unknown number is at the end of the number sentence and identify the missing information. Model rephrasing a number sentence into a question. Use part-part-whole diagrams to emphasize the whole and the parts.

8 – 2 = □	6 + 1 = □	7 – 4 = □
What is two less than eight?	What is the total of six and one?	What is seven take away four?



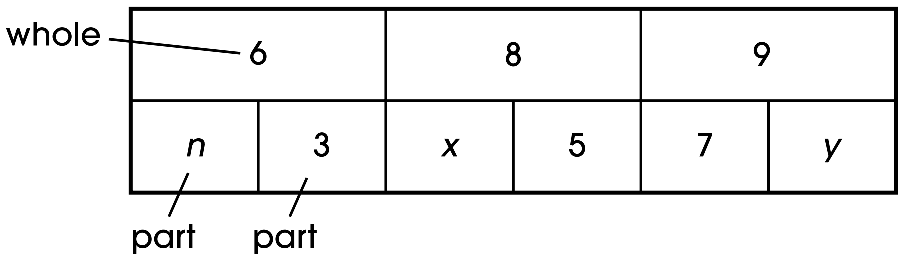
- Explain that sometimes the unknown number is at the beginning or in the middle of a number sentence. Show a variety of number sentences where the unknown number is at the beginning or in the middle of the number sentence and identify the missing information. Model rephrasing a number sentence into a question.

___ + 7 = 9	6 – ___ = 1	___ + 5 = 9
What number plus seven equals nine?	What is taken away from six to get one?	What number added to five makes nine?



- Explain that variables may be used instead of blanks or boxes to show the unknown number. A variable is a letter like *n* or *x*. For example, in the equation *n* + 5 = 8, the variable, *n*, represents a value that is unknown. Model rephrasing an equation with a variable into a question.

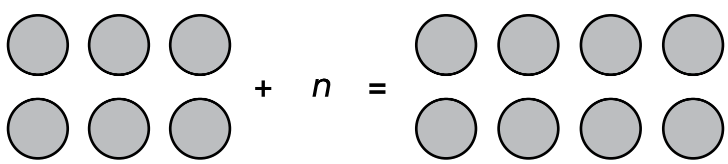
n + 3 = 6	8 – *x* = 5	7 + *y* = 9
What number plus three equals six?	What value is taken away from eight to get five?	What number added to seven makes nine?

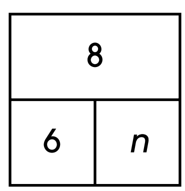


- Ask students to represent the three parts of an equation in a part-part-whole diagram. For example, present the equation 5 – *y* = 2. Students should identify the 5 as the whole and the *y* and the 2 as the parts.

❑ **Solve one-step addition equations with a variable as an addend.**

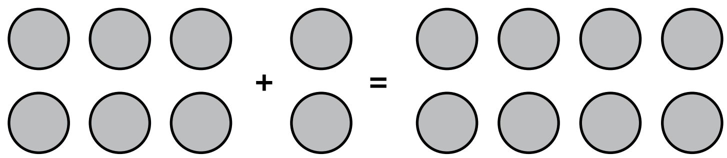
- Demonstrate how to solve 6 + *n* = 8 using manipulatives. Reference a question model and a part-part-whole diagram. What number added to 6 will equal 8? If 6 is a part and 8 is the whole, what is the other part? Model the equation with manipulatives.

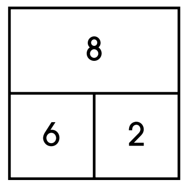




Emphasize that solving the equation means both sides of the equal sign must have the same value. Indicate that there are 8 tokens after the equal sign, so there also need to be 8 tokens before the equal sign. Count the 6 tokens, lay down 2 tokens in the box, and count on: 7, 8. Both sides of the equal sign now equal 8, so *n* = 2.

6 + [2] = 8





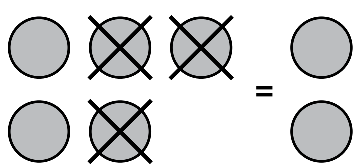
- Continue modeling how to solve one-step addition equations and emphasizing that the value on both sides of the equal sign must be the same. Be sure to include examples in which the variable is the first addend as well as the second addend. Also include examples with the variable as the sum.

$6 + y = 9$	$x + 4 = 5$	$3 + 2 = n$

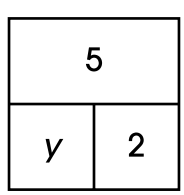
- Ask students to solve one-step addition equations with a variable.

❑ **Solve one-step subtraction equations with a variable as the subtrahend.**

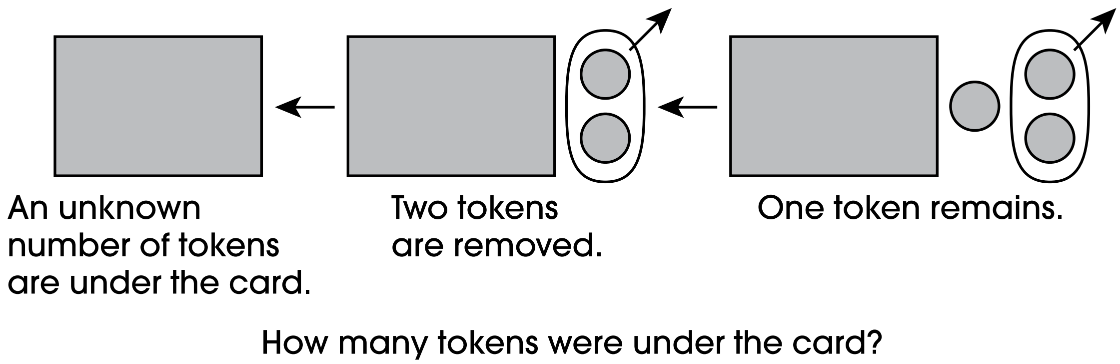
- Demonstrate how to solve 5 – *y* = 2 using manipulatives. Reference a question model. How many are taken away from 5 to equal 2? Model the equation by showing 5 tokens on one side of the equal sign and 2 tokens on the other side of the equal sign. Ask students if the model shows an equal number on both sides of the equal sign. Explain that solving the problem means making the value on both sides of the equal sign the same. Next, demonstrate removing tokens until there are 2 tokens on each side of the equal sign. Then count how many tokens were taken away to find the solution, *y* = 3.



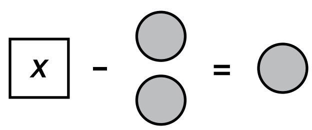
Demonstrate further with a part-part-whole diagram. Starting with a whole of 5, a part is taken away and a part of size 2 remains. What was the size of the part that was taken away?



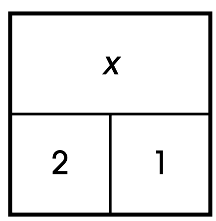
- Explain that subtraction equations may also be presented with a variable at the beginning, as in $x - 2 = 1$. Use manipulatives as one method to solve. Reference a question model. “Starting with some number, 2 are taken away and 1 remains. What was the starting number?” Demonstrate with tokens. First, place three tokens (the unknown) under a card. Next, slide the card to the left to reveal 2 tokens and reference the minus 2 in the equation (2 are taken away). Slide the card to the left again to reveal the last token and reference the answer to the subtraction equation (and 1 remains). How many tokens were under the card? The answer, 3, represents the unknown in this subtraction equation.



Demonstrate with tokens.



Model with a part-part-whole diagram. The whole is unknown, *x*. The part of 2 is removed, and a part of 1 remains. Therefore, the whole must be 3.



- Continue modeling how to solve one-step subtraction equations and emphasizing that the value on both sides of the equal sign must be the same. Be sure to include examples in which the variable is the minuend and the subtrahend. Also include examples with the variable as the difference.

$4 - y = 2$	$n - 6 = 3$	$3 - 2 = n$

- Ask students to solve one-step subtraction equations with a variable.

Prerequisite Extended Indicators

MAE 3.2.2.b—Solve a one-step equation for sums and differences 0–9.

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

addition, equation, subtraction, variable

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-4-topic-g-lesson-28

MA 6.2.2.f

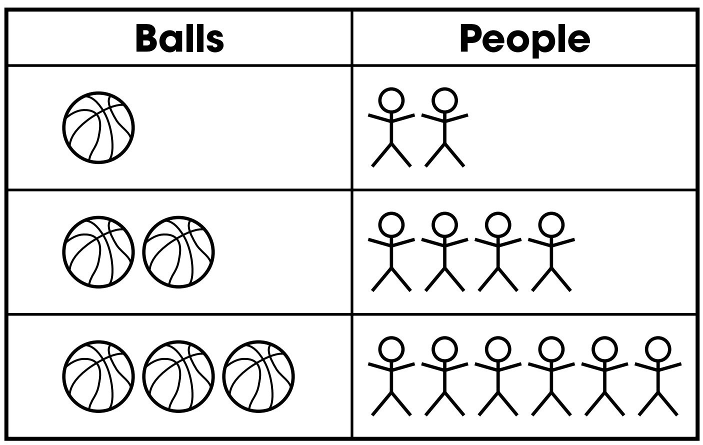
Use equivalent ratios relating quantities with whole numbers to create a table. Find missing values in the table.

Extended: Find a missing number in a table with the ratio of 1:2, 1:3, or 1:10.

Scaffolding Activities for the Extended Indicator

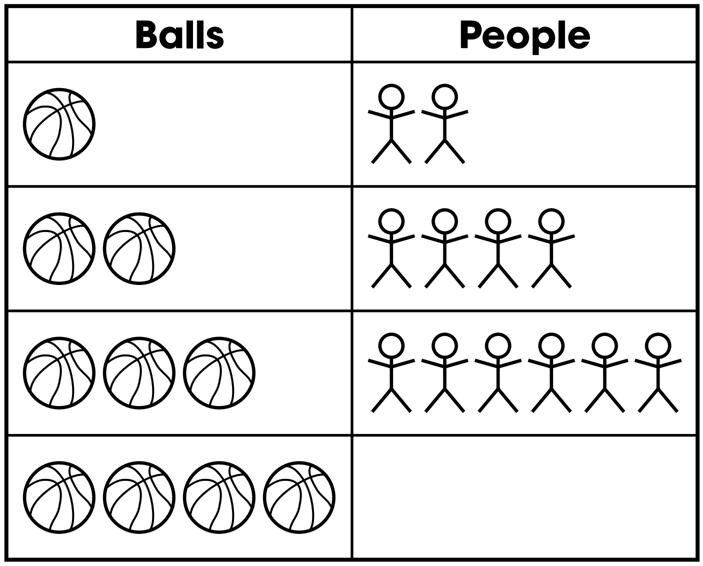
❑ **Find a missing number in a table with a ratio of 1:2.**

- Use manipulatives, pictures, or drawings to show a model of a 1:2 ratio. Explain that a ratio compares the values of two groups. A ratio table is used to organize quantities so patterns can be recognized, and problems can be solved. When a ratio table shows a ratio of 1:2, there is the following pattern: as the value in one group increases by 1, the value of a second group increases by 2. Present the scenario where 1 ball is needed for every 2 people to play catch, and the ratio is 1:2. The ratio table shows the number of balls in one column and the number of people playing catch in the other. The table shows how there is 1 ball for every 2 people, 2 balls for 4 people, 3 balls for 6 people, and so on.



Ratio – 1:2

Present the table with a value missing. Demonstrate how to determine the missing number of people that belongs in the table by drawing in 2 people for every ball.



Ratio – 1:2

Since there is 1 ball for 2 people, the missing value is 8.

- Ask students to identify a 1:2 ratio table that represents another scenario.
- Ask students to find the missing value in a 1:2 ratio table.

❑ **Find a missing number in a table with a ratio of 1:3.**

- Use manipulatives, pictures, or drawings to show a model of a 1:3 ratio. When a ratio table shows a ratio of 1:3, there is the following pattern: as the value of one group increases by 1, the value of the second group increases by 3. Present the scenario of a dance routine in which there is one clap for every 3 stomps. Therefore, the ratio of clapping to stomping is 1:3. Create a ratio table that shows the scenario using tick marks.

<table>
<tr><th>Claps</th><th>Stomps</th></tr>
<tr><td>|</td><td>|||</td></tr>
<tr><td>||</td><td>||||||</td></tr>
<tr><td>|||</td><td>|||||||||</td></tr>
</table>

Ratio – 1:3

Present the table with a missing value. Demonstrate how to determine the missing number of stomps by making three tick marks for every clap.

<table>
<tr><th>Claps</th><th>Stomps</th></tr>
<tr><td>|</td><td>|||</td></tr>
<tr><td>||</td><td></td></tr>
<tr><td>|||</td><td>|||||||||</td></tr>
</table>

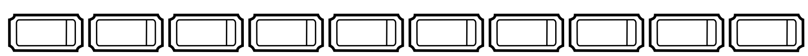
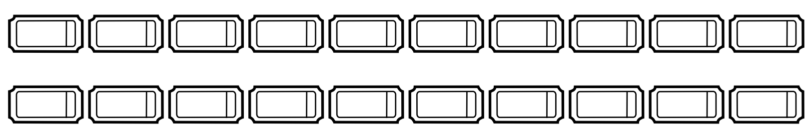
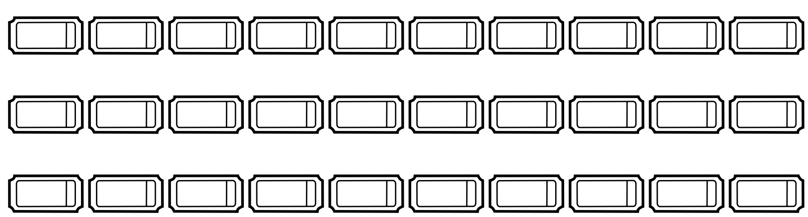
Ratio – 1:3

Since there is 1 clap for 3 stomps, the missing value is 6.

- Ask students to identify a 1:3 ratio table that represents another scenario.
- Ask students to find a missing value in a 1:3 ratio table.

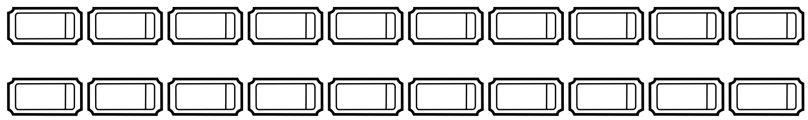
❑ **Find a missing number in a table with a ratio of 1:10.**

- Use manipulatives, pictures, or drawings to show a model of a 1:10 ratio. When a ratio table shows a ratio of 1:10, there is the following pattern: as the value of one group increases by 1, the value of the second group increases by 10.

<table>
<tr><th>Dollars</th><th>Tickets</th></tr>
<tr><td>$1.00</td><td></td></tr>
<tr><td>$2.00</td><td></td></tr>
<tr><td>$3.00</td><td></td></tr>
</table>

Ratio – 1:10

Present the table with a missing value. Demonstrate how to determine the missing number of tickets by drawing a group of 10 tickets for every dollar.

Dollars	Tickets
$1.00	
$2.00	
$3.00	

Ratio – 1:10

Since 1 dollar is needed for every 10 tickets, the missing value is 30.

- Ask students to identify a 1:10 ratio table that represents another scenario.
- Ask students to find a missing value in a 1:10 ratio table.

Prerequisite Extended Indicators

MAE 4.1.2.b—Multiply 2's, 5's and 10's by a single digit number.

MAE 4.1.1.d—Count by twos and fives, and tens with numbers, models, or objects up to 40.

Key Terms

compare, increases, pattern, ratio, ratio table, set

Additional Resources or Links

https://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/61

https://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/2157

MA 6.2.2.g

Represent inequalities on a number line (e.g., graph $x > 3$).

Extended: Identify a solution to an inequality on a number line (–10 to 10).

Scaffolding Activities for the Extended Indicator

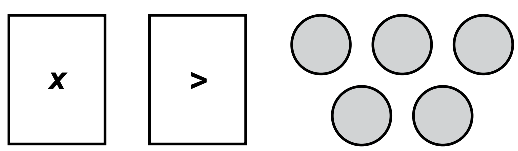
- ❑ **Identify an inequality in the form of *x* > (the integers –10 to 10).**

  - Introduce the concept of an inequality as an expression showing that two quantities are not equal. The quantities are compared using inequality symbols. Use a table to display the inequality symbols, example inequalities, and word forms of the inequalities. Make connections between the symbols and the words that describe the symbols. The symbol “opens” to the greater value. When a variable is on the left side of an inequality and a number is on the right side, the variable is “less than” the number when the symbol opens to the right. The variable is “greater than” the number when the symbol opens to the left.

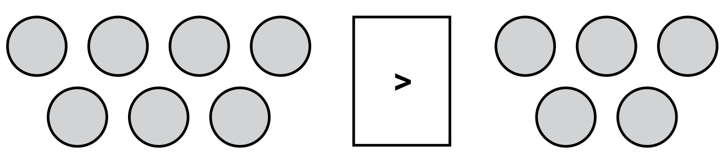
$<$	$>$	$\leq$	$\geq$
$x < 5$	$x > 5$	$x \leq 5$	$x \geq 5$
x is less than 5	*x* is greater than 5	*x* is less than or equal to 5	*x* is greater than or equal to 5

  Explain that reading an inequality uses the specific vocabulary shown. Demonstrate how to read inequalities such as $x > 4$, $x < –9$, $x \geq –2$, and $x \leq 1$.

  - Use manipulatives, a card with a variable, and a set of four cards with an inequality symbol to demonstrate that an inequality can have more than one solution that makes the inequality true.



  Replace the variable card with a set of manipulatives, for example, 7 tokens, that makes the number sentence true. Replace the 7 tokens with 9 tokens and indicate that the number sentence is still true. Be sure to reinforce the idea that there is more than one possible solution to an inequality.



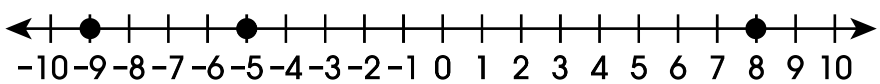
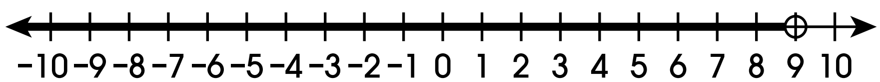
  Continue the process with all the inequality cards.

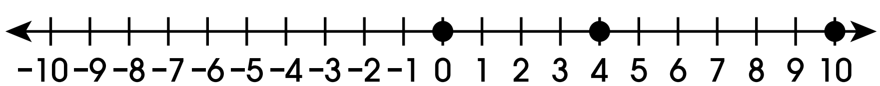
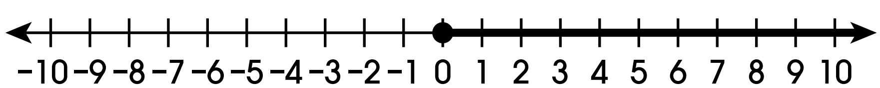
- Ask students to interpret an inequality and to identify a set of manipulatives that is a solution to an inequality.

$x > 6$	$x \geq 10$	$x < -2$	$x \leq 7$

❑ **Identify a solution to an inequality on a number line (–10 to 10).**

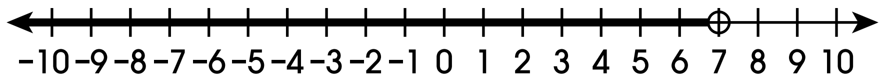
- Use tables to display example inequalities, a description of the solution, graphs of **some** solutions, and graphs of **all** solutions. Explain that the inequalities have too many solutions to list but that solutions can be graphed on a number line to show all the solutions. Some of the solutions may be plotted on the number line. All the solutions are shown with an arrow and either a closed circle or an open circle as the endpoint.

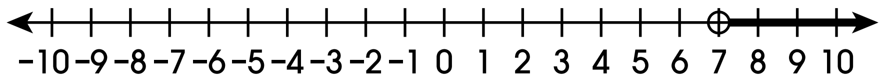
Inequality	$x < 9$
Description of solution	all numbers less than 9
Graph of <u>some</u> values of *x* that are solutions	
Graph of <u>all</u> values of *x* that are solutions	

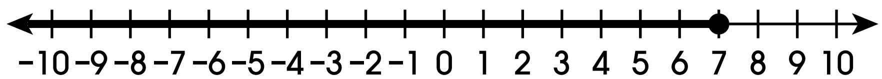
Inequality	$x \geq 0$
Description of solution	all numbers greater than or equal to 0
Graph of <u>some</u> values of *x* that are solutions	
Graph of <u>all</u> values of *x* that are solutions	

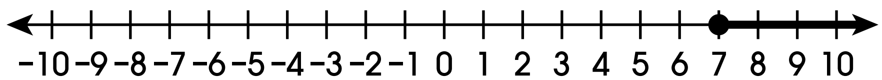
In each graph, there is a circle drawn at the number given in the inequality. When an endpoint is <u>not</u> a solution, an open circle is used. When an endpoint is a solution, a closed circle is used. If the inequality uses < or >, the circle is open. If the inequality uses ≤ or ≥, the circle is closed. If the solutions are less than the number, the arrow points to the left. If the solutions are greater than the number, the arrow points to the right.

Continue to demonstrate number lines with various quantities and inequality symbols. For example, present four possible solutions to $x < 7$. Ask a series of questions to determine the correct solution. Are the solutions less than or greater than 7? Which direction should the arrow point to show solutions less than 7? The answers to the first two questions eliminate the second and fourth number lines. Does the solution include the number 7? The answer to the last question is yes, so the circle should be closed, or filled in. The correct solution to $x < 7$ is the third number line. It may be helpful to create a checklist of questions to ask when determining the solution to an inequality on a number line.









- Ask students to identify the graph of the solution to an inequality.

Prerequisite Extended Indicators

MAE 6.1.1.h—Compare and order integers (–10 to 10) on a number line.

MAE 6.1.1.g—Identify models of integers –10 to 10 using drawings, words, manipulatives, number lines and symbols.

MAE 4.1.1.f—Use symbols <, >, and = to compare whole numbers up to 40.

Key Terms

endpoints, equal, inequality, number line, solution, variable

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-lesson-1

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-overview

https://curriculum.illustrativemathematics.org/MS/students/1/7/8/index.html

MA 6.2.3 Applications

MA 6.2.3.b

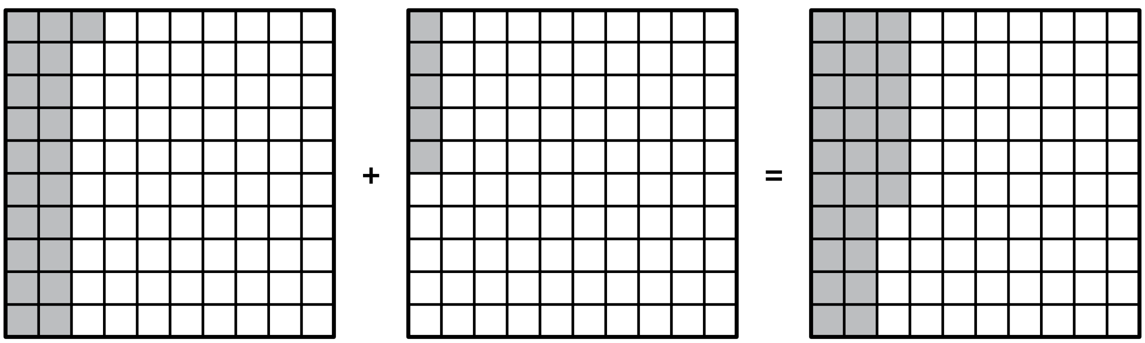
Solve real-world problems involving non-negative rational numbers.

Extended: Solve real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping.

Scaffolding Activities for the Extended Indicator

❑ **Add numbers 0–1 with decimal numbers to the hundredth without regrouping.**

- Use a hundreds grid to demonstrate adding decimal numbers less than 1. Explain that a hundreds grid represents one whole, and each small square is $\frac{1}{100}$ of the whole, or 0.01. Shading in 21 of the squares represents 0.21, and shading in 5 of the squares represents 0.05. The model shown represents 0.21 + 0.05 = 0.26.



0.21 + 0.05 = 0.26

Continue to demonstrate solving a variety of addition problems using hundreds grids and sums less than 1.

- Ask students to identify the sum of two numbers 0–1 with decimal numbers to the hundredth represented on a hundreds grid.
- Ask students to add numbers 0–1 with decimal numbers to the hundredth without regrouping using hundreds grids.
- Use the standard algorithm to demonstrate adding decimal numbers less than 1. Present the addition problem 0.82 + 0.13 as shown.

$$\begin{array}{r} 0.82 \\ +\ 0.13 \\ \hline \end{array}$$

Make note that the decimals in each number must align vertically so that each place value is added to the same place value in the other number. So, for this example, the digits 2 and 3 in the hundredths place are added together to make 5 hundredths, and the digits 8 and 1 in the tenths place are added together to make 9 tenths. There are zeros in the ones place, so the final sum is 0.95. Emphasize that when writing the sum, the decimal point should align with the other decimal points.

$$\begin{array}{r} 0.82 \\ +\ 0.13 \\ \hline 0.95 \end{array}$$

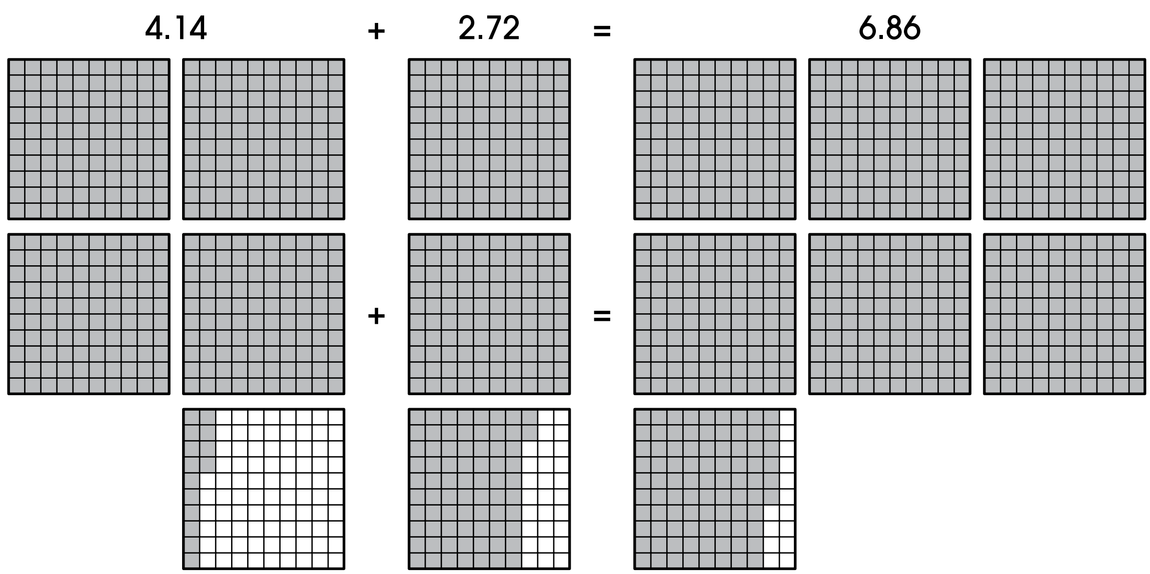
It might be helpful to use visual supports including but not limited to writing the addition problem on grid paper, writing the addition problem on a place value mat, or using base ten blocks to represent the problem on a place value mat.

Continue to demonstrate solving a variety of addition problems involving decimal numbers less than 1, with hundredths and no regrouping.

- Ask students to add numbers 0–1 with decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

❑ **Add numbers greater than 1 with decimal numbers to the hundredth without regrouping.**

- Use strategies such as the hundreds grid and the standard algorithm to demonstrate adding numbers greater than 1 with decimal numbers to the hundredth. Present the problem 4.14 + 2.72, and demonstrate finding the sum using hundreds grids as shown.



As the numbers become greater, it gets more difficult to add them using hundreds grids. It might be helpful to first add together the whole numbers (i.e., the numbers to the left of the decimal point) and then use the hundreds grids for the tenths and hundredths places only. For this example, 4 + 2 = 6, and the bottom row of the figure can be used to find 0.86. Explain that combining the sum of the whole numbers and the sum of the decimals results in 6.86.

Then demonstrate how to use the standard algorithm to add numbers greater than 1 with decimal numbers to the hundredth. Present the problem 14.02 + 3.94 as shown and demonstrate finding the sum of each place value. Be sure to emphasize lining up the decimal points.

$$\begin{array}{r} 14.02 \\ +\ 3.94 \\ \hline 17.96 \end{array}$$

Continue to demonstrate solving a variety of addition problems with numbers greater than 1 with decimal numbers to the hundredth using models and the standard algorithm with visual supports as needed (e.g., grid paper, place value mats, base ten blocks).

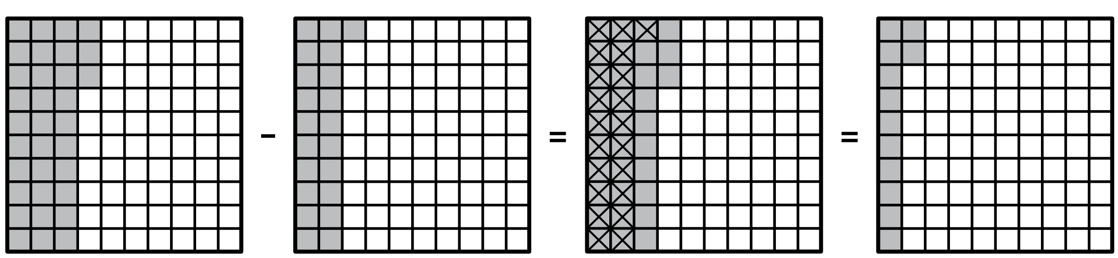
- Ask students to add numbers greater than 1 with decimal numbers to the hundredth without regrouping using a hundreds grid model.
- Ask students to add numbers greater than 1 with decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

❑ **Subtract decimal numbers to the hundredth without regrouping.**

- Use strategies such as the hundreds grid and the standard algorithm to demonstrate subtracting decimal numbers to the hundredth. Present the problem 23.89 – 3.77, and demonstrate using the standard algorithm to subtract 23.89 – 3.77. Emphasize the alignment of the decimal points, as well as the alignment of the digits of the same place value. Demonstrate starting with the place furthest to the right to calculate: 9 – 7 = 2 in the hundredths column, then 8 – 7 = 1 in the tenths column, 3 – 3 = 0 in the ones column, and 2 – 0 = 2 in the tens column. The difference can be written as shown.

$$\begin{array}{r} 23.89 \\ -\ 3.77 \\ \hline 20.12 \end{array}$$

Hundreds grids can also be used to model subtraction, and it is easiest to only focus on the decimal numbers with hundreds grids. For the problem 17.33 – 14.21, first demonstrate subtracting the whole numbers, 17 – 14 = 3. The final answer will start with 3. Then use the hundreds grids to show 0.33 – 0.21.



0.33 0.21 0.12

Explain that putting together the difference of the whole numbers and the difference of the decimals results in 17.33 – 14.21 = 3.12.

Continue to demonstrate solving a variety of subtraction problems with decimal numbers to the hundredth without regrouping.

- Ask students to subtract decimal numbers to the hundredth without regrouping using a hundreds grid model.
- Ask students to subtract decimal numbers to the hundredth without regrouping using the standard algorithm and visual supports as needed.

❑ **Solve real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping.**

- Use story problems to demonstrate how to solve real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping.

Marcy has a new plant. It is 1.10 centimeters tall.
One month later, the plant is 6.35 centimeters tall.
How much did the plant grow in one month?

Explain that this problem involves subtraction because the difference between the two heights is needed. Model solving the subtraction problem using a hundreds grid or the standard algorithm to identify that the plant grew 5.25 centimeters in one month.

$$\begin{array}{r} 6.35 \\ -\ 1.10 \\ \hline 5.25 \end{array}$$

Continue to demonstrate solving a variety of real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping.

- Ask students to solve real-world problems with addition and subtraction of decimal numbers to the hundredth without regrouping. For example, present the problem shown with three answer options.

Brian has $3.24 in his pocket.
He found $1.55 in his room.
What is the total amount of money Brian has now?

$$\begin{array}{r} \$3.24 \\ +\ \$1.55 \\ \hline \end{array}$$

$2.31

$4.00

$4.79

Students should determine that $4.79 is the correct answer.

Prerequisite Extended Indicators

MAE 6.1.2.d—Add and subtract numbers 0–10 with one decimal place without regrouping.

MAE 4.2.3.a—Solve addition and subtraction real-world problems with addition and subtraction up to 40 without regrouping.

MAE 3.2.3.a—Solve a one-step real-world problem using addition or subtraction 0–9.

Key Terms

add, decimal number, difference, hundredth, place value, subtract, sum, tenth

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/NBT/B/7/tasks/1293

https://nysed-prod.engageny.org/resource/grade-5-mathematics-module-1-topic-d-overview

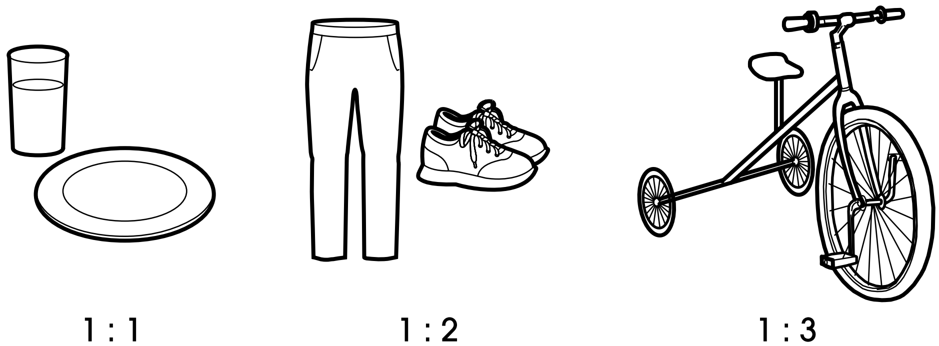
MA 6.2.3.d

Solve real-world problems using ratios and unit rates.

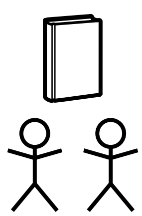
Extended: Solve real-world problems using ratios up to 1:3.

Scaffolding Activities for the Extended Indicator

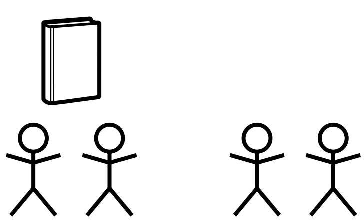
- ❑ **Identify graphic representations that represent the ratios 1:1, 1:2, or 1:3 in a real-world context.**
  - Use real-world objects or pictures to show models of 1:1, 1:2, and 1:3 ratios, and explain that a ratio indicates the quantity of one object compared to another.



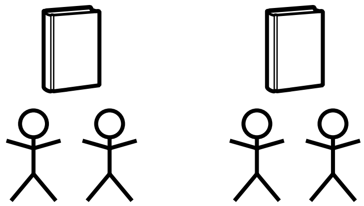
  - Ask students to identify the ratios 1:1, 1:2, and 1:3 using real-world objects or pictures.
- ❑ **Use pictures to solve real-world problems with ratios 1:1, 1:2, and 1:3.**
  - Use a real-world word problem and a picture to model solving a ratio problem. For example, “During partner reading there is 1 book for every 2 students. How many books will 4 students need?” Draw a picture of 1 book and 2 students to represent the given ratio.



Then reread the question and draw two more students.



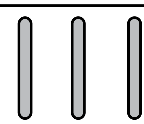
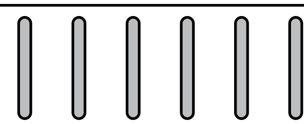
Explain that the ratio 1 book to 2 students needs to be the same for both pairs of students and draw in the book for the second pair of students. Reread the question one more time to identify that the problem is asking for the total number of books, which is 2.



- Ask students to solve a ratio problem given a word problem and a picture that represents the scenario.

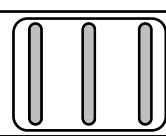
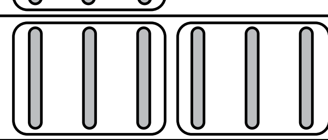
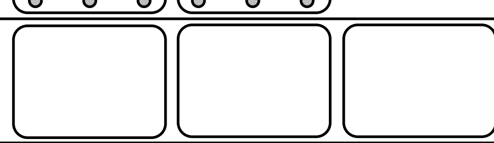
❑ **Solve real-world problems with the ratios 1:1, 1:2, and 1:3.**

- Represent a ratio problem in a table. For example, “Each student will need three craft sticks for the art project. How many craft sticks will be needed for three students?”

Students	Craft Sticks
1	
2	
3	

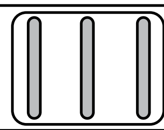
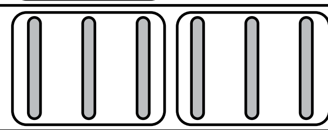
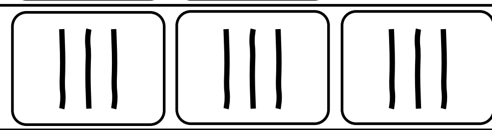
Ratio – 1:3

Model solving the problem by first circling groups of three craft sticks to represent the ratio 1:3. Each circle, or group of 3, is for one student. Then, draw in 3 circles to represent the number of groups of craft sticks needed for three students.

Students	Craft Sticks
1	
2	
3	

Ratio – 1:3

Next, draw 3 craft sticks or tally marks in each circle to represent the ratio 1 student to 3 craft sticks. Reread the question to identify that the problem is asking for the number of craft sticks needed for 3 students, and count to find the answer is 9.

Students	Craft Sticks
1	
2	
3	

Ratio – 1:3

Ask students to solve a ratio problem given a word problem and a ratio table that represents the scenario.

Prerequisite Extended Indicator

MAE 6.2.2.f—Find a missing number in a table with the ratio of 1:2, 1:3, or 1:10.

Key Terms

ratio, ratio table

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-1-topic-overview

http://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/496

http://tasks.illustrativemathematics.org/content-standards/6/RP/A/tasks/61

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Mathematics—Grade 6
MA 6.3 Geometry

MA 6.3.1 Characteristics

MA 6.3.1.a

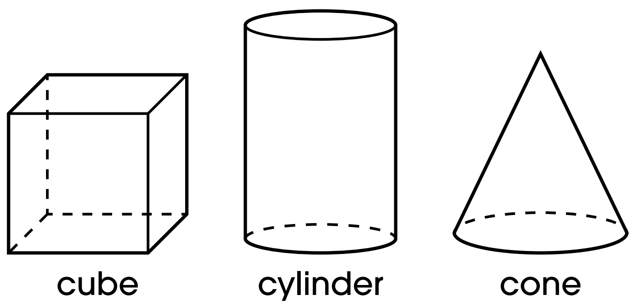
Identify and create nets to represent two-dimensional drawings of prisms, pyramids, cylinders, and cones.

Extended: Identify a cube, cylinder, or cone from a given two-dimensional representation.

Scaffolding Activities for the Extended Indicator

❑ **Identify a cube, a cylinder, or a cone from a drawing.**

- Present purchased or created geometric solids of a cube, a cylinder, and a cone to compare to drawings of a cube, a cylinder, and a cone. Explain that a cube has only square faces, a cylinder has two circular faces, and a cone has one circular face and comes to a point opposite the circle. Be sure to show drawings of cubes, cylinders, and cones in different sizes and positions.



  Present drawings of rectangles, squares, and cubes, and emphasize the differences in the drawings that represent two-dimensional figures and the drawings that represent the cubes. Present drawings of circles, triangles, rectangles, cubes, and cones, and emphasize the differences in the drawings that represent the two-dimensional figures and the drawings that represent the cones and the cylinders.

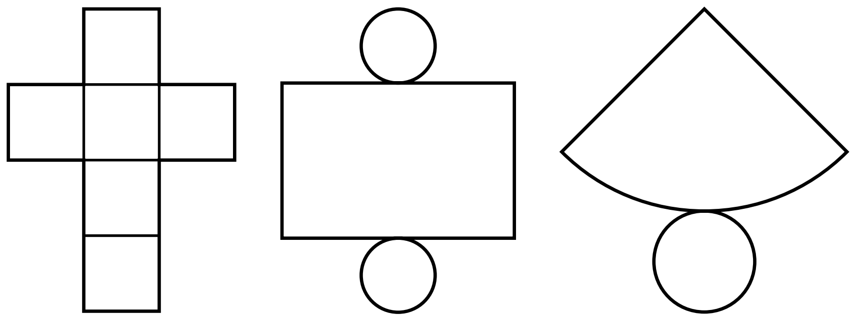
- Ask students to identify a cube from a collection of drawings of a square, a rectangle, and a cube.

- Ask students to identify a cylinder or a cone from a collection of drawings of a circle, a triangle, a rectangle, a cylinder, and a cone.

- Ask students to identify a cube, a cylinder, or a cone from a collection of drawings of three-dimensional figures.

- [ ] **Identify a cube, a cylinder, or a cone from a net.**

  - Use a net to make a three-dimensional model of a cube, a cylinder, and a cone. Then show how each shape can be unfolded into the net shown. The net is a two-dimensional representation of the three-dimensional model when it is unfolded. Emphasize the shape of each face. For the cube, each square of the net is a face of the cube. For the cylinder, the two circles are the top and bottom faces. For the cone, the circle shows the face of the cone that is flat.



    When appropriate, show examples and offer opportunities to form three-dimensional models of cubes, cylinders, and cones using a variety of sizes and two-dimensional net patterns.

  - Ask students to identify a cube, a cylinder, or a cone from a net.

Prerequisite Extended Indicators

MAE 5.3.1.a—Identify three-dimensional models limited to cube, cylinder, and cone.

MAE 3.3.1.b—Identify two-dimensional shapes, circles, triangles, rectangles, or squares from a collection of circles, rectangles, and squares.

Key Terms

cone, cube, cylinder, face, net, three-dimensional, two-dimensional

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB1SUP-C1_Geometry3D-201304.pdf

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB1SUP-C7_MarDesc_3-D_Shapes-201304.pdf

MA 6.3.2 Coordinate Geometry

MA 6.3.2.c

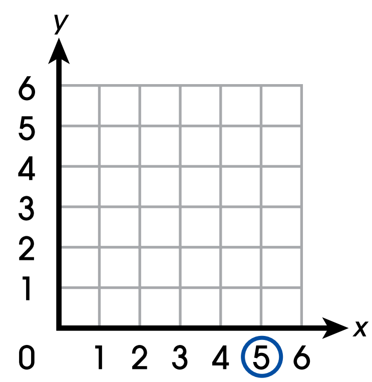
Identify the quadrant of a given point in the coordinate plane.

Extended: Identify a point on a 4-by-4 grid in quadrant 1.

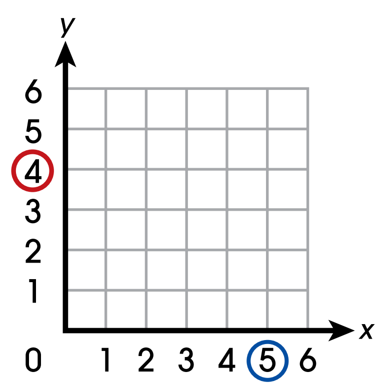
Scaffolding Activities for the Extended Indicator

❑ **Identify the location of an ordered pair on a coordinate plane.**

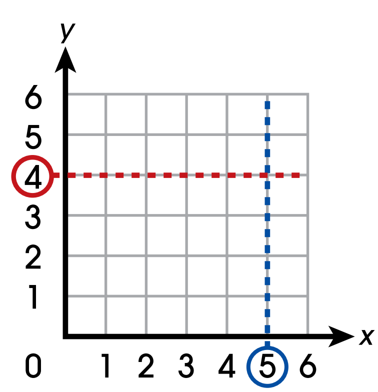
- Use an ordered pair to show how to place an object in the correct location on a graph. Demonstrate locating the ordered pair (5, 4). Begin by locating the 5 on the *x*-axis.



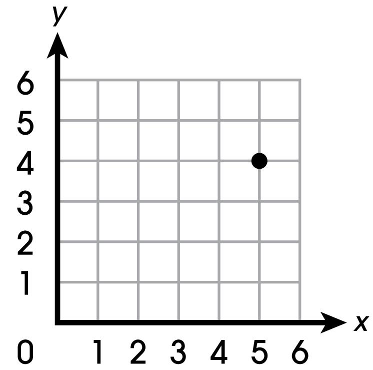
Then locate the 4 on the *y*-axis.



Follow the grid lines to find the location where they intersect.



The place of intersection is represented by (5, 4), so that is where a point is placed.

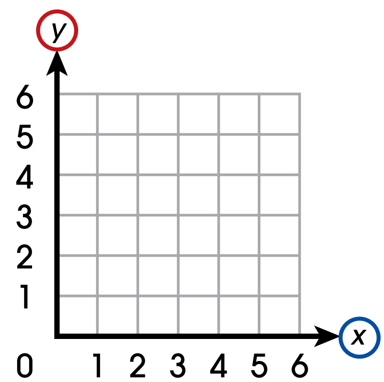


Show this same process for a variety of whole-numbered ordered pairs in quadrant 1.

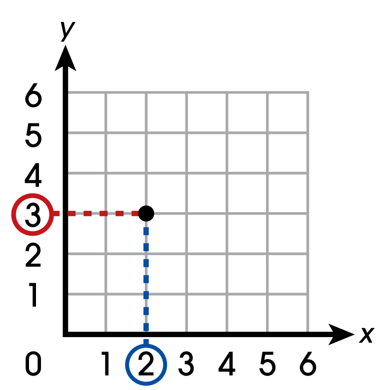
- Ask students to find the location of an ordered pair on a coordinate plane in quadrant 1.

❑ **Identify the ordered pair for a point on a coordinate plane.**

- Use a graph to demonstrate how to find the *x*- and *y*-coordinates of a point. Indicate that the *x*-coordinate comes from the *x*-axis and the *y*-coordinate comes from the *y*-axis.



Follow the grid lines from the point to each axis and explain that the number that aligns to the point shows the coordinate of that point. It may be helpful to trace or highlight the vertical grid line down to the *x*-axis and the number on the *x*-axis in one color and then use a different color to locate the *y*-coordinate.



For this point, the *x*-coordinate is 2 and the *y*-coordinate is 3. An ordered pair is always written as (*x*, *y*), with the appropriate numbers put in the place of the *x* and *y*. The location of the point can be written as the ordered pair (2, 3). Repeat this example with points at other locations.

- Ask students to identify the ordered pair for a point on a coordinate plane, keeping the points in the first quadrant of the graph.

Prerequisite Extended Indicators

MAE 5.3.2.b—Identify the *x*- or *y*-coordinate of whole-numbered points in quadrant I.

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

Key Terms

coordinate plane, intersect, point, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-2/lesson.html

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-c-lesson-14

MA 6.3.2.d

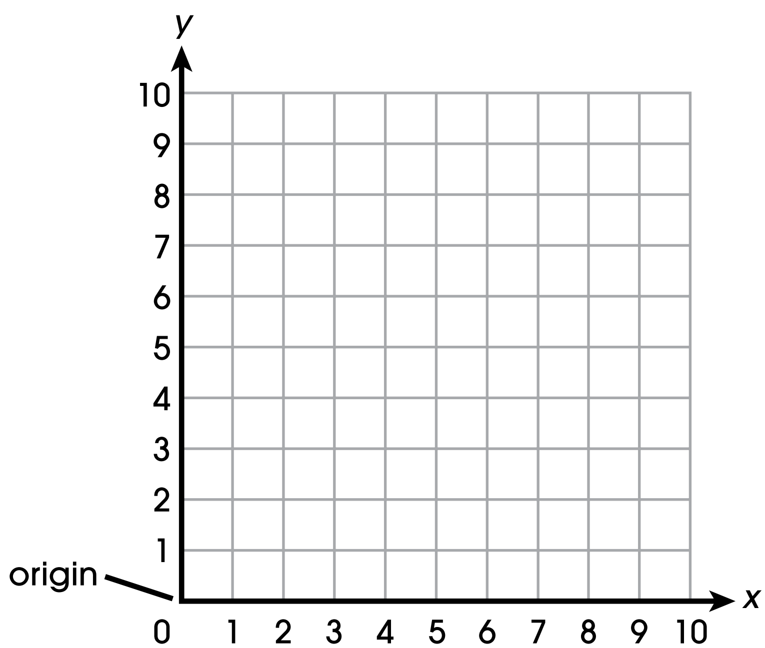
Draw polygons in the coordinate plane given coordinates for the vertices.

Extended: Identify the location of one vertex of a triangle in quadrant 1 with one vertex on the origin.

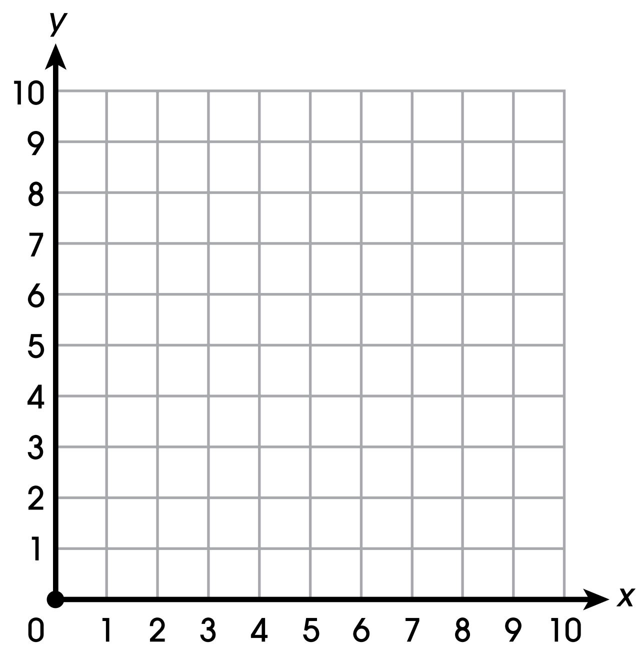
Scaffolding Activities for the Extended Indicator

❑ **Identify the origin on a coordinate plane.**

- Use a coordinate plane to show the location of the origin on a graph.



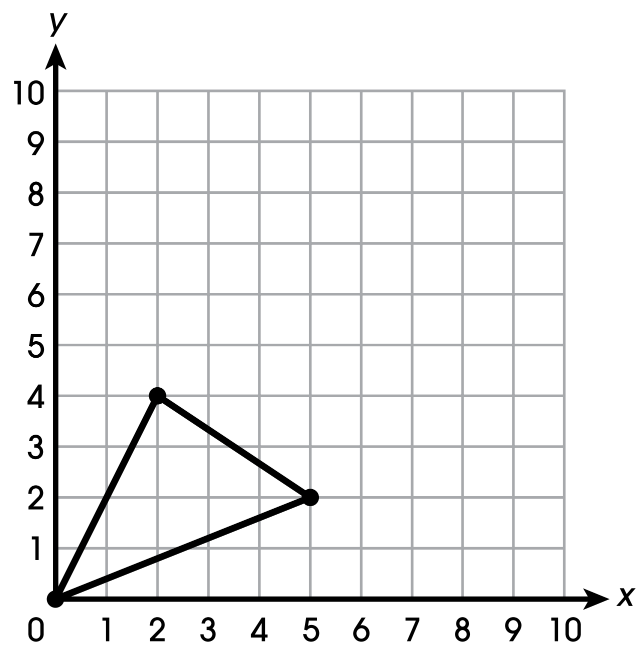
The point where the *x*-axis and *y*-axis meet is called the origin. It can be represented by the ordered pair (0, 0). A point that is plotted at the origin looks like the point on the graph shown.



Make note that the origin is always located at (0, 0), even in graphs of different sizes.

- Ask students to identify the origin on a given coordinate graph.

- ❑ **Identify the vertices of a triangle on a coordinate plane.**
  - Use a coordinate graph to show the locations of the three vertices of a triangle.

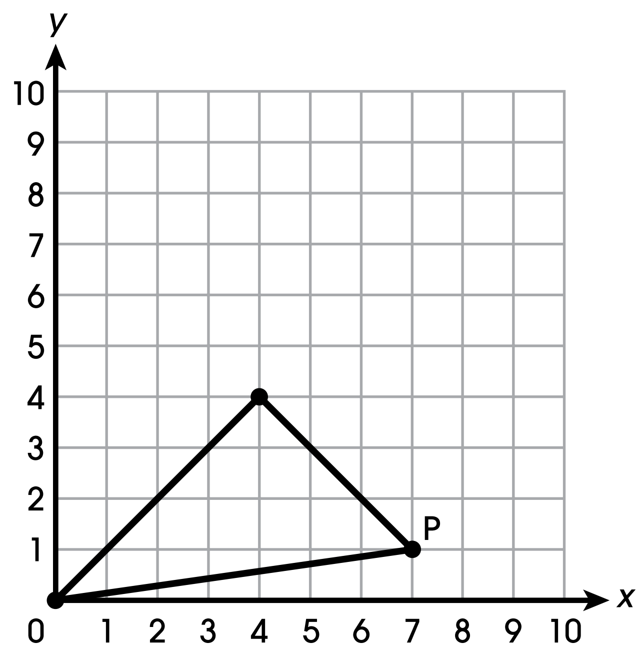


In the triangle shown, each vertex has a point plotted, and one of the vertices is at the origin. Point out where the other two vertices, or corners, of the triangle are located. Demonstrate finding the three vertices of a triangle and identifying which vertex is at the origin using a variety of different-size triangles.

  - Ask students to identify the three vertices of a triangle drawn on a coordinate graph.

- ❑ **Identify the location of a vertex of a triangle on a coordinate plane.**

  - Use a coordinate graph to show how to find the coordinates of a given vertex of a triangle. For example, demonstrate identifying the coordinates of the vertex labeled P on the graph shown.



The vertex at point P has the *x*-coordinate 7 and the *y*-coordinate 1, so the ordered pair for point P is (7, 1). Show a variety of triangles graphed in the first quadrant of the coordinate plane and demonstrate finding the ordered pair for each vertex.

  - Ask students to identify the coordinates of a vertex of a triangle on a coordinate plane.

Prerequisite Extended Indicator

MAE 6.3.2.c—Identify a point on a 4-by-4 grid in quadrant 1.

Key Terms

coordinate plane, coordinates, origin, point, triangle, vertex, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-7/lesson-2/lesson.html

https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-3/file/69596

MA 6.3.3 Measurement

MA 6.3.3.a

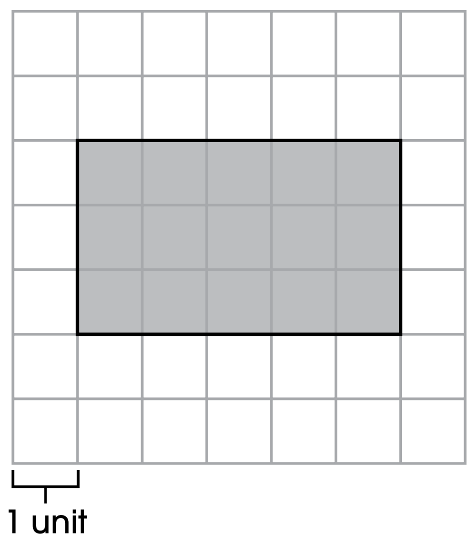
Determine the area of quadrilaterals, including parallelograms, trapezoids, and triangles by composition and decomposition of polygons as well as application of formulas.

Extended: Find the area of a rectangle using its whole number side lengths.

Scaffolding Activities for the Extended Indicator

❑ **Find the area of a rectangle by counting whole number unit squares.**

- Use a rectangle drawn on a grid to count the number of unit squares that cover a rectangle. Draw and shade in a rectangle that is 3 units tall and 5 units wide on a grid. Explain that area is the number of unit squares that cover a shape.



Demonstrate creating a 3 × 5 array that represents the square units shaded to form the rectangle.

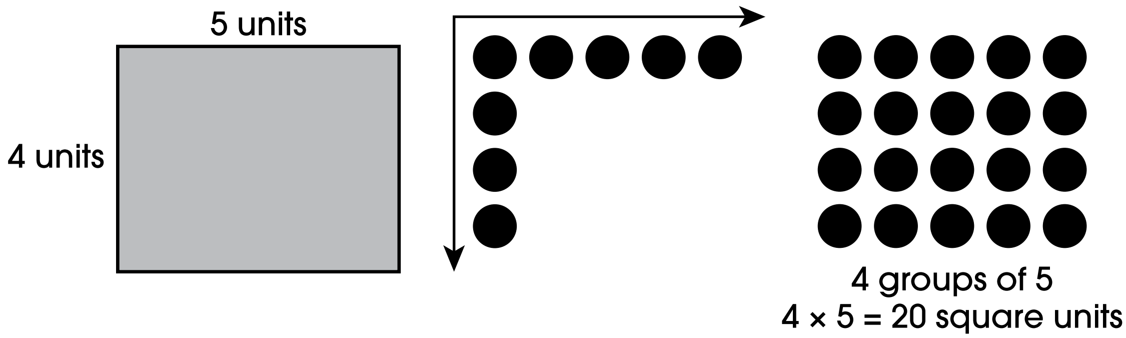
x x x x x 5
x x x x x 5
x x x x x + 5
15 square units

Determine the area of the rectangle or the size of the array by counting, skip counting by 5, or repeated addition. Identify 15 square units as the area of the rectangle.

- Ask students to construct arrays that represent the areas of rectangles presented on a grid.
- Ask students to calculate the areas of rectangles presented on a grid.

- ❑ **Find the area of a rectangle using its whole number side lengths.**

  - Present a 4-by-5 rectangle labeled with side lengths. Explain that the side lengths indicate the number of rows and columns of square units that cover the rectangle. Create an array of dots that represent the square units.



  - Demonstrate calculating the area of the rectangle using the side lengths with an appropriate computation strategy, including but not limited to a calculator, skip counting by 5, repeated addition, automaticity of multiplication facts, or a multiplication chart.
  - Continue to demonstrate finding the area of a rectangle given the side lengths. Continue to use arrays when necessary to support computation strategies and progress to finding the area without visual representations when appropriate.
  - Ask students to create arrays when given the side lengths of a rectangle.
  - Ask students to calculate the area of a rectangle given whole number side lengths.

Prerequisite Extended Indicators

MAE 5.1.2.a—Multiply a two-digit number by a single-digit number.

MAE 4.3.3.a—Identify the area of a rectangle by counting unit squares.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

Key Terms

array, length, multiply, rectangle, sides, square unit, width

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-4-topic-lesson-3

https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-c-overview

MA 6.3.3.b

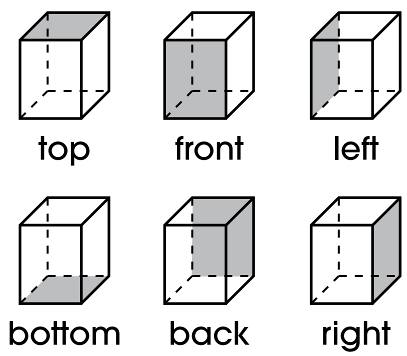
Determine the surface area of rectangular prisms and triangular prisms using nets. **Extended:**

Find the surface area of a rectangular prism by counting unit squares in a net.

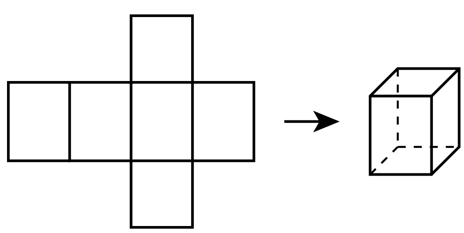
Scaffolding Activities for the Extended Indicator

❑ **Identify the six faces (sides) of a rectangular prism represented in a net figure.**

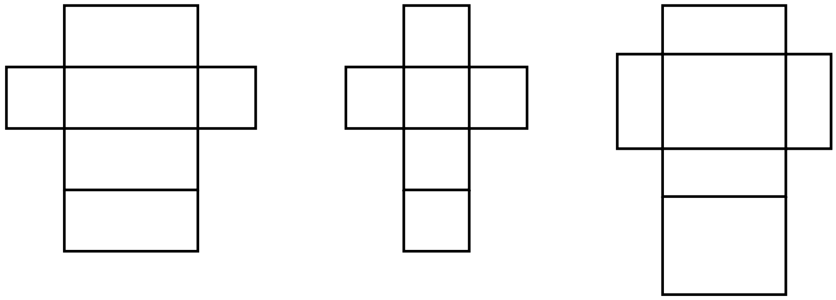
- Explain that three-dimensional shapes are solid shapes. A rectangular prism is one type of three-dimensional shape. Show real-life examples and images of rectangular prisms (e.g., tissue box, cereal box, fish tank).
- Use a rectangular prism manipulative or one made from a net to show the six faces. Explain that faces are the flat sides of the rectangular prism and that each face in a rectangular prism is a rectangle. Label each of the faces with tape or sticky paper.



- Ask students to identify the six faces on a model of a rectangular prism.
- Demonstrate folding the net of a rectangular prism into a three-dimensional model. Repeat with several examples of nets. Be sure to emphasize the rectangular shape of each section of the flat figure (net) and the rectangular shape of each face of the three-dimensional figure as the nets are folded and unfolded.

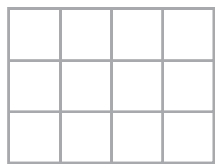


- Ask students to identify the six rectangles in a net of a rectangular prism.



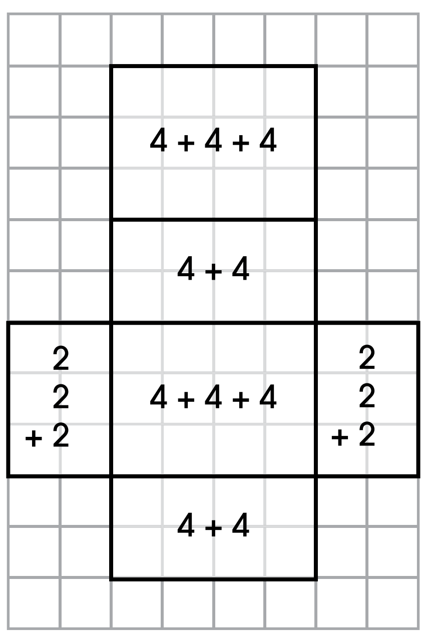
☐ **Find the surface area of a rectangular prism by counting unit squares.**

- Explain that the area of a rectangle is the count of how many square units cover the surface of the rectangle. Demonstrate counting the squares of the rectangle shown one row at a time and adding to find the total area equals 12 square units.



4 + 4 + 4 = 12

- Ask students to find the area of a rectangle by counting unit squares.
- Explain that the area of all the faces of a rectangular prism combined is called the surface area. Use the example below to demonstrate finding the surface area of a rectangular prism when given a net on a grid.



2 + 2 + 2 = 6
2 + 2 + 2 = 6
4 + 4 + 4 = 12
4 + 4 = 8
4 + 4 + 4 = 12
4 + 4 = 8
52

Count the left side and the right side, then count the four remaining faces starting at the top and moving down. Record the results of each side and then add to find the total surface area of 52 square units. Another strategy could be placing tick marks or dots in each square as it is counted and recording the results for each face to be added with a calculator. Continue modeling finding the surface areas of rectangular prisms using appropriate counting and computation strategies, including counting, skip counting, repeated addition, and arrays.

- Ask students to find the surface area of a rectangular prism by counting unit squares in a net for each of the faces.

Prerequisite Extended Indicators

MAE 5.3.1.a—Identify three-dimensional models limited to cube, cylinder, and cone.

MAE 4.3.3.a—Identify the area of a rectangle by counting unit squares.

Key Terms

area, face, rectangle, rectangular prism, side, square unit, surface area, three-dimensional shape

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-d-lesson-15

https://www.engageny.org/resource/grade-6-mathematics-module-5-topic-d-lesson-17

MA 6.3.3.c

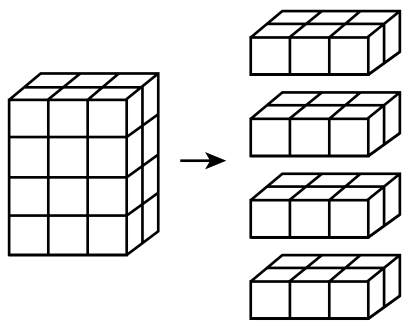
Apply volume formulas for rectangular prisms.

Extended: Find the volume of a rectangular prism using the volume formula.

Scaffolding Activities for the Extended Indicator

❑ **Find the volume of a rectangular prism by counting unit cubes.**

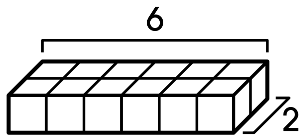
- Use a square or rectangular box to demonstrate that volume can be found by filling the box with unit cubes. Explain that volume is the amount of space inside an object. Stack unit cubes on top of one another and next to each other to fill the box. The number of unit cubes that fit into the box is the volume of the box.
- Use models of rectangular prisms built from unit cubes to show how to count individual unit cubes to find the volume. Model finding the volume of a rectangular prism by counting the unit cubes in one layer and then skip counting to find the total. Explain that each layer has the same number of unit cubes.



- Ask students to determine the number of unit cubes in one layer of a rectangular prism.
- Ask students to determine the number of layers, or height, of a rectangular prism.
- Ask students to determine the total number of unit cubes, or volume, of a rectangular prism by counting the unit cubes. Encourage skip counting when appropriate.

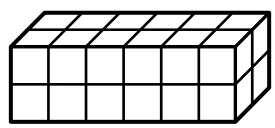
❑ **Find the volume of a rectangular prism using the volume formula.**

- Use unit cube models of rectangular prisms to point out that each layer is a rectangle. Demonstrate finding the area of a rectangle by multiplying the length and the width.



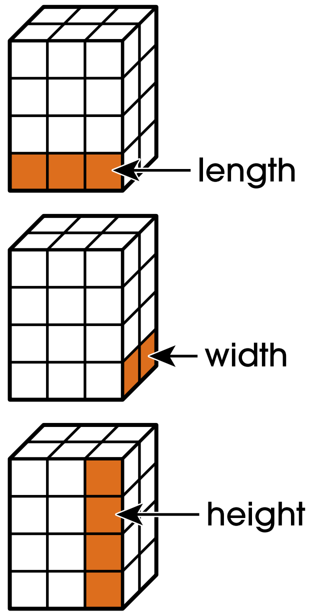
$$\begin{array}{r} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ +\ 2 \\ \hline 12 \end{array} \qquad 2 \times 6 = 12$$

Next, explain that all the layers of rectangular prisms have stacked identical rectangular prisms. Model how the volume of a rectangular prism can be found by adding the area of the different rectangle layers that make up the prism. For example, a 2 × 2 × 6 rectangular prism has two layers, each with an area of 2 × 6, or 12 square units. To find the volume, add 12 + 12. The volume is found by repeated addition of the layers. Therefore, the volume is 12 + 12, or 24 cubic units.



- Ask students to determine the area of a rectangular face (or one layer) of a rectangular prism.
- Ask students to determine the number of layers, or height, of the rectangular prism.
- Ask students to determine the volume of the rectangular prism by using repeated addition of the area of one of the layers.

- Use unit cube models of rectangular prisms to demonstrate finding the volume of a rectangular prism using the formula length × width × height. Model counting unit cubes to find the length, the width, and the height. Colored tape can be used to identify each measurement on a rectangular prism.



  Model appropriate computation strategies including the use of a calculator. It may be helpful to check calculated answers by counting individual unit cubes after a rectangular prism has been deconstructed.

- Ask students to determine the length, width, and height of a rectangular prism by counting unit cubes.

- Ask students to determine the volume of a rectangular prism using the volume formula.

Prerequisite Extended Indicators

MAE 6.3.3.a—Find the area of a rectangle using its whole number side lengths.

MAE 5.3.3.b—Find the volume of a rectangular prism by counting unit cubes.

MAE 5.1.2.a—Multiply a two-digit number by a single-digit number.

Key Terms

cubic unit, face, height, layer, length, multiply, rectangle, rectangular prism, volume, width

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-5/file/117906

Mathematics—Grade 6
MA 6.4 Data

MA 6.4.2 Analysis & Applications

MA 6.4.2.a

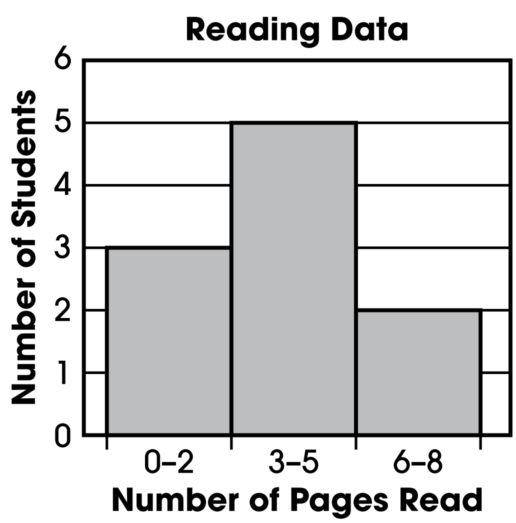
Solve problems using information presented in line plots, dot plots, box plots, and histograms.

Extended: Interpret a histogram that matches a data set.

Scaffolding Activities for the Extended Indicator

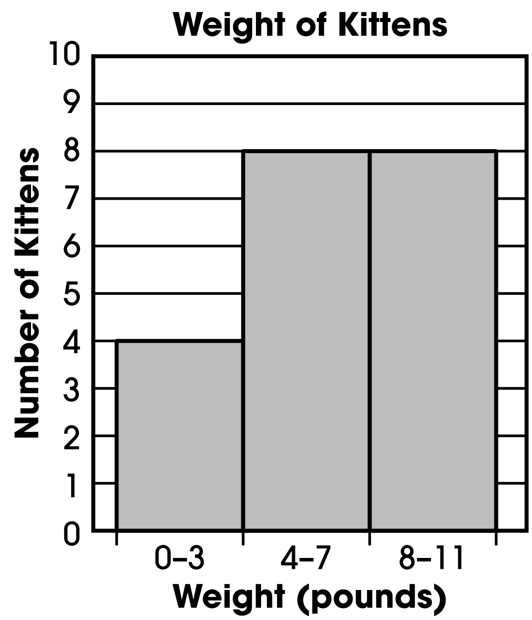
❑ **Interpret information from a histogram.**

- Explain that a histogram is a chart that shows data grouped by intervals. Intervals include all the values between two numbers. Demonstrate an interval on a number line. For example, 0–2 is an interval that includes the numbers 0, 1, and 2. The interval 3–5 includes the numbers 3, 4, and 5.
- Present a histogram and show that the intervals are found on the *x*-axis. Explain that reading the labels on both axes will show what the intervals represent. Identify the intervals and describe them in the context of the labeling. For example, "This histogram has intervals of 0–2, 3–5, and 6–8. The label on the *x*-axis is 'number of pages read' and the label on the *y*-axis is 'number of students.' The interval 0–2 indicates how many students read 0, 1, or 2 pages. The interval 3–5 indicates how many students read 3, 4, or 5 pages."



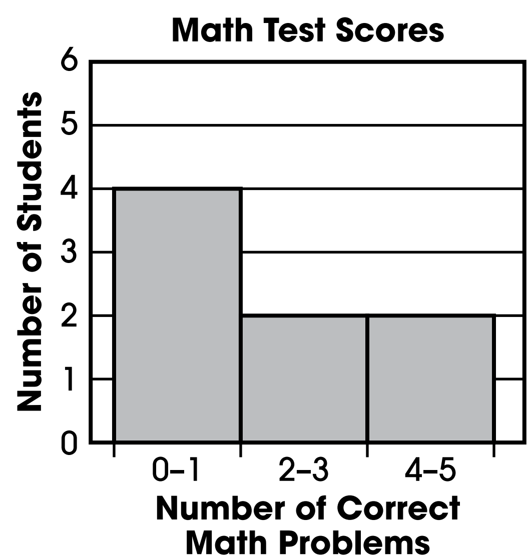
- Ask students to identify the intervals and axis labels on a histogram.

- Ask students to answer questions about the intervals of a histogram within the context of the labeling. For example, present the histogram shown and ask the following questions: “Which interval would data be added to if a kitten weighs 3 pounds?” and “Will this histogram tell us the weight of a dog?”



❑ **Interpret a histogram that matches a data set.**

- Demonstrate how to determine whether a histogram matches a data set. Present a histogram as shown.



Present a table that matches the data in the histogram. Demonstrate matching the values in the table to the values on the histogram. Progress to presenting two tables (one table that correctly matches the histogram and one table that does not match the histogram) and asking a series of questions to determine which table matches the histogram. Repeat the process with a choice of three tables.

Correct

Number of Correct Math Problems	Number of Students
0–1	4
2–3	2
4–5	2

Not Correct

Number of Correct Math Problems	Number of Students
0–1	1
2–3	2
4–5	3

Not Correct

Number of Correct Math Problems	Number of Students
0–1	4
2–3	4
4–5	4

- Given a histogram and a set of three tables with the same numbers in all cells but different column headings (one table with column headings that match the histogram labels), ask students to identify which table matches the given histogram.
- Given a histogram and a set of three tables with column headings that match the histogram labels but numbers that are different from table to table (one table with numbers that match the histogram), ask students to identify which table matches the given histogram.

Prerequisite Extended Indicators

MAE 5.4.2.a—Interpret information in a bar graph using at least two data points.

MAE 3.4.2.a—Solve a problem using a bar graph or a pictograph.

MAE 3.4.1.a—Identify a characteristic of a bar graph or a pictograph. (e.g., quantities, comparisons).

Key Terms

data set, interval, label, range, title, *x*-axis, *y*-axis

Additional Resources or Links

https://www.engageny.org/file/45621/download/math-g6-m6-topic-a-lesson-4-teacher.pdf?token=j114C_-P

https://curriculum.illustrativemathematics.org/MS/students/1/8/6/index.html

MA 6.4.2.b

Compare and interpret data sets based upon their graphical representations (e.g., center, spread, and shape).

Extended: Solve basic problems using histograms (e.g., How many times did Sara knock down 9 pins? How many more students have 1 pet than have 2 pets?).

Scaffolding Activities for the Extended Indicator

❑ **Identify the components of a histogram.**

- Present a histogram and a bar graph and explain the difference between the two graphic representations. Explain that a bar graph is a graph where the information on the *x*-axis is grouped by categories (e.g., names, colors, places, objects) and shows single values, while a histogram is a graph that shows data values grouped by intervals. An interval includes all the values between two numbers. Intervals can be modeled on a number line. For example, 0–2 is an interval that includes the numbers 0, 1, and 2. The interval 3–5 is an interval that includes the numbers 3, 4, and 5. Point to the intervals on a histogram to show that the intervals are found on the *x*-axis. Explain that for both a bar graph and a histogram, the information on the graph is interpreted by reading the title and the labels on the *x*- and *y*-axes.

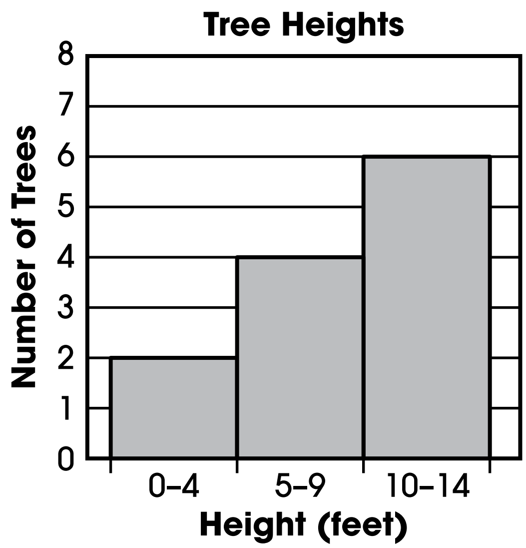
  Present the histogram shown. Identify the intervals on the histogram and describe them in the context of the labeling. For example, "This histogram has intervals of 0 to 1, 2 to 3, and 4 to 5. The label on the *x*-axis is 'Number of Goals Scored,' and the label on the *y*-axis is 'Number of Soccer Games.' So when the interval is 0 to 1, that shows how many games a soccer team scored 0 or 1 goals. When the interval is 2 to 3, that shows how many games the soccer team scored 2 or 3 goals."

- Ask students to identify on a given bar graph or histogram whether the data on the *x*-axis are categories or intervals.

- Ask students to answer questions about the labels and intervals on a histogram. For example, “What numerical values are included in the first interval?”

❑ **Solve basic problems using histograms.**

- Demonstrate solving problems based on data found in a histogram. Present the histogram shown. Model answering the following questions: “How many trees are between 5 and 9 feet tall?” and “Which interval has the least number of trees?”



- Ask students to answer questions based on intervals, such as “How many ______ are in the interval ______?” within context.
- Ask students to answer questions regarding which intervals or values are represented more/most or less/least.

Prerequisite Extended Indicators

MAE 6.4.2.a—Interpret a histogram that matches a data set.

MAE 5.4.2.b—Solve a problem with addition or subtraction of whole numbers using information from a bar graph.

MAE 5.4.2.a—Interpret information in a bar graph using at least two data points.

MAE 3.4.1.a—Identify a characteristic of a bar graph or a pictograph. (e.g., quantities, comparisons).

Key Terms

bar graph, category, histogram, interval, label, less, more, title, *x*-axis, *y*-axis

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-6-topic-lesson-4

https://curriculum.illustrativemathematics.org/MS/students/1/8/6/index.html

MA 6.4.2.c

Find and interpret the mean, median, mode, and range for a set of data.

Extended: Find the mode of a set of ordered whole number data.

Scaffolding Activities for the Extended Indicator

- **Identify the object that occurs the most in a set.**
  - Present a set of four objects in which three objects are the same and one object is different. For example, present three pencils and one stapler. Identify the pencil as the object that appears the most in this set of school supplies. Present two pencils, one marker, and one stapler, and again identify the pencil as the object that appears the most.

    Repeat the process with a drawing of one circle and four squares, and identify the square as the shape that appears the most. Present a drawing of one circle, one triangle, and two squares, and again identify the square as the shape that occurs the most. Continue to demonstrate finding the object that appears the most in a set with larger sets and a variety of objects and drawings.
  - Ask students to identify the object that appears the most in a set of four objects.
  - Ask students to identify the object that appears the most in a set of five or more objects.
- **Find the mode of a set of ordered whole number data.**
  - Describe the mode of a data set as the number that is listed the most often. Make connections between “mode” and “most.” Demonstrate finding the mode in a whole number ordered set of data when the frequency of the mode is much greater than the frequency of the other numbers in the data set. For example, present the set of numbers shown and identify the number 8 as the mode.

    {8, 8, 8, 8, 14}

    Continue to demonstrate finding the mode when the frequency of the mode is closer to the frequency of the other numbers in the data set. For example, present the data set {12, 14, 17, 17, 17} and then present the data sets {15, 19, 20, 20, 23} and {3, 3, 7, 7, 7, 12, 15}.
  - Ask students to identify the mode of a set of four numbers when three of the numbers are the same.
  - Ask students to identify the mode of a set of ordered whole number data.

Prerequisite Extended Indicators

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

MAE 3.1.1.a—Read, write, and demonstrate whole numbers up to 20 that are equivalent representations including visual models, standard form, and word form.

Key Terms

data, mode, most, set

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/pick%20a%20pocket.pdf

https://www.insidemathematics.org/sites/default/files/materials/through%20the%20grapevine.pdf

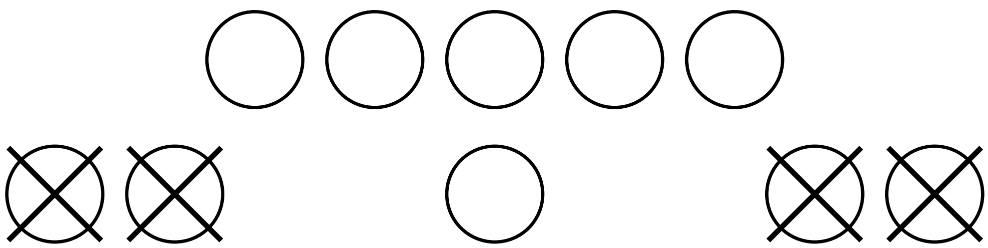
MA 6.4.2.d

Compare the mean, median, mode, and range from two sets of data.

Extended: Find the median of a set of ordered whole number data.

Scaffolding Activities for the Extended Indicator

- ❑ **Locate the middle object in a set of objects aligned horizontally.**
  - Identify the middle of a group of objects. Present three objects and indicate the object in the middle. Continue to demonstrate identifying the middle object in a set of five objects and a set of seven objects. Explain that when there are more objects, a strategy can be used to locate the middle.
  - Demonstrate finding the middle circle in a drawing of five circles aligned horizontally. For example, draw five circles as shown and cross out the circles one at a time from each end to find the middle circle of the set. Repeat the process with drawings of a set of seven circles and a set of nine circles.



  - Ask students to locate the middle object in groups of three, five, seven, and nine objects aligned horizontally.
  - Ask students to locate the middle shape in drawings of three, five, seven, and nine shapes aligned horizontally.

- ❑ **Find the median of a set of ordered whole number data.**
  - Describe the median of a data set as the number in the middle of the set when the numbers are ordered from least to greatest. Present the numbers 6, 7, and 8, and identify 7 as the median or the number in the middle. Present the numbers 5, 6, 7, 8, and 9, and explain that 7 is also the median in this set of numbers.

    Continue to demonstrate finding the median by presenting larger sets of ordered data. For example, in the set below, draw a line through the four numbers on each side of 18 to show 18 as the median of the set of data.

    6, 9, 13, 16, 18, 20, 21, 25, 28
    ~~6, 9, 13, 16~~, 18, ~~20, 21, 25, 28~~

Another strategy is to use two pieces of paper to cover one number at a time from the left and one number at a time from the right of the data set until only the number 18 remains in the middle.

6	9	13	16	18	20	21	25	28

Present the data set 14, 15, 19, 24, and 25. Demonstrate crossing one number off at a time from the left and from the right until only 19 remains. Explain that the strategy of crossing off numbers can be used to find the median of a set of numbers that are ordered from least to greatest. Continue to model with ordered data sets of three, five, seven, and nine numbers.

- Ask students to identify the median in an ordered data set of three numbers.
- Ask students to identify the median in an ordered data set of five, seven, or more numbers.

Prerequisite Extended Indicators

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

MAE 3.1.1.a—Read, write, and demonstrate whole numbers up to 20 that are equivalent representations including visual models, standard form, and word form.

Key Terms

data, median, middle, set

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/pick%20a%20pocket.pdf

https://www.insidemathematics.org/sites/default/files/materials/through%20the%20grapevine.pdf

Alternate Mathematics
Instructional Supports
for
NSCAS Mathematics Extended Indicators
Grade 6