NEBRASKA

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 5

for
Students with the Most Significant Cognitive Disabilities
who take the
Statewide Mathematics Alternate Assessment

NEBRASKA DEPARTMENT OF EDUCATION

Table of Contents

Overview

Introduction

Mathematics standards apply to all students, regardless of age, gender, cultural or ethnic background, disabilities, aspirations, or interest and motivation in mathematics (NRC, 1996).

The mathematics standards, extended indicators, and instructional supports in this document were developed by Nebraska educators to facilitate and support mathematics instruction for students with the most significant intellectual disabilities. They are directly aligned to the Nebraska's College and Career Ready Standards for Mathematics adopted by the Nebraska State Board of Education.

The instructional supports included here are sample tasks that are available to be used by educators in classrooms to help instruct students with significant intellectual disabilities.

The Role of Extended Indicators

For students with the most significant intellectual disabilities, achieving grade-level standards is not the same as meeting grade-level expectations, because the instructional program for these students addresses extended indicators.

It is important for teachers of students with the most significant intellectual disabilities to recognize that extended indicators are not meant to be viewed as sufficient skills or understandings. Extended indicators must be viewed only as access or entry points to the grade-level standards. The extended indicators in this document are not intended as the end goal but as a starting place for moving students forward to conventional reading and writing. Lists following "e.g." in the extended indicators are provided only as possible examples.

Students with the Most Significant Intellectual Disabilities

In the United States, approximately 1% of school-aged children have an intellectual disability that is "characterized by significant impairments both in intellectual and adaptive functioning as expressed in conceptual, social, and practical adaptive domains" (U.S. Department of Education, 2002 and American Association of Intellectual and Developmental Disabilities, 2013). These students show evidence of cognitive functioning in the range of severe to profound and need extensive or pervasive support. Students need intensive instruction and/or supports to acquire, maintain, and generalize academic and life skills in order to actively participate in school, work, home, or community. In addition to significant intellectual disabilities, students may have accompanying communication, motor, sensory, or other impairments.

Alternate Assessment Determination Guidelines

The student taking a Statewide Alternate Assessment is characterized by significant impairments both in intellectual and adaptive functioning which is expressed in conceptual, social, and practical adaptive domains and that originates before age 18 (American Association of Intellectual and Developmental Disabilities, 2013). It is important to recognize the huge disparity of skills possessed by students taking an alternate assessment and to consider the uniqueness of each child.

Thus, the IEP team must consider all of the following guidelines when determining the appropriateness of a curriculum based on Extended Indicators and the use of the Statewide Alternate Assessment.

- The student requires extensive, pervasive, and frequent supports in order to acquire, maintain, and demonstrate performance of knowledge and skills.
- The student's cognitive functioning is significantly below age expectations and has an impact on the student's ability to function in multiple environments (school, home, and community).
- The student's demonstrated cognitive ability and adaptive functioning prevent completion of the general academic curriculum, even with appropriately designed and implemented modifications and accommodations.
- The student's curriculum and instruction is aligned to the Nebraska College and Career Ready Mathematics Standards with Extended Indicators.
- The student may have accompanying communication, motor, sensory, or other impairments.

The Nebraska Department of Education's technical assistance documents "***IEP Team Decision Making Guidelines—Statewide Assessment for Students with Disabilities***" and "***Alternate Assessment Criteria/Checklist***" provide additional information on selecting appropriate statewide assessments for students with disabilities. School Age Statewide Assessment Tests for Students with Disabilities—Nebraska Department of Education.

Instructional Supports Overview

The mathematics instructional supports are scaffolded activities available for use by educators who are instructing students with significant intellectual disabilities. The instructional supports are aligned to the extended indicators in grades three through eight and in high school. Each instructional support includes the following components:

- Scaffolded activities for the extended indicator
- Prerequisite extended indicators
- Key terms
- Additional resources or links

The scaffolded activities provide guidance and suggestions designed to support instruction with curricular materials that are already in use. They are not complete lesson plans. The examples and activities presented are ready to be used with students. However, teachers will need to supplement these activities with additional approved curricular materials. The scaffolded activities adhere to research that supports instructional strategies for mathematics intervention, including explicit instruction, guided practice, student explanations or demonstrations, visual and concrete models, and repeated, meaningful practice.

Each scaffolded activity begins with a learning goal, followed by instructional suggestions that are indicated with the inner level, circle bullets. The learning goals progress from less complex to more complex. The first learning goal is aligned with the extended indicator but is at a lower achievement level than the extended indicator. The subsequent learning goals progress in complexity to the last learning goal, which is at the achievement level of the extended indicator.

The inner level, bulleted statements provide instructional suggestions in a gradual release model. The first one or two bullets provide suggestions for explicit, direct instruction from the teacher. From the teacher's perspective, these first suggestions are examples of “I do.” The subsequent bullets are suggestions for how to engage students in guided practice, explanations, or demonstrations with visual or concrete models, and repeated, meaningful practice. These suggestions start with “Ask students to . . .” and are examples of moving from “I do” activities to “we do” and “you do” activities. Visual and concrete models are incorporated whenever possible throughout all activities to demonstrate concepts and provide models that students can use to support their own explanations or demonstrations.

The prerequisite extended indicators are provided to highlight conceptual threads throughout the extended indicators and show how prior learning is connected to new learning. In many cases, prerequisites span multiple grade levels and are a useful resource if further scaffolding is needed.

Key terms may be selected and used by educators to guide vocabulary instruction based on what is appropriate for each individual student. The list of key terms is a suggestion and is not intended to be an all-inclusive list.

Additional links from web-based resources are provided to further support student learning. The resources were selected from organizations that are research based and do not require fees or registrations. The resources are aligned to the extended indicators, but they are written at achievement levels designed for general education students. The activities presented will need to be adapted for use with students with significant intellectual disabilities.

Mathematics—Grade 5
MA 5.1 Number

MA 5.1.1 Numeric Relationships

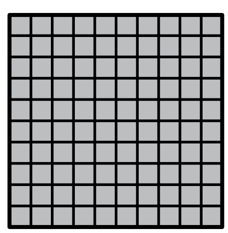
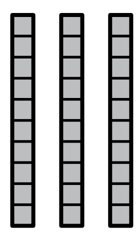
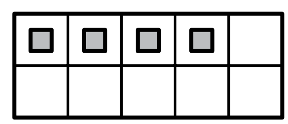
MA 5.1.1.a

Determine multiple equivalent representations for whole numbers and decimals through the thousandths place using standard form, word form, and expanded notation.

Extended: Identify representations of whole numbers up to 200.

Scaffolding Activities for the Extended Indicator

- ❑ **Use base-ten blocks and base-ten mats to represent whole numbers up to 200.**
  - Use base-ten blocks and a mat to demonstrate the connection between standard form and expanded notation. Indicate that the 1 in the hundreds column represents 1 hundred, the 3 in the tens column represents 3 tens, and the 4 in the ones column represents 4 ones.

Hundreds	Tens	Ones
		
1 hundred	3 tens	4 ones
134		

  - Ask students to use base-ten blocks and a mat to represent other whole numbers up to 200, such as 192 and 85.
  - Ask students to identify a whole number represented with base-ten blocks on the base-ten mat.

- ❑ **Represent a whole number up to 200 in expanded notation.**
  - Using a table such as the one shown, display the number 146. Decompose the whole number in the table to show the word form and expanded notation: (1 × 100) + (4 × 10) + (6 × 1).

146		
1 hundred	4 tens	6 ones
(1 × 100) 1 group of size 100	(4 × 10) 4 groups of size 10	(6 × 1) 6 groups of size 1

- Ask students to complete the middle row of missing information when given a whole number value up to 200 and the expanded notation.

124		
____ hundred	**____ tens**	**___ ones**
(1 × 100) 1 group of size 100	(2 × 10) 2 groups of size 10	(4 × 1) 4 groups of size 1

- Ask students to identify the missing values in expanded notation in the bottom row when given a whole number value up to 200 and the second row completed.

138		
1 hundred	3 tens	8 ones
(___ × 100) 1 group of size 100	**(___ × 10)** 3 groups of size 10	**(___ × 1)** 8 groups of size 1

Prerequisite Extended Indicators

MAE 4.1.1.a—Identify representations of numbers 0–100.

MAE 3.1.1.a—Read, write, and demonstrate whole numbers up to 20 that are equivalent representations including visual models, standard form, and word form.

Key Terms

expanded notation, standard form, word form

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/2/NBT/A/1/tasks/1236

https://www.engageny.org/resource/grade-4-mathematics-module-1-topic-lesson-4

MA 5.1.1.b

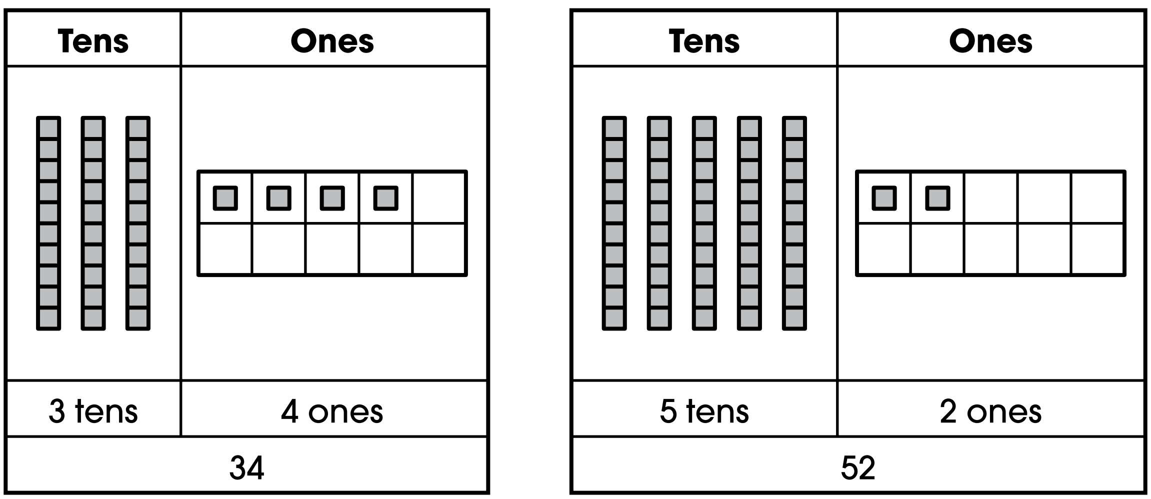
Compare whole numbers, fractions, mixed numbers, and decimals through the thousandths place and represent comparisons using symbols <, >, or =.

Extended: Compare and order whole numbers using symbols <, >, and = up to 200.

Scaffolding Activities for the Extended Indicator

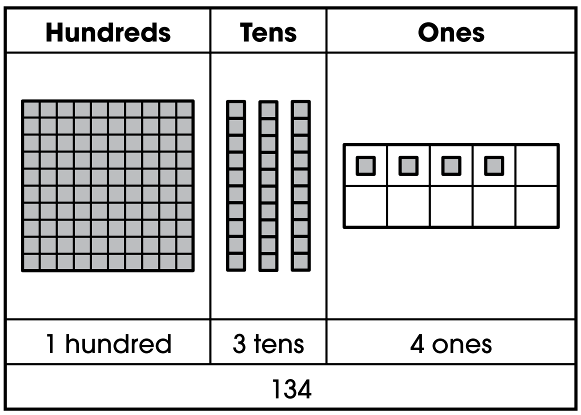
❑ **Compare whole numbers using symbols <, >, and = up to 200.**

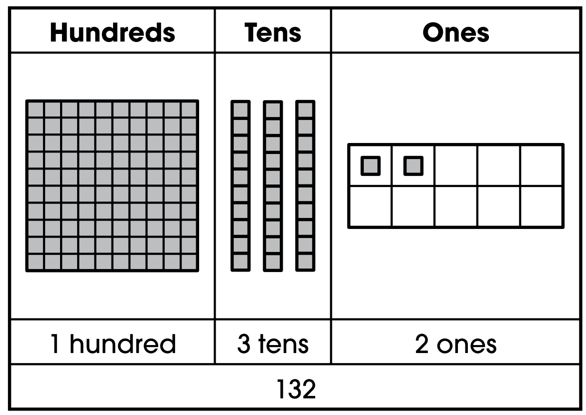
- Use base ten blocks and mats to demonstrate the connection between standard form and expanded notation. For example, present the number 34. Indicate that the 3 in the tens column represents 3 tens and the 4 in the ones column represents 4 ones. Then, present the number 52 and indicate that the 5 in the tens column represents 5 tens and the 2 in the ones column represents 2 ones. Explain that since 34 has fewer tens than 52, 34 is less than 52.



34 < 52

Similarly, present the number 134. Indicate that the 1 in the hundreds column represents 1 hundred, the 3 in the tens column represents 3 tens, and the 4 in the ones column represents 4 ones. Present the number 132 and indicate that the 1 in the hundreds column represents 1 hundred, the 3 in the tens column represents 3 tens, and the 2 in the ones column represents 2 ones. Explain that since both numbers have the same number of hundreds, 1 hundred, the next place to compare the values is the tens. Since both numbers have the same number of tens, 3 tens, the next place to compare the values is the ones. Compare the ones in each number; 4 ones is greater than 2 ones. Finally, explain that since 134 has the same hundreds, the same tens, but more ones than 132, 134 is greater than 132.



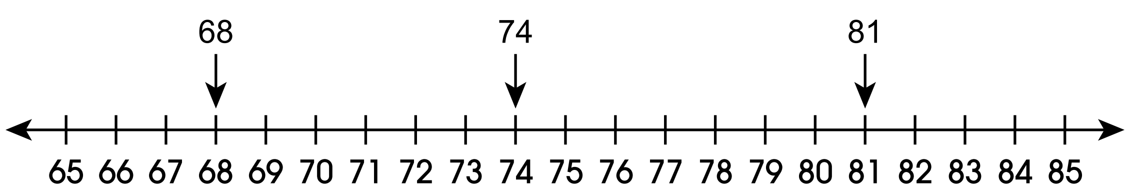


134 > 132

- Ask students to compare numbers using the symbols <, >, and = when given values represented with base ten blocks on mats.
- Ask students to represent two numbers on place value mats and then compare the numbers using the symbols <, >, or =.

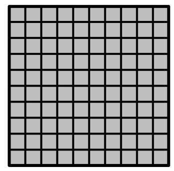
❑ **Order whole numbers up to 200.**

- Use a number line to indicate the location of the numbers 74, 68, and 81. Explain that the number with the least value is located farthest to the left on the number line (or closest to 0) and the number with the greatest value is located farthest to the right of the number line (or farthest from 0). Demonstrate writing the numbers in order from least to greatest by writing the numbers from left to right as they appear on the number line.



Numbers in order from least to greatest: 68, 74, 81

- Compare the place values of whole numbers to help order them. For example, write the numbers 184, 169, and 172 on a place value mat. Explain that, similar to comparing base ten blocks, each number can be compared by its place value.

Hundreds	Tens	Ones
		
1	8	4
1	6	9
1	7	2

The numbers in the hundreds places are all the same, 1. The numbers in the tens places are 8, 6, and 7. Since the tens place values are different, the numbers may be ordered without comparing the ones values. Therefore, the numbers in order are 169, 172, 184.

Continue to model comparing and ordering a variety of two- and three-digit whole number combinations. Be sure to use examples in which the digit that is different is represented in all three place value positions: hundreds, tens, and ones.

- Ask students to order three numbers up to 200 from least to greatest using number lines or place value mats.
- Ask students to order three numbers up to 200 from least to greatest without using number lines or other visual supports.

Prerequisite Extended Indicators

MAE 5.1.1.a—Identify representations of whole numbers up to 200.

MAE 4.1.1.f—Use symbols <, >, and = to compare whole numbers up to 40.

Key Terms

compare, greater than, greatest, least, less than, order, symbols

Additional Resources or Links

https://www.engageny.org/resource/grade-6-mathematics-module-3-topic-b-lesson-9

http://tasks.illustrativemathematics.org/content-standards/2/NBT/A/1

MA 5.1.1.c

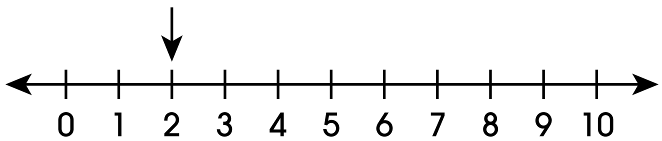
Round whole numbers and decimals to any given place.

Extended: Round whole numbers to the nearest tens place up to 200.

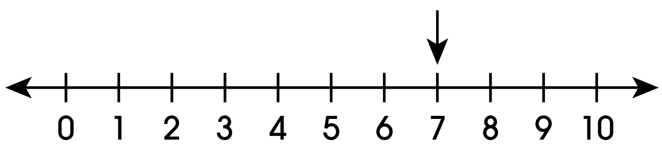
Scaffolding Activities for the Extended Indicator

❑ **Round a single-digit number to 0 or 10 using a number line.**

- Discuss the concept of rounding. Explain that sometimes an exact number is not needed, only an estimate or about how much is needed. Provide relevant real-world examples of using an estimate or approximate number.
- Use a number line to demonstrate that the location of a single-digit number on a number line can be closer to 0 or 10.

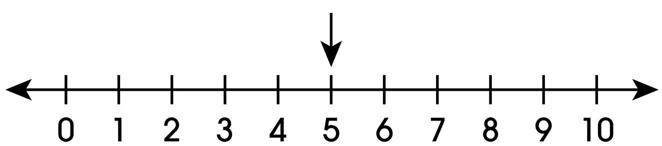


The number 2 is closer to 0 than 10, so it rounds to 0.



The number 7 is closer to 10 than 0, so it rounds to 10.

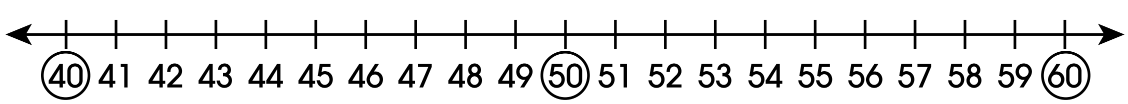
- Explain that the number 5 always rounds to 10, even though it is the same distance from 10 as it is from 0.



- Ask students to round a single-digit number to 0 or 10 when given a number on a number line.

❑ **Round a two- or three-digit number to the nearest ten using a number line.**

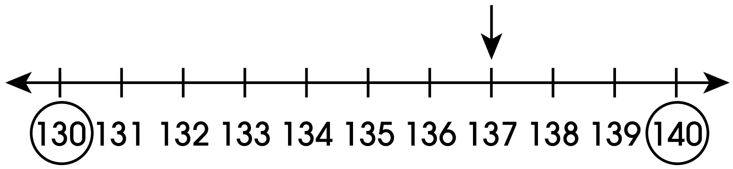
- Demonstrate how to find the multiples of 10 on a number line. Any two- or three-digit number that is a multiple of 10 will end with 0.



Relate skip counting by 10 to finding the multiples of 10 on a number line.

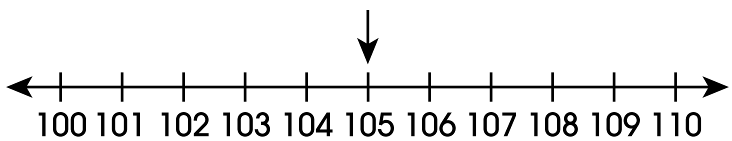
- Ask students to identify all the multiples of 10 when provided with a number line from 0 to 200 or sections of a number line from 0 to 200.

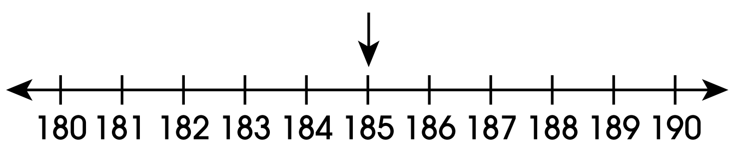
- Demonstrate finding the number 137 on a number line and then finding the multiples of 10 that 137 is located in between.



The number 137 is in between 130 and 140, so those numbers can be circled or highlighted.

- Ask students to find the multiples of 10 that are on either side of various two- or three-digit numbers when provided a number line from 0 to 200 or sections of a number line from 0 to 200.
- Demonstrate that once the two multiples of 10 on either side of the given number are found, the next step to rounding is to choose the multiple of 10 that is nearest to that number on the number line. From the example above, 137 is nearest to 140, so it rounds to 140.
- Explain that since 5 rounds to 10, any number ending in 5 will round to the multiple of 10 to the right of that number on a number line or the multiple of 10 that has a greater value. In other words, a number ending in 5 will always round to the multiple of 10 that is greater.





- Ask students to round various two- and three-digit numbers to the nearest ten using a number line from 0 to 200 or a section of a number line from 0 to 200. Be sure to include numbers with a 5 in the ones place and numbers that will round to 100 and 200, such as 95, 103, and 198.

Prerequisite Extended Indicators

MAE 4.1.1.g—Round a 2-digit number, 1–100, to the nearest ten using a number line.

MAE 3.1.1.c—Identify a number closer to a given number on a number line, 1–20.

Key Terms

estimate, greater, multiple, nearest, number line, round, skip counting, tens place

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/3/NBT/A/1/tasks/1805

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB2SUP-A4_NumPlVal-201304.pdf

MA 5.1.1.d

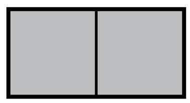
Recognize and generate equivalent forms of commonly used fractions, decimals, and percents (e.g., halves, thirds, fourths, fifths, and tenths).

Extended: Use models to identify equivalent fractions between thirds, fourths, halves, and one whole.

Scaffolding Activities for the Extended Indicator

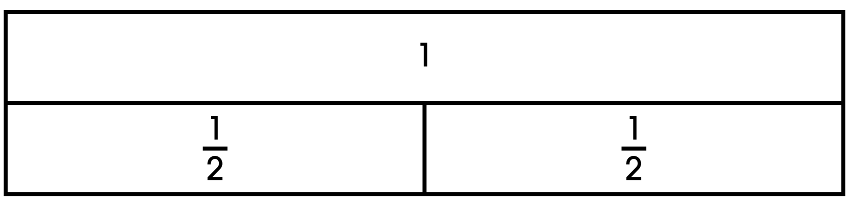
❑ **Identify when a fraction is equivalent to one whole.**

- Use models to show that two-halves is equivalent to one whole. For example, the following model of one whole rectangle is divided into halves that are shaded. Make note that shading “two-halves” results in the entire whole being shaded, so $\frac{2}{2} = 1$.



$\frac{2}{2}$

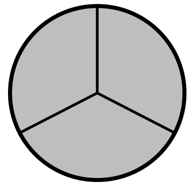
Show a variety of fraction models that represent that “two-halves” is equal to one whole. Another model is fraction strips, which use rectangles to compare fractional pieces to the whole. The following figure shows the whole, labeled with a 1, and then two halves that are the same size as the whole when put together.



- Ask students to select the model that represents one whole when given a model that represents $\frac{1}{2}$ and a model that represents $\frac{2}{2}$.

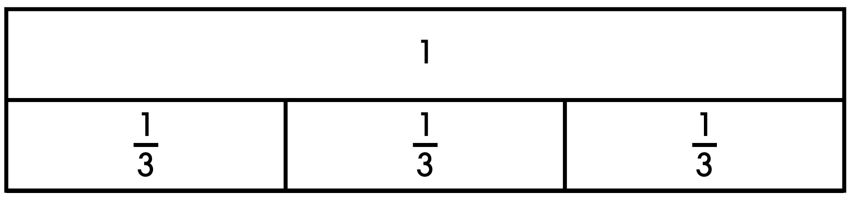
- Ask students to complete a model to represent one whole when given a model that represents $\frac{1}{2}$. For example, present a rectangle with one-half shaded and ask students to identify how to show one whole on the rectangle.

- Use models to show that three-thirds is equivalent to one whole. For example, the following circle model shows thirds, all shaded. Explain that “three-thirds” is the whole circle, so $\frac{3}{3}$ = 1.



$\frac{3}{3}$

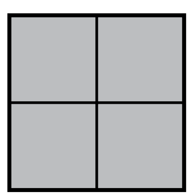
Fraction strips can be used to show the same result. This fraction strip is the same size as the fraction strip used above and can show that $\frac{3}{3}$ is also equivalent to $\frac{2}{2}$, since they are both equal to 1.



- Ask students to select the model that represents one whole when given a model that represents $\frac{1}{3}$ and a model that represents $\frac{3}{3}$.

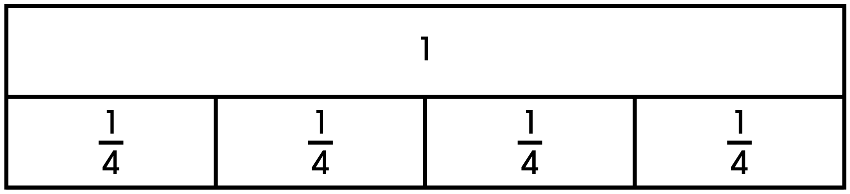
- Ask students to complete a model to represent one whole when given a model that represents $\frac{1}{3}$. For example, present the fraction strip model for $\frac{1}{3}$ and ask students to identify how to show one whole on the model.

- Use models to show that four-fourths is equivalent to one whole. For example, the following model of a large square is divided into fourths. Explain that when “four-fourths” is shaded, the model shows that $\frac{4}{4}$ = 1.



$\frac{4}{4}$

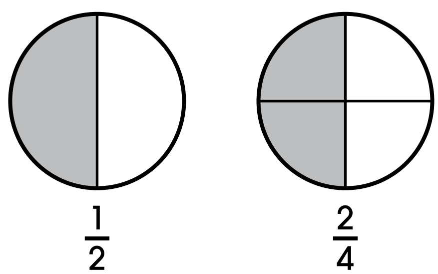
Fraction strips can be used to show the same result. Make note that since $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ are all equivalent to one whole, they are also all equivalent to each other.



- Ask students to select the model that represents one whole when given a model that represents $\frac{1}{4}$ and a model that represents $\frac{4}{4}$.
- Ask students to complete a model to represent one whole when given a model that represents $\frac{1}{4}$. For example, present the fraction strip model for $\frac{1}{4}$ and ask students to identify how to show one whole on the model.

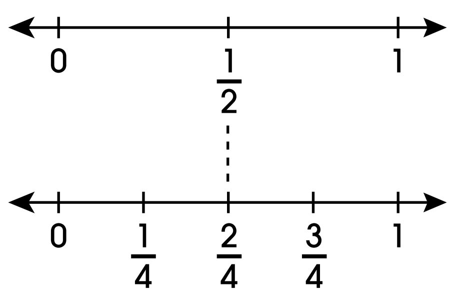
❑ **Identify that two-fourths is equivalent to one-half.**

- Use models to show that two-fourths is equivalent to one-half. For example, the figure shown uses a circle as the whole.



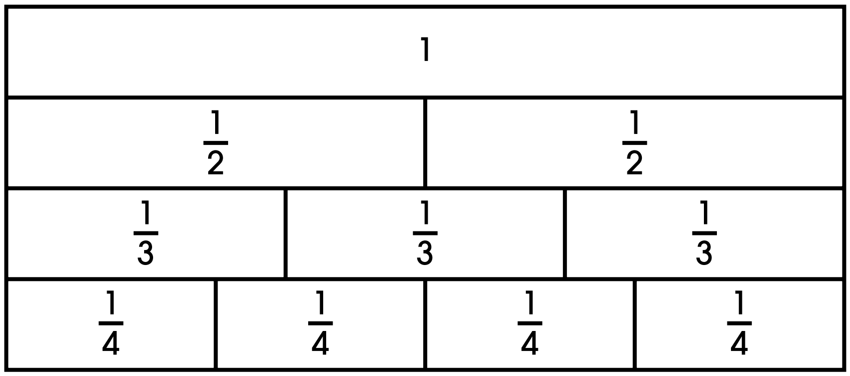
Explain that the circle on the left is divided into two equal parts, or halves, with one shaded, representing $\frac{1}{2}$. The circle on the right is divided into four equal parts, or fourths, with two shaded, representing $\frac{2}{4}$. Indicate that the shaded portion of each circle is the same, so $\frac{1}{2}$ is equal to $\frac{2}{4}$.

Another way to demonstrate that two-fourths is equivalent to one-half is with number lines from 0 to 1.



The number lines are aligned so that the 0 and the 1 line up. The dashed lined going through both $\frac{1}{2}$ and $\frac{2}{4}$ shows that both fractions are in the same location on the number line, so they must be equivalent.

Fraction strips can also be used to show that $\frac{1}{2}$ and $\frac{2}{4}$ are equal. The figure shown is the same fraction strip that has been used in the previous examples. Make note that two $\frac{1}{4}$ strips take up the same amount of space as one $\frac{1}{2}$ strip. This figure is also a helpful way to note that $\frac{2}{2} = \frac{3}{3} = \frac{4}{4} = 1$.



- Ask students to select the model that represents one-half when given a model that represents $\frac{2}{4}$ and a model that represents $\frac{4}{4}$.
- Ask students to identify how to show $\frac{1}{2}$ on a shape that has been divided into four equal sections.

Prerequisite Extended Indicators

MAE 3.1.1.i—Use a model to compare unit fractions one-half, one-third, and one-fourth.

MAE 3.1.1.e—Given a model, represent a whole number (1–3) as a fraction with a denominator of 2, 3, or 4.

Key Terms

equivalent, fourth, fraction, half, third, whole

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/3/NF/A/3/tasks/1502

http://tasks.illustrativemathematics.org/content-standards/3/NF/A/3/tasks/2108

https://www.insidemathematics.org/sites/default/files/assets/classroom-videos/formative-re-engaging-lessons/4th-grade-math-understanding-fractions/4th_grade_understanding_and_interpreting_fractions_lesson_plan.pdf

MA 5.1.2 Operations

MA 5.1.2.a

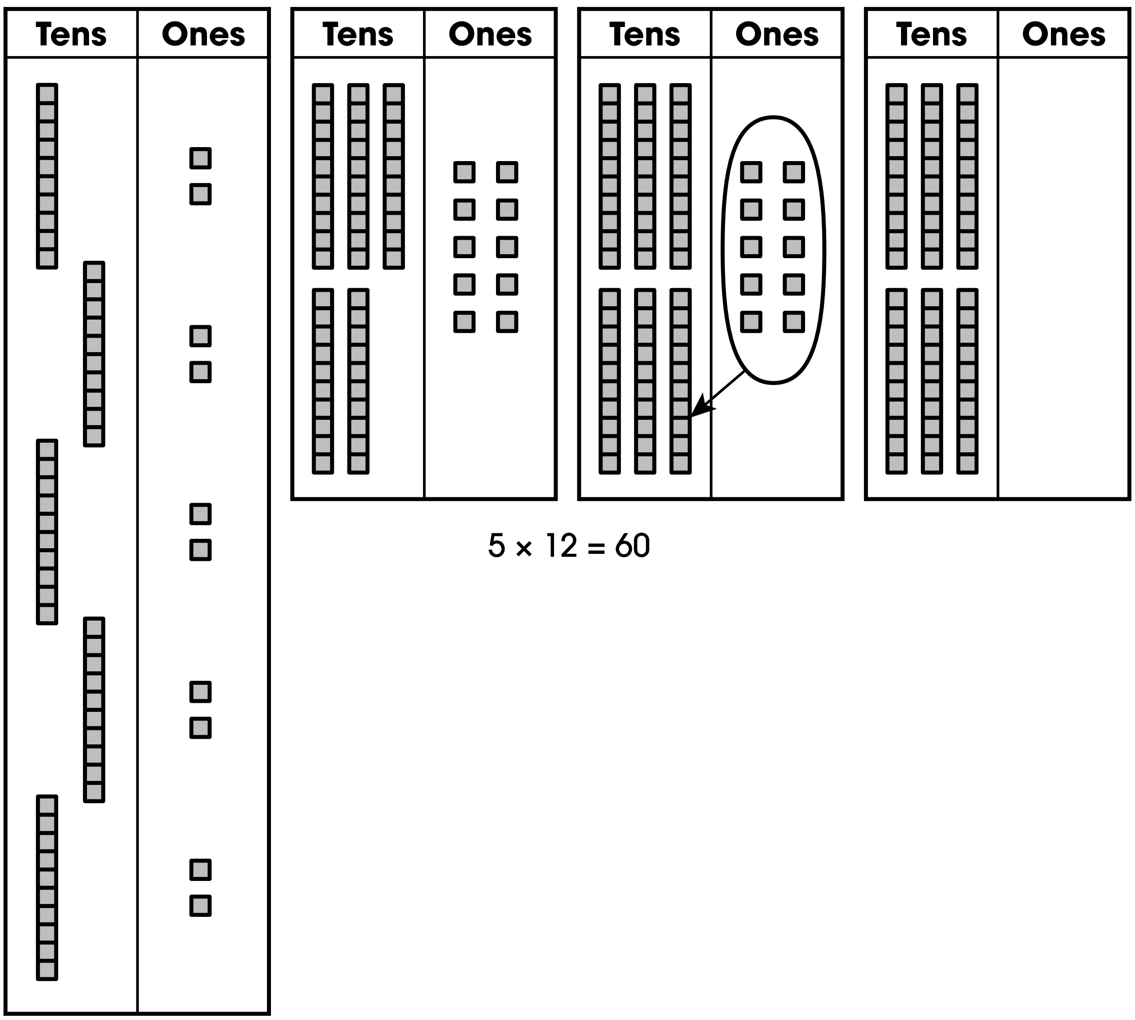
Multiply multi-digit whole numbers using the standard algorithm.

Extended: Multiply a two-digit number by a single-digit number.

Scaffolding Activities for the Extended Indicator

❑ **Multiply a two-digit number by a single-digit number with manipulatives.**

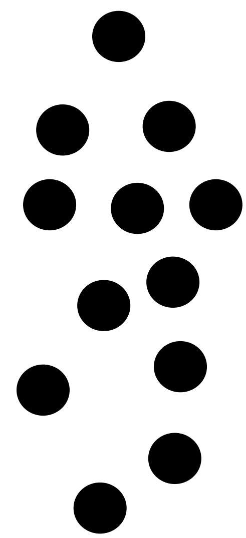
- Use a place value mat and base ten blocks to model 5 × 12. First, represent 5 groups of 12. Next, combine the tens rods and unit cubes. Model regrouping (or exchanging) 10 unit cubes for 1 tens rod. Find the product by skip counting to 10 (and adding ones when applicable) or placing the tens rods (and the unit cubes when applicable) on a hundreds chart.



- Continue modeling other multiplication problems (e.g., 11 × 5 and 12 × 3).
- Ask students to use a place value mat and base ten blocks to multiply a two-digit number by a single-digit number.

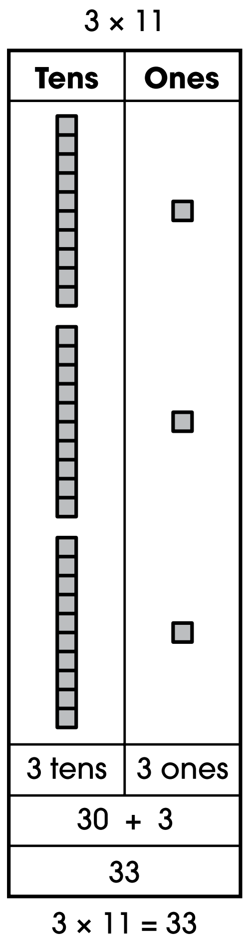
- Use a hundreds chart and tokens to model 15 × 4. Count out 15 tokens. Place the tokens on the hundreds chart. Highlight the number 15. Continue the same process three more times. Indicate the connection between 4 highlighted squares on the hundreds chart and counting to 15 a total of 4 times to find the answer: 15 × 4 = 60.

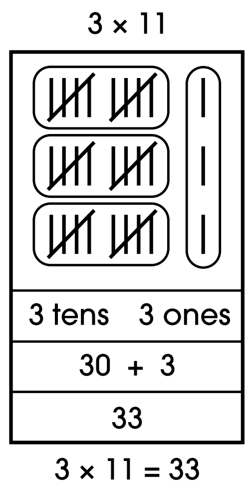
●	●	●	4	5	6	7	8	9	10
11	12	13	14	**15**	16	17	18	19	20
21	22	23	24	25	26	27	28	29	**30**
31	32	33	34	35	36	37	38	39	40
41	42	43	44	**45**	46	47	48	49	50
51	52	53	54	55	56	57	58	59	**60**
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



- Continue modeling other multiplication problems (e.g., 13 × 3 and 17 × 2).
- Ask students to use a hundreds chart and tokens to multiply a two-digit number by a single-digit number.

- ❑ **Multiply a two-digit number by a single-digit number.**
  - Use drawings to represent 3 × 11. Use lines and dots to represent 3 groups of 11. Use tally marks to represent 3 groups of 11 and then group the tens and ones.





  - Ask students to use drawings to multiply a two-digit number by a single-digit number.

Prerequisite Extended Indicators

MAE 5.1.1.a—Identify representations of whole numbers up to 200.

MAE 4.1.2.c—Multiply two-digit multiples of 10 by 2 or 5.

MAE 4.1.2.b—Multiply 2s, 5s and 10s by a single-digit number.

Key Terms

ones, product, regrouping, tally, tens

Additional Resources or Links

https://www.engageny.org/resource/grade-3-mathematics-module-3-topic-f-lesson-19

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-b-lesson-5

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-c-lesson-7

MA 5.1.2.b

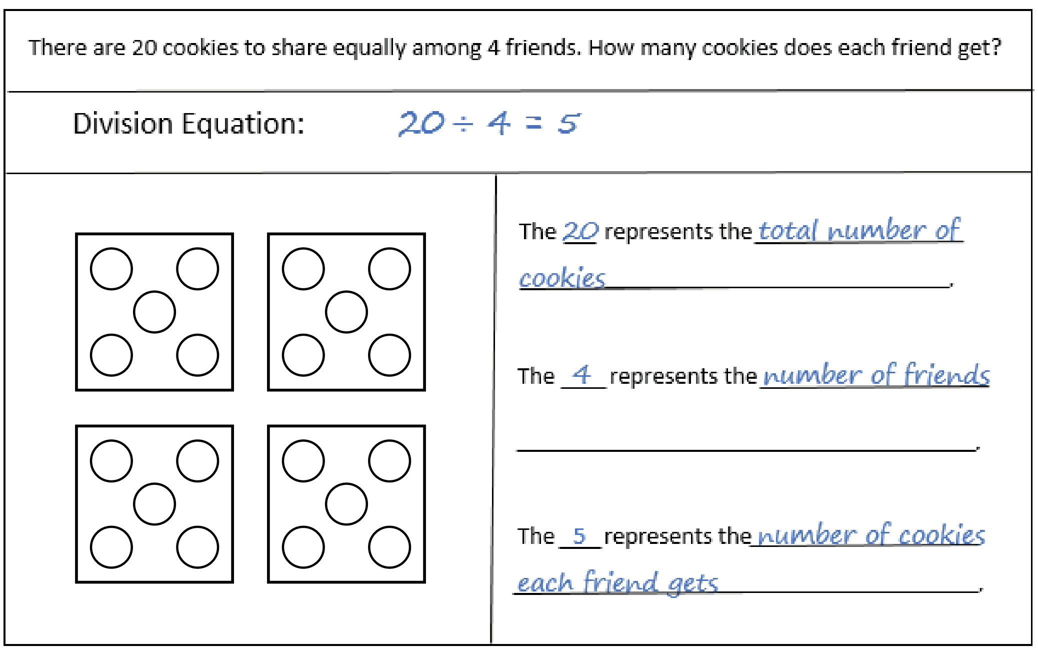
Divide four-digit whole numbers by a two-digit divisor, with and without remainders using the standard algorithm.

Extended: Divide a two-digit whole number by a single-digit number with no remainder.

Scaffolding Activities for the Extended Indicator

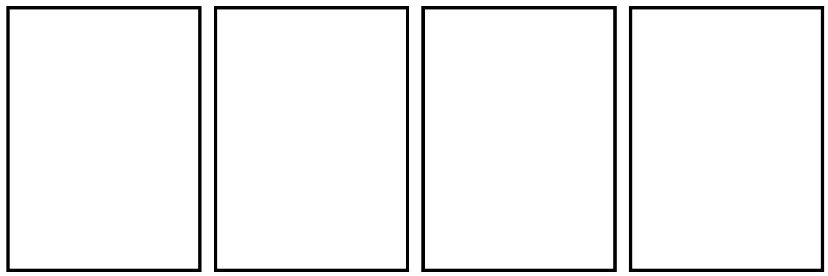
❑ **Determine the size of a group when the number of groups is known.**

- Use manipulatives to demonstrate partition division using fair sharing. Demonstrate solving the following problem: “There are 20 cookies to share equally among 4 friends. How many cookies does each friend get?” Count out 20 manipulatives, and explain that they represent the whole set of 20 cookies. Next, explain that because the cookies will be shared equally among four friends, there will be four groups with the same number of cookies in each group. Demonstrate dividing the manipulatives into four groups. Count to show that there are five manipulatives in each group, and explain that this means each friend will get five cookies. Write the division equation that models the problem, 20 ÷ 4 = 5. Emphasize how each number in the division problem is represented in the model. The template shown can be used to represent the division problem.



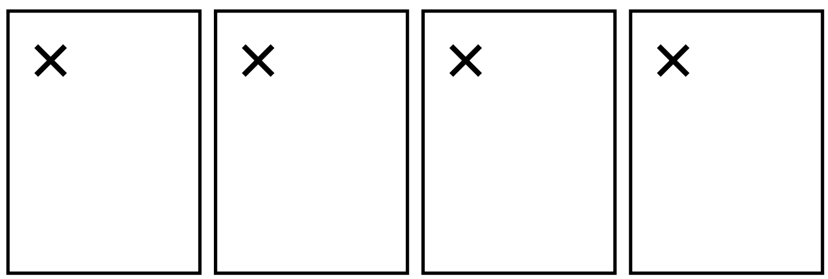
- Use a division mat to demonstrate partition division using fair sharing. Use the same problem: “There are 20 cookies to share equally among 4 friends. How many cookies does each friend get?” Draw four rectangles below the problem, and explain that each rectangle represents one person.

  $20 \div 4 =$

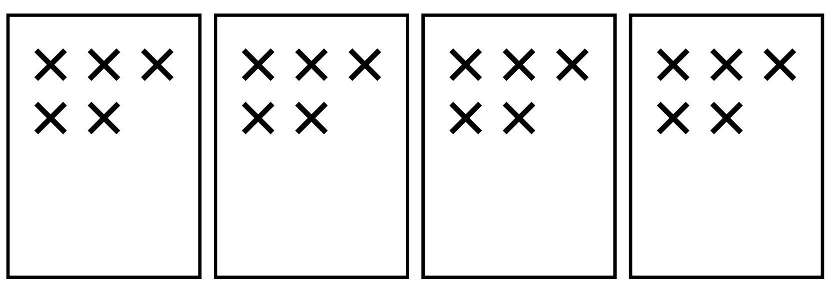
  Next, explain that each of the 20 cookies can be represented with an “x” as they are shared with the four friends. Mark one “x” in each rectangle.

  $20 \div 4 =$

  When each rectangle has one “x” marked, repeat, starting at the first rectangle and marking one more “x” in each rectangle. Continue marking “x’s” and counting the “x’s” as they are marked until all 20 cookies are represented. Write the answer to the problem, 5, and emphasize how each number in the division problem is represented in the model.

  $20 \div 4 =$

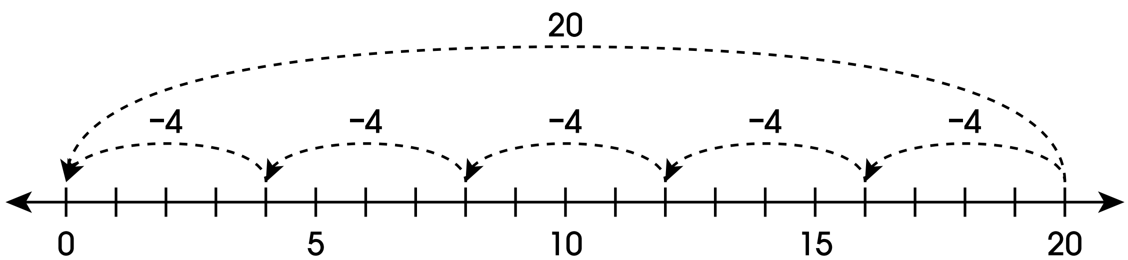
  Continue to demonstrate solving division problems with no remainder when the number of groups is given using a variety of numbers and a model.

- Ask students to use manipulatives to solve a division problem with no remainder when the number of groups is given. For example, present the following problem: “There are 15 apples placed equally into 3 baskets. How many apples are in each basket?”

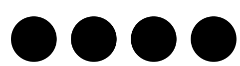
- Ask students to use a division mat or other drawing to solve a division problem with no remainder when the number of groups is given. For example, present the following problem: “There are 12 apples placed equally into 3 baskets. How many apples are in each basket?”

❑ **Determine the number of groups when the size of the group is known.**

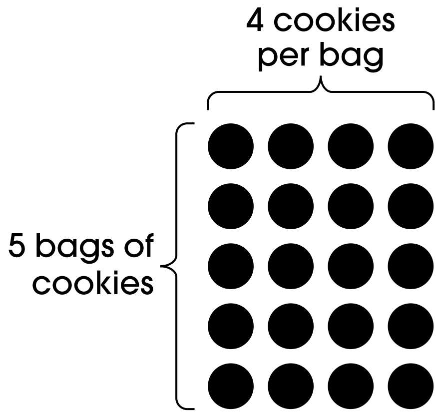
- Use a number line to demonstrate measurement division using repeated subtraction. Present the expression 20 ÷ 4 and the following problem: “There are 20 cookies to put into bags. There will be 4 cookies in each bag. How many bags can be filled with cookies?” Display a number line from 0 to 20. Point to the 20 and explain that 20 is the starting number because there are 20 cookies. Then “jump” four units to the left. Explain that one “jump” represents one bag and the four units represent the four cookies in each bag. “Jump” four more units to the left to represent another bag filled with four cookies. Continue “jumping” four units to the left until you reach 0. Count the five jumps to show that the 20 represents the total number of cookies, the four units within each “jump” represent the number of cookies in each bag, and the five “jumps” represent the number of bags that can be filled. Write the division equation that models the problem, 20 ÷ 4 = 5. Emphasize how each number in the division problem is represented in the model.



- Use an array to demonstrate measurement division. Present the expression 20 ÷ 4, and use the same problem: “There are 20 cookies to put into bags. There will be 4 cookies in each bag. How many bags can be filled with cookies?” Begin with a row of four dots, as shown, and explain that each row of four dots represents one bag filled with four cookies.



Continue adding dots to each row and counting the dots as they are drawn until all 20 dots have been added, representing the 20 cookies. The 20 dots are arranged in a five-by-four array. Show that the 20 dots represent the total number of cookies, the four columns represent the number of cookies in each bag, and the five rows represent the number of bags that can be filled. Write the division equation that models the problem, 20 ÷ 4 = 5. Emphasize how each number in the division problem is represented in the model.



Continue to demonstrate solving division problems with no remainder when the size of the group is given using a variety of numbers and a model.

- Ask students to use a number line to solve a division problem with no remainder when the size of the group is given. For example, present the following problem: “There are 18 apples placed equally into baskets. Each basket will have 6 apples. How many baskets can be filled with apples?”
- Ask students to use an array to solve a division problem with no remainder when the size of the group is given. For example, present the following problem: “There are 9 apples placed equally into baskets. Each basket will have 3 apples. How many baskets can be filled with apples?”

Prerequisite Extended Indicators

MAE 4.1.2.d—Identify numbers 2–20 in equal-size groups.

MAE 3.1.2.f—Count the number of twos in four, six, and eight and the number of threes in six and nine using a model.

Key Terms

array, divide, equal groups, factor, number line, quotient

Additional Resources or Links

https://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons/3rd-grade-math-interpreting-multiplication-and-division

https://www.engageny.org/resource/grade-4-math-represent-and-solve-division-problems-a-three-digit-dividend-4nbt6

MA 5.1.2.c

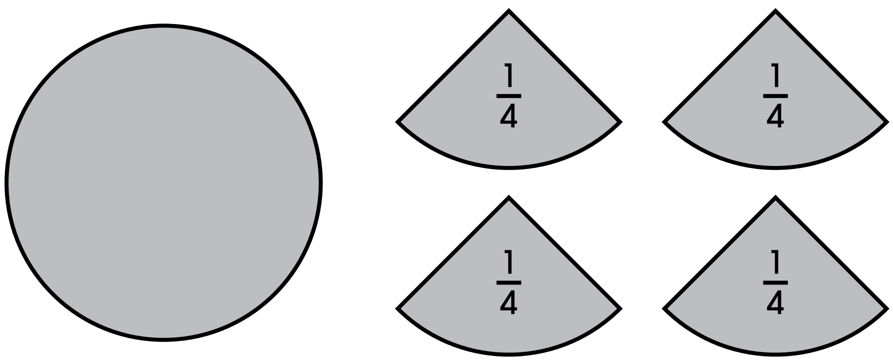
Multiply a whole number by a fraction or a fraction by a fraction using models and visual representations.

Extended: Multiply $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ by 2, 3, and 4.

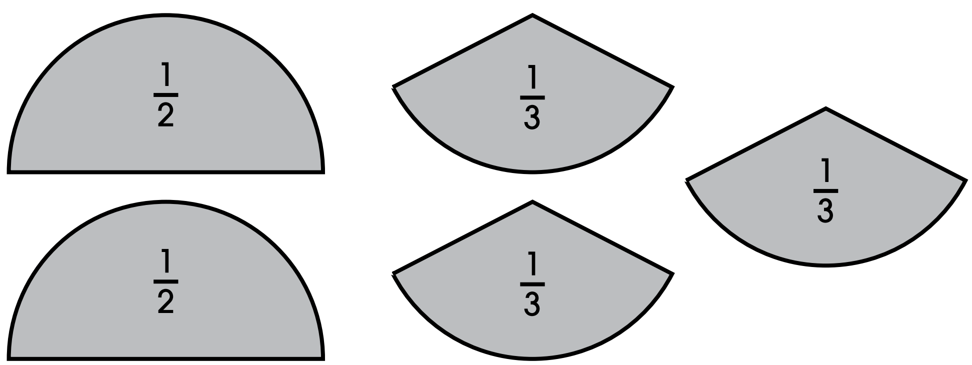
Scaffolding Activities for the Extended Indicator

❑ **Use a fraction model to multiply fractions with a product of 1 or less.**

- Use a fraction model that includes a template for 1 whole and 4 pieces that each represent $\frac{1}{4}$ to multiply fractions by 2, 3, and 4.



Place 2 of the $\frac{1}{4}$ pieces on the 1 whole to represent $2 \times \frac{1}{4}$. Then show that 3 of the $\frac{1}{4}$ pieces cover $\frac{3}{4}$ of the whole, which represents $3 \times \frac{1}{4}$. Note that it takes 4 of the $\frac{1}{4}$ pieces to cover the entire circle, so $4 \times \frac{1}{4}$ is the same as 1 whole. Follow this same process with pieces that are $\frac{1}{3}$ of the circle and $\frac{1}{2}$ of the circle.

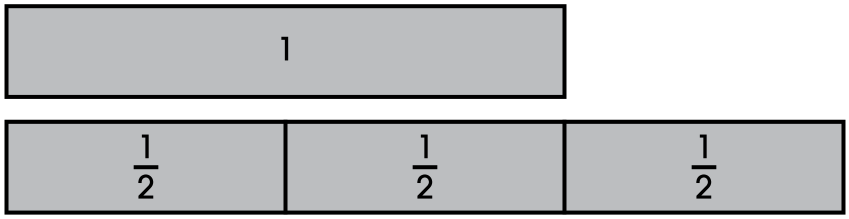


With each size fraction piece, connect the number of pieces to the appropriate multiplication sentence (e.g., $2 \times \frac{1}{2} = 1$, $2 \times \frac{1}{3} = \frac{2}{3}$ and $3 \times \frac{1}{3} = 1$).

- Ask students to multiply $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ by 2, 3, and 4 with products of 1 or less. Use manipulatives or visual representations of fraction models as needed.

MA 5.1.2 Operations

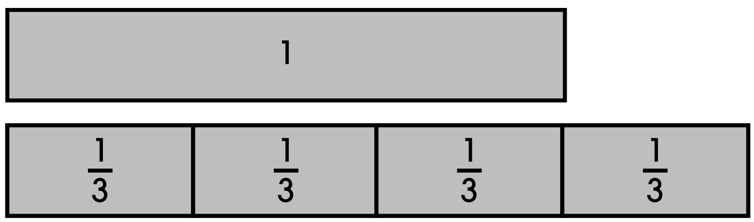
- [ ] **Use a fraction model to multiply fractions with a product greater than 1.**



- Show that $3 \times \frac{1}{2}$ is greater than 1. This product can be written as $\frac{3}{2}$ or $1\frac{1}{2}$. Write $3 \times \frac{1}{2} = \frac{3}{2}$ and $3 \times \frac{1}{2} = 1\frac{1}{2}$.

  Repeat the process to represent $4 \times \frac{1}{2}$, which is equal to $\frac{4}{2}$ or 2 wholes. Give students the opportunity to layer the fraction pieces on top of the whole and identify when there are more fractional parts than will fit on top of 1 whole, which represents a product greater than 1.

  Repeat this same process with fraction strips or other manipulatives that represent $\frac{1}{3}$.



  With each size fraction piece, connect the number of pieces to the appropriate multiplication sentence $4 \times \frac{1}{3} = \frac{4}{3}$ and $4 \times \frac{1}{3} = 1\frac{1}{3}$.

- Ask students to multiply $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ by 2, 3, and 4. Use manipulatives or visual representations of fraction models as needed.

Prerequisite Extended Indicators

MAE 4.1.2.f—Add and subtract halves to halves, thirds to thirds, fourths to fourths, and fifths to fifths . . . to a whole.

MAE 3.1.1.e—Given a model, represent a whole number (1–3) as a fraction with a denominator of 2, 3, or 4.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

Key Terms

fraction, multiplication, product, whole number

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/4/NF/B/4/tasks/2076

https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-g-overview/file/84606

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB4SUP-A10_NumMultWhlFrac-201309.pdf

MA 5.1.2.d

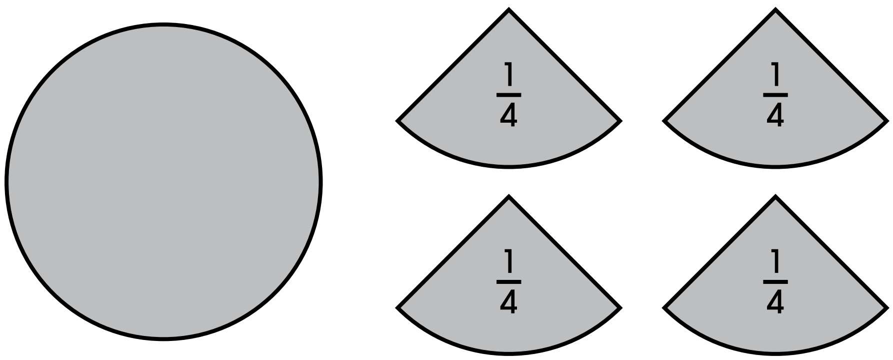
Divide a unit fraction by a whole number and a whole number by a unit fraction.

Extended: Divide a whole number by $\frac{1}{3}$, $\frac{1}{2}$, or $\frac{1}{4}$ using a visual model (e.g., 3 divided by one-half).

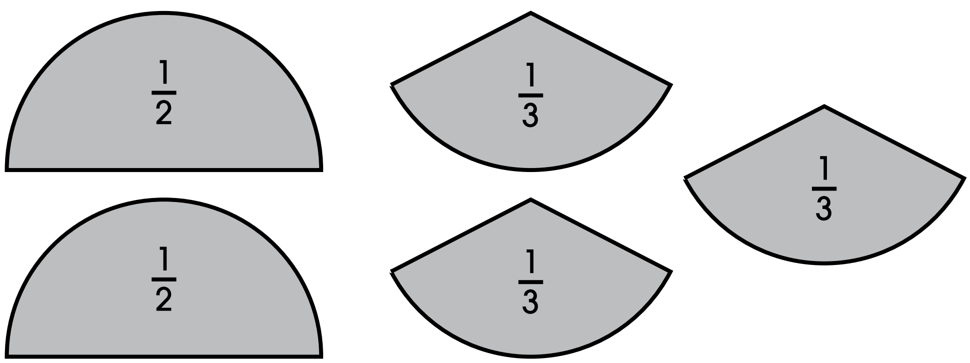
Scaffolding Activities for the Extended Indicator

- ❑ **Use a fraction model to divide 1 by $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.**

  - Use a fraction model that includes one full circle to represent one whole and four pieces to represent four quarters when dividing 1 by $\frac{1}{4}$. Place the four $\frac{1}{4}$ pieces on the one whole to represent $1 \div \frac{1}{4}$. Show that four $\frac{1}{4}$ pieces are needed to cover the one whole, so $1 \div \frac{1}{4} = 4$.



Follow this process with pieces that are $\frac{1}{2}$ of the circle and $\frac{1}{3}$ of the circle.



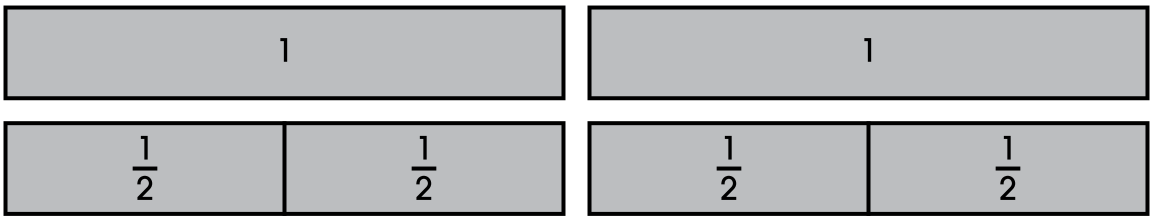
With each size fraction piece, connect the number of pieces to the appropriate division sentence (i.e., $1 \div \frac{1}{2} = 2$ and $1 \div \frac{1}{3} = 3$).

  - Ask students to identify how many pieces of fraction size $\frac{1}{4}$ it takes to cover a whole. Repeat for $\frac{1}{3}$ and $\frac{1}{2}$.

- Ask students to use manipulatives or other visual representations of fraction models (e.g., number lines, paper folding, fraction rods, fraction strips) to divide 1 by $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.

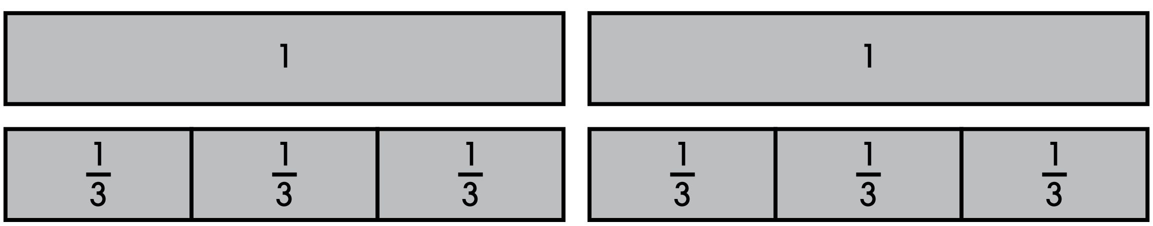
❑ **Use a fraction model to divide a whole number greater than 1 by $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.**

- Use fraction strips to demonstrate division of two wholes by $\frac{1}{2}$. Place two of the whole pieces above four of the $\frac{1}{2}$ size fraction pieces. Show students that four of the $\frac{1}{2}$ pieces are the same size as two of the whole pieces, so $2 \div \frac{1}{2} = 4$.



Repeat the process to represent $3 \div \frac{1}{2}$, which is equal to 6 because six $\frac{1}{2}$ pieces are the same size as three whole pieces. Give students the opportunity to layer the fraction pieces on top of the wholes and to identify how many fraction pieces are needed to create the given number of wholes.

Repeat this process with fraction strips or manipulatives of fraction size $\frac{1}{3}$. With each fraction piece, connect the number of pieces to the appropriate multiplication sentence (i.e., $2 \div \frac{1}{3} = 6$ and $3 \div \frac{1}{3} = 9$).



- Ask students how many pieces of fraction size $\frac{1}{4}$ are needed to cover two wholes. Repeat for $\frac{1}{3}$ and $\frac{1}{2}$.
- Ask students to use manipulatives or other visual representations of fraction models (e.g., number lines, paper folding, fraction rods, area models) to divide whole numbers greater than 1 by $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.

Prerequisite Extended Indicators

MAE 4.1.2.f—Add and subtract halves to halves, thirds to thirds, fourths to fourths, and fifths to fifths . . . to a whole.

MAE 3.1.1.i—Use a model to compare unit fractions one-half, one-third, and one-fourth.

MAE 3.1.1.e—Given a model, represent a whole number (1–3) as a fraction with a denominator of 2, 3, or 4.

Key Terms

division, fourths, fraction, halves, thirds, whole number

Additional Resources or Links

https://www.mathlearningcenter.org/sites/default/files/pdfs/SecB5SUP-A12_DivFracWhlNum-201304.pdf

http://tasks.illustrativemathematics.org/content-standards/5/NF/B/7

MA 5.1.2.h

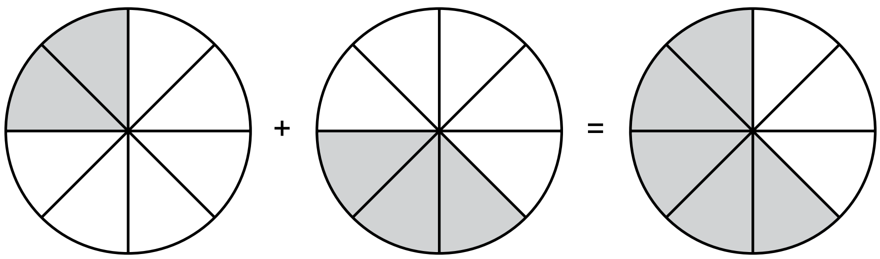
Add and subtract fractions and mixed numbers with unlike denominators.

Extended: Add and subtract fractions with like denominators using a visual model without regrouping.

Scaffolding Activities for the Extended Indicator

❑ **Add fractions with like denominators using a visual model without regrouping.**

- Use models to demonstrate adding fractions with like denominators. For example, add $\frac{2}{8} + \frac{3}{8}$ by using a circle to represent the whole, with each whole divided into 8 equal parts. The model shows $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$.

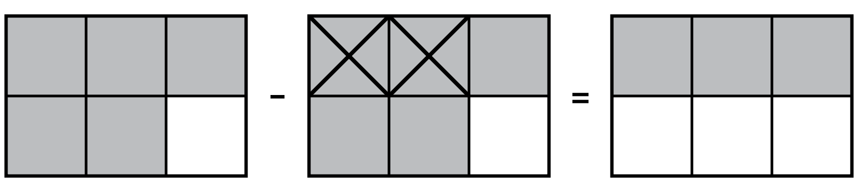


  Continue to demonstrate how to add fractions with like denominators using a variety of shapes and models. Emphasize that the denominator stays the same when adding fractions with like denominators and only the numerators are added.

- Ask students to identify the sum (the answer) when given a visual model that represents adding two fractions with like denominators.
- Ask students to add fractions with like denominators using a visual model without regrouping.

❑ **Subtract fractions with like denominators using a visual model without regrouping.**

- Use models to demonstrate subtracting fractions with like denominators. Present the model shown for $\frac{5}{6} - \frac{2}{6}$. The first rectangle shows 5 of the 6 equal parts shaded, and then 2 of those sixths are removed, leaving the answer of $\frac{3}{6}$. So $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$.



  Continue to demonstrate how to subtract fractions with like denominators using a variety of shapes and models. Emphasize that the denominator stays the same when subtracting fractions with like denominators and only the numerators are subtracted.

MA 5.1.2 Operations

- Ask students to identify the difference (the answer) when given a visual model that represents subtracting two fractions with like denominators.
- Ask students to subtract fractions with like denominators using a visual model without regrouping.

Prerequisite Extended Indicators

MAE 4.1.2.f—Add and subtract halves to halves, thirds to thirds, fourths to fourths, and fifths to fifths . . . to a whole.

MAE 3.1.2.a—Add and subtract, through 20 without regrouping.

Key Terms

add, denominator, difference, fraction, numerator, subtract, sum

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-5

https://www.insidemathematics.org/common-core-resources/3rd-grade

MA 5.1.2.j

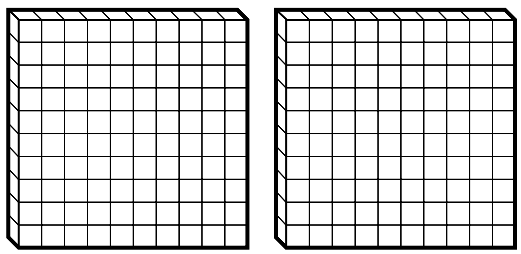
Multiply and divide by powers of 10.

Extended: Multiply a one-digit whole number by 100.

Scaffolding Activities for the Extended Indicator

❑ **Multiply a one-digit whole number by 100 using manipulatives.**

- Use base ten blocks or other manipulatives to demonstrate multiplying 100 by the numbers 1 through 9. For example, 2 × 100 can be modeled with two base ten “flats,” each representing the number 100. Demonstrate using skip counting or repeated addition strategies to determine that 2 × 100 = 200.



- Repeat the process and multiply 100 by all the numbers 1 through 9. Other ideas for manipulatives include tiles, counting sticks in bundles of 100, or cutout hundred grids.
- Ask students to select the manipulatives needed to show 1 × 100, 2 × 100, 3 × 100 . . . 9 × 100.
- Ask students to multiply a one-digit whole number by 100 using manipulatives.

❑ **Multiply a one-digit whole number by 100 using equations.**

- Use a table and number sentences to show math patterns for multiplying one-digit whole numbers by 100. For example, present the following table. Emphasize the pattern of the products. Each one-digit whole number is now in the hundreds place and there are zeros in the tens and ones places. Each row of the table can be made into an equation—for example, 8 × 100 = 800.

number sentence	product
1 × 100	100
2 × 100	200
3 × 100	300
4 × 100	400
5 × 100	500
6 × 100	600
7 × 100	700
8 × 100	800
9 × 100	900

- Use a template as shown to demonstrate that the one-digit whole number is in the hundreds place in the product. Create the template with empty boxes and then demonstrate multiplying one-digit whole numbers by 100 by writing or placing cards with the one-digit whole number in the empty boxes.

$$\begin{array}{r} 100 \\ \times \quad \square \\ \hline \square 00 \end{array} \qquad \begin{array}{r} 100 \\ \times \quad \boxed{4} \\ \hline \boxed{4}00 \end{array}$$

- Ask students to multiply a one-digit whole number by 100 using a table or a template to find the product.
- Ask students to identify the product of a one-digit whole number times 100 when given three choices. For example, present the multiplication problem 6 × 100 = _____ and the choices 6, 60, or 600. Students should determine the correct answer is 600.

Prerequisite Extended Indicators

MAE 4.1.2.b—Multiply 2s, 5s, and 10s by a single-digit number.

MAE 4.1.1.d—Count by twos and fives, and tens with numbers, models, or objects up to 40.

MAE 3.1.2.e—Multiply one and two by ten, twenty, and thirty up to 60.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

Key Terms

hundred, multiply, product

Additional Resources or Links

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-b-lesson-4

https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-b-lesson-5

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Mathematics—Grade 5
MA 5.2 Algebra

MA 5.2.1 Algebraic Relationships

MA 5.2.1.a

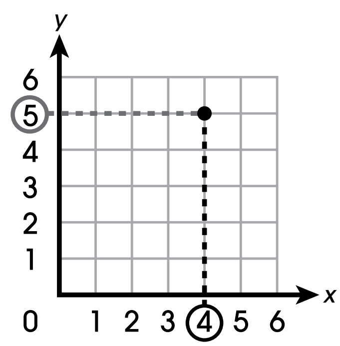
Form ordered pairs from a rule such as $y = 2x$, and graph the ordered pairs on a coordinate plane.

Extended: Identify the location of the ordered pairs on a coordinate plane (1st quadrant).

Scaffolding Activities for the Extended Indicator

❑ **Identify the *x*- and *y*-coordinates of a point on a coordinate plane.**

- Use a coordinate plane to demonstrate finding the *x*- and *y*-coordinates of a point. Color coordinating references to the *x*-axis and the *y*-axis can be helpful to some students. For example, trace vertical lines toward the *x*-axis in blue and circle *x*-coordinates in blue. Likewise, trace horizontal lines toward the *y*-axis in red and circle *y*-coordinates in red.



x-coordinate is 4

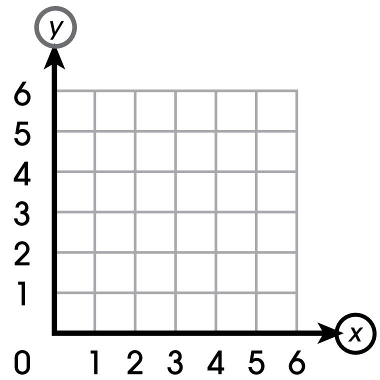
y-coordinate is 5

Explain that the *x*-coordinate is found using the numbers along the horizontal, or bottom in this case, axis. The *y*-coordinate is found using the numbers along the vertical, or left-side, axis. Also make note of the *x* and *y* labels on the ends of the axes, which serve as reminders about which axis is which. Repeat the process of finding the coordinates for a variety of points in a variety of locations. Keep the points at whole number locations and demonstrate identifying the locations by circling or highlighting the correct numbers on the axes. Avoid using the (*x*, *y*) notation at this level of instruction.

- Ask students to identify the *x*- and *y*-coordinates of a point on a coordinate plane (without expecting the use of (*x*, *y*) notation).

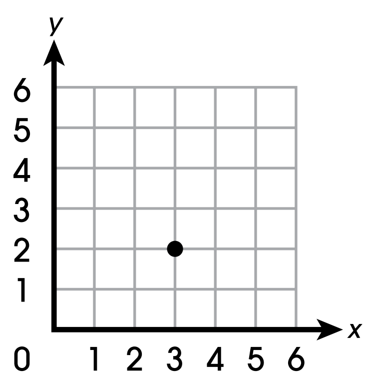
- **Identify the ordered pair of a point on a coordinate plane.**

    - Use ordered pair notation, (x, y), to show the x- and y-coordinates of a point on a coordinate plane.



(x, y)

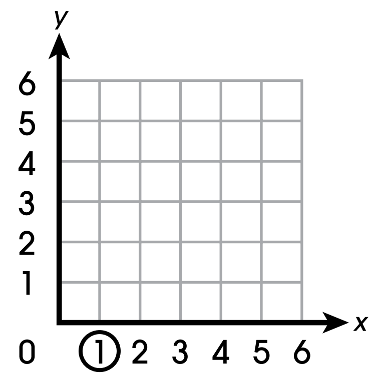
Be sure to point out that the axes have the labels x and y to remind students which number is the x-coordinate and which number is the y-coordinate. Ordered pairs are always in the form (x, y), so it is important to keep in mind which coordinate comes first.



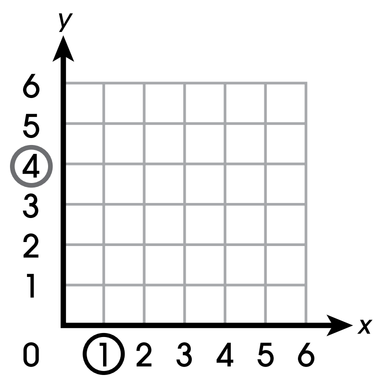
The point on the graph is located at the 3 on the x-axis and the 2 on the y-axis, so those are its coordinates. The ordered pair for the location of the point is (3, 2). Repeat this process with a variety of points in a variety of locations on the graph.

- Ask students to use ordered pairs to show the locations of points on a graph.

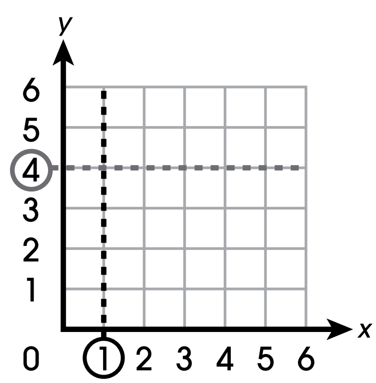
- ❑ **Identify the location of an ordered pair on a coordinate plane.**
  - Use an ordered pair to show students how to place a point in the correct location on a graph. For example, use the ordered pair (1, 4). Begin by locating the 1 on the *x*-axis.



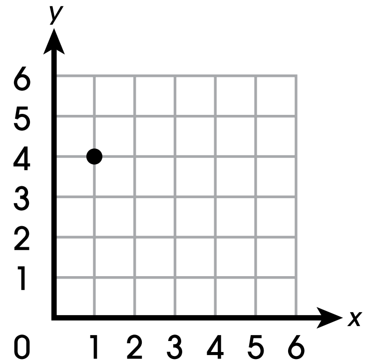
Then locate the 4 on the *y*-axis.



Follow the grid lines to find the location where they intersect.



A line drawn from both axes intersects at (1, 4), so that is where the point is placed.



Students commonly find the number on the axes and then place the point on the axis instead of following the grid lines to the appropriate point of intersection. Be sure to emphasize that the location of the point needs to align with **both** coordinates from the ordered pair.

- Ask students to find the location of an ordered pair on a coordinate plane.

Prerequisite Extended Indicators

MAE 5.3.2.b—Identify the *x*- or *y*-coordinate of whole-numbered points in quadrant I.

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

Key Terms

coordinate plane, horizontal, intersection, ordered pair, point, vertical, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/G/A/1/tasks/489

https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-overview

MA 5.2.2 Algebraic Processes

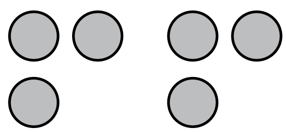
MA 5.2.2.a

Interpret and evaluate numerical or algebraic expressions using order of operations (excluding exponents).

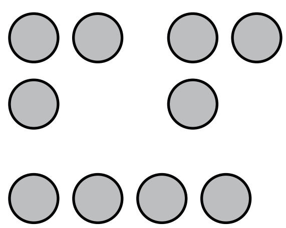
Extended: Evaluate a numerical expression with addition or subtraction and multiplication, 1–5.

Scaffolding Activities for the Extended Indicator

- ❑ **Evaluate numerical expressions with addition or subtraction and multiplication using the numbers 1–5.**
  - Explain that a numerical expression (math problem) can have more than one operation. The expression 4 + 2 × 3 has two operations, addition and multiplication. When an expression has more than one operation, and multiplication is one of the operations, there are special rules to follow called order of operations. The multiplication is always done before the addition or subtraction.
  - Model solving 4 + 2 × 3. Since multiplication always comes first, make 2 groups of 3 tokens.

    Then add 4 more tokens.

    Count the total number of tokens. The answer is 10. Therefore 4 + 2 × 3 = 10.
  - Continue modeling how to evaluate additional examples of expressions with addition or subtraction and multiplication. Be sure to model examples with multiplication as the second operation and multiplication as the first operation, e.g., 5 – 3 × 1 and 2 × 4 + 3.
  - Ask students to evaluate numerical expressions with addition or subtraction and multiplication.

MA 5.2.2 Algebraic Processes

- ❑ **Evaluate a numerical expression with addition or subtraction and multiplication that includes parentheses using the numbers 1–5.**

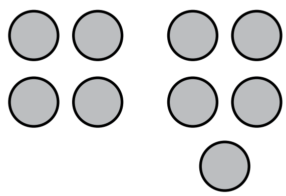
  - Explain that when parentheses are used in a numerical expression, the parentheses are always evaluated first. Demonstrate identifying the parentheses and what is evaluated first by highlighting, underlining, or circling the first step in solving numerical expressions.

2 × (3 + 4)	(5 – 1) × 2	4 × (2 + 2)	4 – (2 + 1)

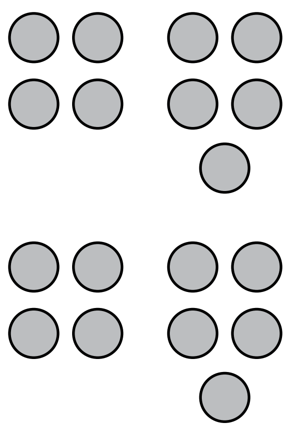
  - Ask students to identify the parentheses and what is evaluated first by highlighting, underlining, or circling the first step in solving numerical expressions.

  - Model solving (4 + 5) × 2. Explain that parentheses are always the first operation within a numerical expression regardless of what expression is within them. In this case, 4 + 5 is within the parentheses and therefore the first step to solve.

    Model starting with 4 tokens and then adding 5 more tokens.



    This gives a total of 9 tokens. The second step in the expression is to multiply by 2, so two sets of 4 + 5 are needed.



    Count the total number of tokens. The answer is 18. Therefore (4 + 5) × 2 = 18.

  - Continue modeling how to solve numerical expressions that have parentheses.

(2 × 3) + 4	(5 – 1) × 2	4 × (2 + 2)	4 – (2 + 1)

  - Ask students to evaluate numerical expressions that have parentheses.

- Ask students to solve numerical expressions created with the cutout number cards.

1	2	3	4
5	-	+	×
1	2	3	4
5	-	+	×
(	)		

Prerequisite Extended Indicators

MAE 4.2.2.a—Evaluate numerical expressions using order of operations using numbers 1 through 5.

MAE 3.1.2.c—Use a model to show multiplication as repeat addition with a product no greater than 20.

MAE 3.1.2.a—Add and subtract, through 20, without regrouping.

Key Terms

add, multiply, numerical expression, operation, order of operations, parentheses, subtract

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_189_g_2_t_2.html?open=activities&from=category_g_2_t_2.html

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.nctm.org/Classroom-Resources/Illuminations/Lessons/Exploring-Krypto/

https://www.engageny.org/resource/grade-5-mathematics-module-2-topic-b-lesson-3

MA 5.2.3 Applications

MA 5.2.3.a

Solve real-world problems involving addition and subtraction of fractions and mixed numbers with like and unlike denominators.

Extended: Solve real-world problems with addition or subtraction of fractions limited to like denominators without regrouping involving halves, thirds, and fourths.

Scaffolding Activities for the Extended Indicator

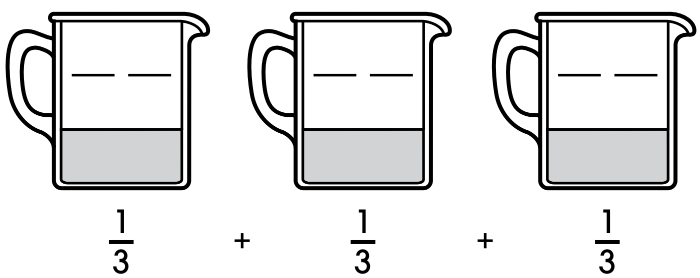
❑ **Solve real-world problems with addition of fractions with halves, thirds, or fourths.**

- Use models or manipulatives to demonstrate adding fractions with halves, thirds, or fourths. Present the problem shown below and demonstrate solving the problem using an actual measuring cup or other model. Explain that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$, or 1 cup in all.

  Rita is making fruit punch.

  She puts $\frac{1}{3}$ cup of apple juice, $\frac{1}{3}$ cup of orange juice, and $\frac{1}{3}$ cup of cranberry juice in a glass.

  How much fruit punch is in the glass?



  Continue to demonstrate solving a variety of real-world problems involving adding fractions with like denominators of 2, 3, or 4. Use manipulatives if possible, or models such as circles or rectangles, to show the part of the whole each fraction represents. When appropriate, progress to solving problems without using models.

- Ask students to solve real-world problems with addition of fractions with halves, thirds, or fourths using models or manipulatives.

- Ask students to solve real-world problems with addition of fractions with halves, thirds, or fourths without using models or manipulatives.

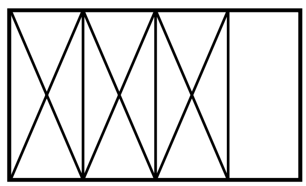
❑ **Solve real-world problems with subtraction of fractions with halves, thirds, or fourths.**

- Use models or manipulatives to demonstrate subtracting fractions with halves, thirds, or fourths. Present the problem shown below and a rectangle model divided into fourths. Explain that the rectangle represents 1 mile, with each part representing $\frac{1}{4}$ mile. Demonstrate crossing off three parts of the rectangle to show the distance already walked, $\frac{3}{4}$ mile. So, the subtraction equation $1 - \frac{3}{4} = \frac{1}{4}$ means that Jesse has $\frac{1}{4}$ mile left to walk.

  Jesse lives 1 mile away from school.

  He walks $\frac{3}{4}$ of the way to school and stops to tie his shoe.

  What fraction of the mile does he have left to walk?



  Continue to demonstrate solving a variety of real-world problems involving subtracting fractions with like denominators of 2, 3, or 4. Use manipulatives if possible, or models such as circles or rectangles, to show the part of the whole each fraction represents. When appropriate, progress to solving problems without using models.

- Ask students to solve real-world problems with subtraction of fractions with halves, thirds, or fourths using models or manipulatives.
- Ask students to solve real-world problems with subtraction of fractions with halves, thirds, or fourths without using models or manipulatives.

Prerequisite Extended Indicators

MAE 5.1.2.h—Add and subtract fractions with like denominators using a visual model without regrouping.

MAE 4.1.2.f—Add and subtract halves to halves, thirds to thirds, fourths to fourths, and fifths to fifths. . . . to a whole.

Key Terms

add, denominator, fourth, fraction, half, numerator, subtract, third

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/4/NF/B/3/tasks/968

https://www.engageny.org/resource/grade-4-mathematics-module-5-topic-d-overview/file/77296

Mathematics—Grade 5
MA 5.3 Geometry

MA 5.3.1 Characteristics

MA 5.3.1.a

Identify three-dimensional figures including cubes, cones, pyramids, prisms, spheres, and cylinders.

Extended: Identify three-dimensional models limited to cube, cylinder, and cone.

Scaffolding Activities for the Extended Indicator

❑ **Identify characteristics of three-dimensional objects.**

- Use pattern blocks or cutout shapes of squares, circles, and triangles to demonstrate the concept of something being two-dimensional. Explain that the shapes are called “two-dimensional” because they have length and width but no depth (i.e., they are flat). Another way to think of two-dimensional is something that can be easily drawn on a piece of paper. Draw the shapes on paper and indicate the length and width.

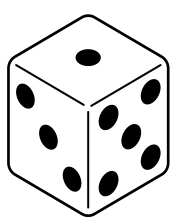
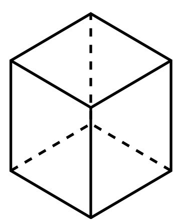
  Explain to students that “three-dimensional” means a shape has length, width, and depth. Present three-dimensional real-world objects or geometric solid figures and indicate the three dimensions.

- Compare a variety of objects that represent two-dimensional shapes with three-dimensional shapes and demonstrate identifying a shape as two-dimensional or three-dimensional. For example, compare a cutout square with a die or a cutout circle with a can.

- Present students with a collection of figures that represent two-dimensional shapes and three-dimensional objects and ask students to select the three-dimensional objects.

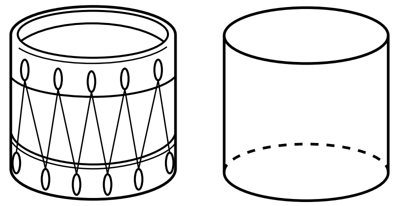
❑ **Identify three-dimensional models of cubes, cylinders, and cones.**

- Use a drawing of a cube and a real-world object to show how a cube is represented in a drawing.

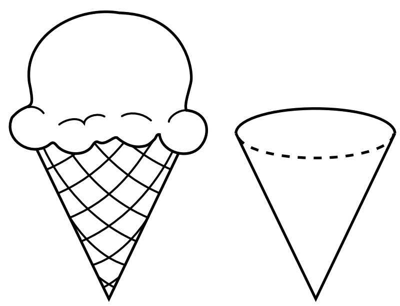
  A cube is a three-dimensional shape that has square faces. The faces are the six flat sides of the cube. Since it is difficult to draw something three-dimensional on paper, drawings of three-dimensional shapes usually have dashed lines or faded lines to help show the edges of the shape that cannot be seen without turning the shape over. Some examples of cubes are building blocks, ice cubes, and cardboard boxes with square faces.

- Use a drawing of a cylinder and a real-world object to show how a cylinder is represented in a drawing.



A cylinder is a three-dimensional shape that has two faces that are circles. In the drum shown above, the circles are the top and bottom. Some other examples of cylinders are buckets, cans of soup, and batteries.

- Use a drawing of a cone and a real-world object to show how a cone is represented in a drawing.



A cone is a three-dimensional shape that has one circle as a face and comes to a point (vertex) opposite of the circle. Some examples of cones are traffic cones, party hats, and megaphones.

- Ask students to identify a cube from a collection that includes objects that represent two-dimensional shapes (pattern blocks or cutout shapes) and other three-dimensional objects that don’t represent a specific geometric figure (e.g., computer mouse, coffee cup, headphones).
- Ask students to identify a cylinder or a cone from a collection that includes objects that represent two-dimensional shapes and other three-dimensional objects that don’t represent a specific geometric figure.
- Ask students to identify a cube, a cylinder, and a cone from a collection of geometric figures.

Prerequisite Extended Indicator

MAE 3.3.1.b—Identify two-dimensional shapes, circles, triangles, rectangles, or squares from a collection of circles, rectangles, and squares.

Key Terms

cone, cube, cylinder, depth, length, three-dimensional, two-dimensional, vertex, width

Additional Resources or Links

https://www.engageny.org/resource/kindergarten-mathematics-module-2-topic

https://www.engageny.org/resource/kindergarten-mathematics-module-2-topic-b

MA 5.3.1.b

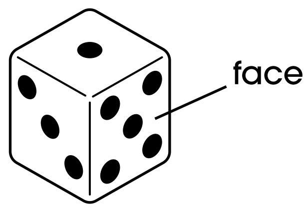
Identify faces, edges, and vertices of rectangular prisms.

Extended: Identify the faces, edges, and vertices of a cube.

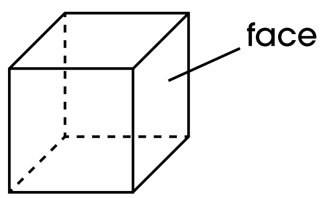
Scaffolding Activities for the Extended Indicator

❑ **Identify the faces of a cube.**

- Indicate the faces on an object that is in the shape of a cube. Note that the faces are equal-size squares, and there are always six faces on a cube, no matter how big or small the cube is. It might be helpful to cut out six squares that can be taped to each face of the cube as it is counted. Demonstrate counting the six faces of a cube on a variety of real-world objects and manipulatives in a variety of sizes.



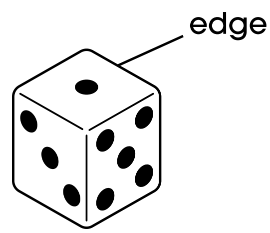
Compare a real-world object that is a cube to a drawing of a cube and indicate the faces of the cube on the drawing. Indicate that the dashed or faded lines in the drawing of a three-dimensional shape show the hidden sides of a three-dimensional shape when it is drawn on a piece of paper.



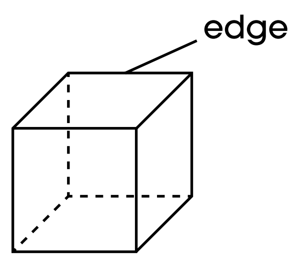
- Ask students to identify the six faces of a cube on a three-dimensional object. For example, present six square sticky notes and ask students to place them on the six faces of a cube.
- Ask students to identify the faces of a cube on a drawing of a cube.

❑ **Identify the edges of a cube.**

- Indicate the edges of a cube on a manipulative or other object. Explain that an edge is the part where two faces meet. Note that the edges on a cube are all the same length, and there are always twelve edges on a cube. It might be helpful to use a model of a cube that can be drawn on to highlight the edges. Another option is to precut sticky string and place sticky string on the edges of the cube.



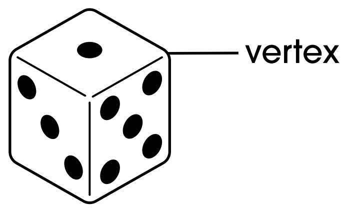
Compare the edges on a real-world object to the edges on a drawing of a cube. Emphasize that there are twelve edges on a cube and explain that the dashed lines or faded lines indicate the hidden edges of a three-dimensional shape when it is drawn on a piece of paper.



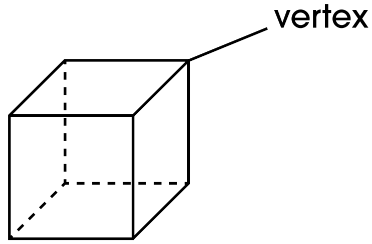
- Ask students to identify the edges of a cube on a three-dimensional object.
- Ask students to identify the edges of a cube on a drawing of a cube.

- ❑ **Identify the vertices of a cube.**

  - Indicate the vertices of a cube on a manipulative or other real-world object. Explain that a vertex is the part where three faces meet to make a corner. Note that there are eight vertices on a cube. It might be helpful to cover three faces of a cube with a sticky note and identify the corner where all three pieces of paper touch. Another option is to form eight small balls of clay to adhere to each vertex.



Compare the vertices on the real-world object to the vertices on a drawing of a cube.



  - Ask students to identify the vertices of a cube on a three-dimensional object.
  - Ask students to identify the vertices of a cube on a drawing of a cube.

Prerequisite Extended Indicators

MAE 5.3.1.a—Identify three-dimensional models limited to cube, cylinder, and cone.

MAE 3.3.1.b—Identify two-dimensional shapes, circles, triangles, rectangles, or squares from a collection of circles, rectangles, and squares.

MAE 3.3.1.a—Identify the number of sides or angles in a regular polygon.

Key Terms

cube, edge, face, three-dimensional, vertex

Additional Resources or Links

https://www.engageny.org/resource/grade-1-mathematics-module-5-topic-lesson-3/file/50211

https://www.insidemathematics.org/sites/default/files/materials/cutting%20a%20cube.pdf

MA 5.3.1.c

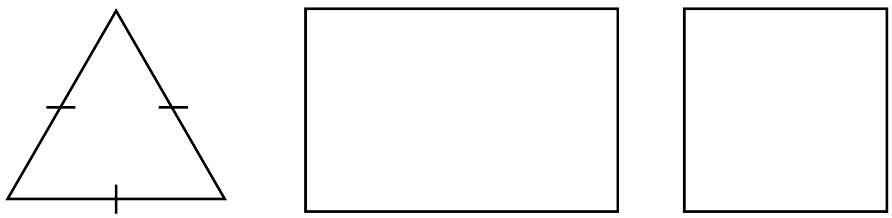
Justify the classification of two-dimensional figures based on their properties.

Extended: Sort triangles, rectangles, and squares by number of sides and/or angles.

Scaffolding Activities for the Extended Indicator

❑ **Identify the number of sides and angles in triangles, rectangles, and squares.**

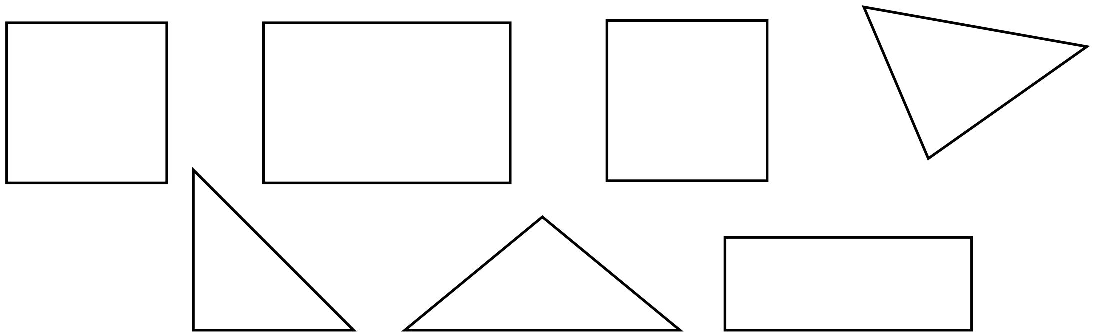
- Explain that sides are the line segments that form a shape. Model a counting method (e.g., circling the tick mark as it is counted, making a tick mark on each side as it is counted, tracing over or highlighting a side as it is counted) for counting the number of sides in a triangle, a rectangle, and a square.



- Ask students to use a counting method to count the number of sides in a triangle, a rectangle, and a square.
- Explain that angles are where the sides meet at the corner or vertex. Model a method (e.g., circling the angle as it is counted, highlighting the angle as it is counted) for counting the number of angles in a triangle, a rectangle, and a square.

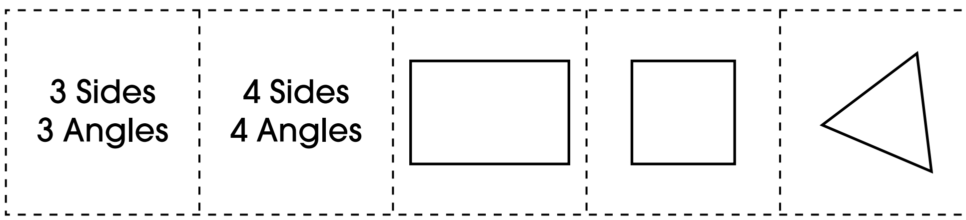
- Ask students to use a counting method to count the number of angles in a triangle, a rectangle, and a square.

- Ask students to count the numbers of sides and angles in triangles, rectangles, and squares. Create a chart to document the students' results and to reveal the pattern (rule) for the numbers of sides and angles in triangles, rectangles, and squares.

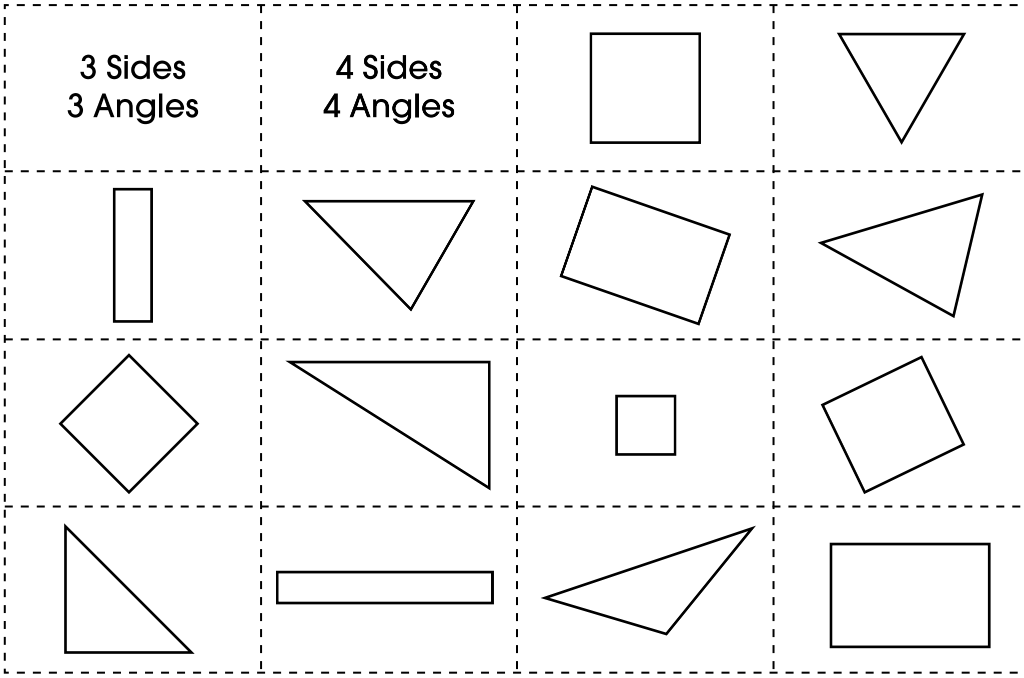


❑ **Sort triangles, rectangles, and squares by number of sides and number of angles.**

- Demonstrate sorting triangles, rectangles, and squares into two categories based on the number of sides and the number of angles, with one category labeled 3 sides/3 angles and the other category labeled 4 sides/4 angles.



- Ask students to sort triangles, rectangles, and squares into two categories with one category labeled 3 sides/3 angles and the other category labeled 4 sides/4 angles.



Prerequisite Extended Indicator

MAE 3.3.1.a—Identify the number of sides or angles in a regular polygon.

Key Terms

angle, category, rectangle, shape, side, square, triangle

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/which%20shape.pdf

https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-b-lesson-4

https://www.engageny.org/resource/grade-3-mathematics-module-7-topic-b-lesson-5

MA 5.3.2 Coordinate Geometry

MA 5.3.2.b

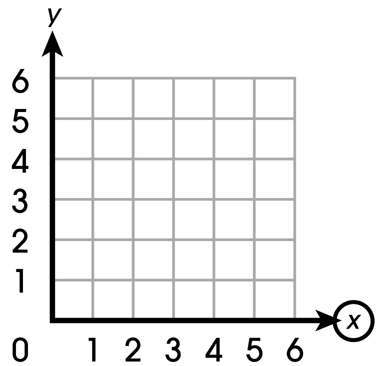
Graph and name points in the first quadrant of the coordinate plane using ordered pairs of whole numbers.

Extended: Identify the *x*- or *y*-coordinate of whole-numbered points in quadrant I.

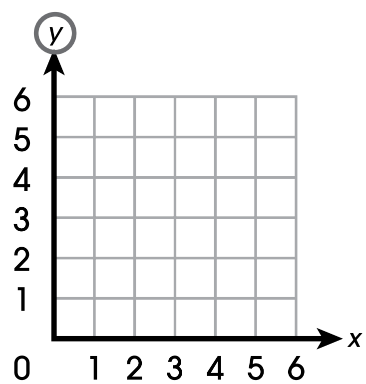
Scaffolding Activities for the Extended Indicator

❑ **Identify the *x*- or *y*-axis on a coordinate plane.**

- Use the labels on the axes of a coordinate graph to identify which axis is the *x*-axis and which is the *y*-axis. Color coordinating references to the *x*-axis and the *y*-axis can be helpful to some students.



Point out the *x* label next to the lower axis and circle the *x* in blue. For this coordinate graph, the *x*-axis begins with the number 0 and goes to the number 6.

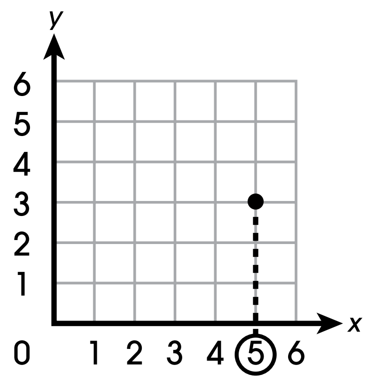


Point out the *y* label above the vertical axis and circle the *y* in red. For this coordinate graph, the *y*-axis begins with the number 0 and goes to the number 6.

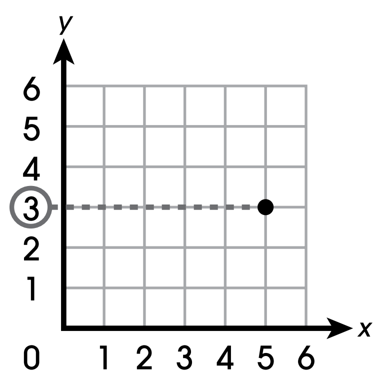
- Ask students to identify the *x*- and *y*-axis on a graph.

❑ **Identify the *x*- or *y*-coordinate of a point on a coordinate plane.**

- Use the labels along the *x*- and *y*-axis to show the location of a point on a graph. Use the graph below to demonstrate finding the *x*-coordinate of a point: use the color blue to trace the grid line down from the point to the *x*-axis and circle the 5 in blue.



  The point aligns with the 5 on the *x*-axis, so the *x*-coordinate of this point is 5. The same process can be done to find the *y*-coordinate: use the color red to trace the grid line to the left to where it intersects with the *y*-axis and circle the 3 in red.



  The point aligns with the 3 on the *y*-axis, so the *y*-coordinate of this point is 3. Repeat this demonstration with points in a variety of locations.

- Ask students to identify the *x*- or *y*-coordinate of a point in the first quadrant of a coordinate graph (i.e., only positive, whole number coordinates).

Prerequisite Extended Indicator

MAE 3.1.1.b—Compare and order whole numbers, 1–20.

Key Terms

coordinate graph, coordinate plane, horizontal, point, vertical, *x*-axis, *x*-coordinate, *y*-axis, *y*-coordinate

Additional Resources or Links

http://tasks.illustrativemathematics.org/content-standards/5/G/A/1/tasks/489

https://www.engageny.org/resource/grade-5-mathematics-module-6-topic-lesson-3/file/69596

MA 5.3.3 Measurement

MA 5.3.3.b

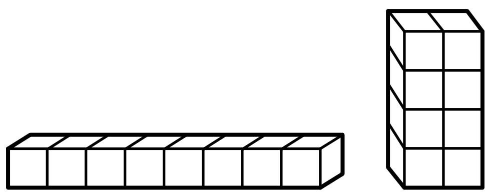
Use concrete models to measure the volume of rectangular prisms in cubic units by counting cubic units.

Extended: Find the volume of a rectangular prism by counting unit cubes.

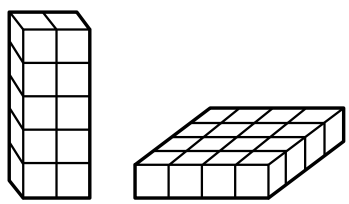
Scaffolding Activities for the Extended Indicator

❑ **Find the volume of a rectangular prism when one of the dimensions has a value of 1 unit.**

- Describe volume as the amount of space inside an object. Use real-life objects to demonstrate the space inside an object (for example, a box, a toilet paper tube, an empty soup can, or an ice-cream cone).
- Use two sets of 8 unit cubes to model two rectangular prisms with a volume of 8 cubic units. Create a 1 × 1 × 8 rectangular prism and a 1 × 2 × 4 rectangular prism. Explain that each rectangular prism is made with 8 unit cubes, so the volume of each rectangular prism is 8 cubic units.



Continue to create other single-layer rectangular prisms with unit cubes and demonstrate finding the volume by counting the unit cubes.

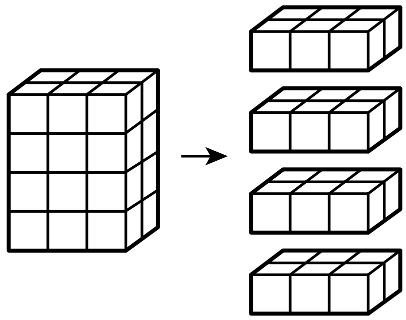


- Ask students to identify the volume of single-layer rectangular prisms by counting the unit cubes. When appropriate, encourage students to use skip-counting to find the total number of unit cubes.

❑ **Find the volume of a rectangular prism by counting unit cubes.**

- Use a small square or rectangular box to explain that volume can be found by filling the box with unit cubes. Demonstrate stacking unit cubes on top of one another and next to each other to fill the box. The number of cubes that fit into the box is the volume of the rectangular prism.

- Use unit cubes to create a 2 × 3 × 4 rectangular prism. Demonstrate finding the volume by counting unit cubes. It might be helpful to glue together the unit cubes in each 1 × 2 × 3 layer to emphasize counting the unit cubes in layers. Another strategy is to use a different-color unit cube for each layer of the rectangular prism. Reference the three dimensions as the length, width, and height of the rectangular prism.



- Use manipulatives and drawings to demonstrate finding the volume of rectangular prisms by counting unit cubes. Demonstrate counting the number of cubes in one layer of the rectangular prism and using skip-counting strategies to find the total number of cubes.
- Ask students to determine the volume of a rectangular prism by counting unit cubes.

Prerequisite Extended Indicators

MAE 5.3.1.a—Identify three-dimensional models limited to cube, cylinder, and cone.

MAE 4.3.3.a—Identify the area of a rectangle by counting unit squares.

Key Terms

face, height, layer, length, rectangle, rectangular prism, volume, width

Additional Resources or Links

https://www.engageny.org/resource/grade-5-mathematics-module-5-topic-lesson-1/file/67421

https://www.engageny.org/resource/grade-5-mathematics-module-5-topic-lesson-2/file/67431

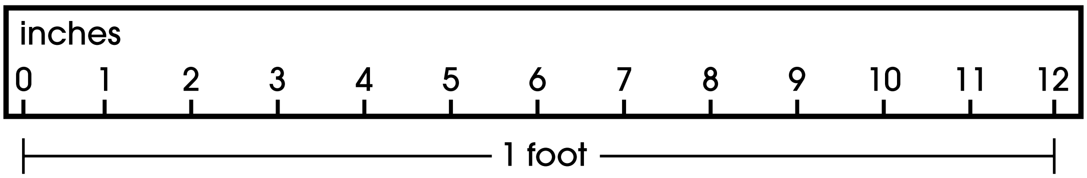
MA 5.3.3.c

Generate conversions within the customary and metric systems of measurement.

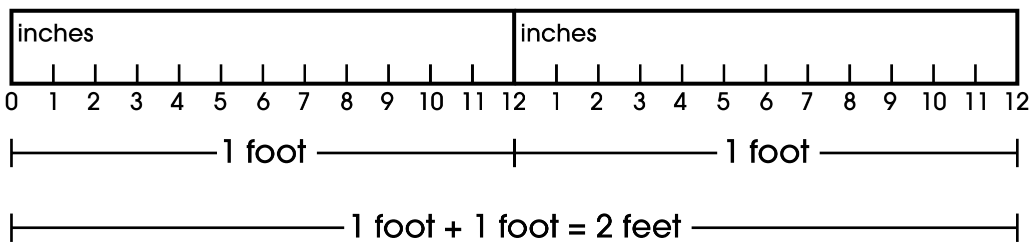
Extended: Convert whole-numbers of feet to inches using a model.

Scaffolding Activities for the Extended Indicator

- ❑ **Recognize lengths that are whole numbers of feet.**
  - Use a 12-inch ruler to show that one ruler is equal to 1 foot.



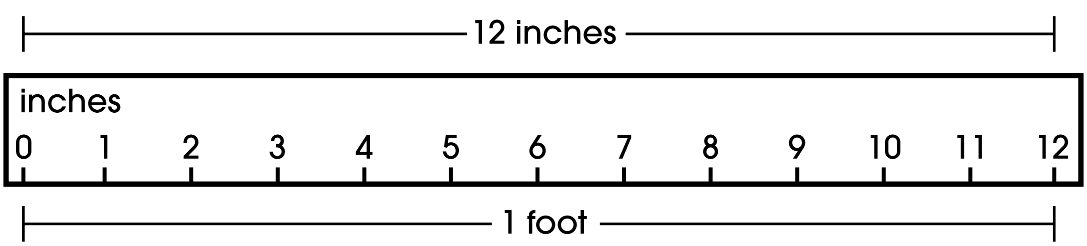
Use two rulers to show that each ruler is equal to 1 foot and when combined they equal 2 feet.



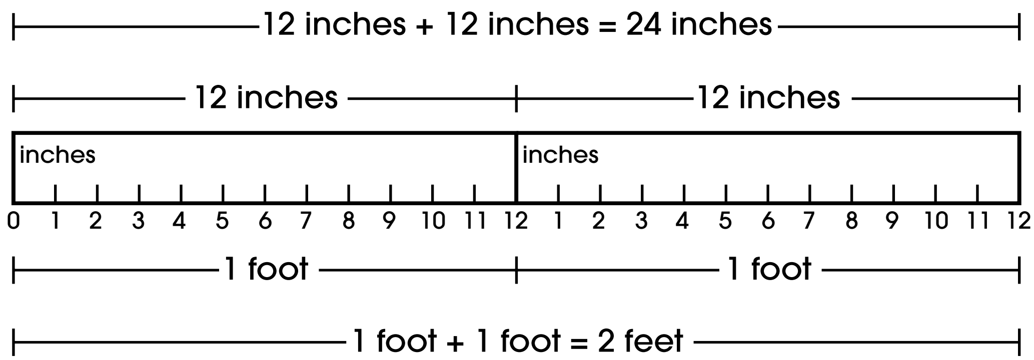
Repeat with three rulers to show 3 feet.

  - Ask students to identify which model shows 2 feet when given three options (e.g., a model of one 12-inch ruler, a model of two 12-inch rulers, and a model of three 12-inch rulers).
  - Ask students to identify how many feet are shown when presented with three 12-inch rulers.

- ❑ **Convert whole numbers of feet to inches using a model.**
  - Use a 12-inch ruler to demonstrate counting each inch on the ruler to show that it is 12 inches long. Explain that this length is equivalent to 1 foot.



Use two rulers to show that putting two rulers of 12 inches in length next to each other makes a total length of 24 inches. Demonstrate this by adding 12 + 12, counting on from 12 to count the 12 inches on the second ruler, or multiplying 12 × 2. Therefore, 2 feet equals 24 inches.



Repeat the process using three rulers to show that putting three rulers of 12 inches in length next to each other makes a total length of 36 inches. Demonstrate this by adding 12 + 12 + 12, counting on, or multiplying 12 × 3. Therefore, 3 feet equals 36 inches.

- Ask students to identify the length of 2 feet in inches when presented with a model of two 12-inch rulers and three choices as shown.

  A. 12 inches
  B. 24 inches
  C. 36 inches

- Ask students to identify which model shows 36 inches when given three options (e.g., a model of one 12-inch ruler, a model of two 12-inch rulers, and a model of three 12-inch rulers).

Prerequisite Extended Indicator

MAE 4.3.3.c—Identify the number of inches in one or two feet using a model of a ruler.

Key Terms

foot, inch, ruler

Additional Resources or Links

http://nlvm.usu.edu/en/nav/frames_asid_272_g_2_t_4.html?open=instructions&from=search.html?qt=ffeet

(Note: Java required for website. Most recent version recommended, but not needed.)

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-c-lesson-15

Mathematics—Grade 5
MA 5.4 Data

MA 5.4.2 Analysis and Applications

MA 5.4.2.a

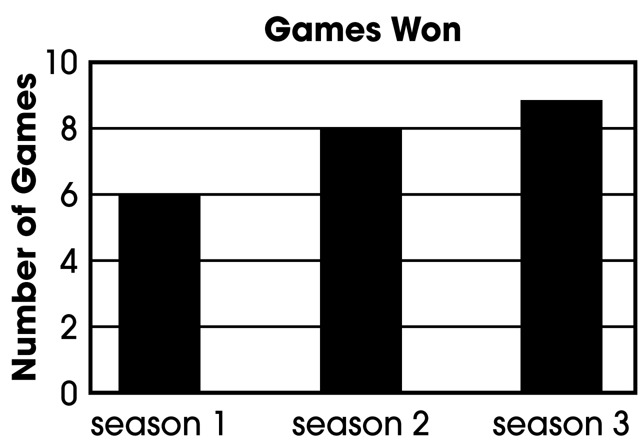
Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (e.g., frequency charts) and bar graphs.

Extended: Interpret information in a bar graph using at least two data points.

Scaffolding Activities for the Extended Indicator

❑ **Identify characteristics of bar graphs.**

- Describe key elements of bar graphs and how the information is interpreted. Present a graph as shown and identify characteristics of the graph, including the title, the categories on the horizontal axis, the label on the vertical axis, and the scale (numbering) on the vertical axis.



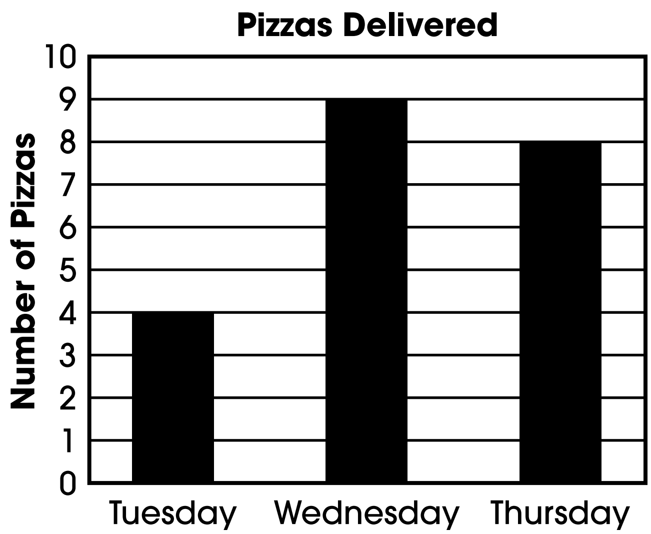
Demonstrate finding the value of the data in each category using a scale of 2 and determining the value of a bar that ends between two lines. Contrast a scale of 2 with scales of 1 and 5, including estimating a value between the labeled lines on a scale of 5.

- Ask students to identify the characteristics of a bar graph.
- Ask students to determine the value of the data displayed in each category on a bar graph.

MA 5.4.2 Analysis and Applications

❑ **Interpret information in a bar graph using at least two data points.**

- Compare category values presented in the bar graph shown. Demonstrate how to determine the data for each day using the scale. Present questions that require interpretation of the data. “Were more pizzas delivered on Tuesday or Wednesday?” “On which day were the most pizzas delivered?” Avoid questions that require the student to add or subtract for this standard.



- Continue to model interpreting information on bar graphs with scales of 2 and 5.
- Ask students to interpret and compare data between categories on a bar graph with a scale of 1.
- Ask students to interpret and compare data between categories on bar graphs with scales of 2 and 5.

Prerequisite Extended Indicators

MAE 3.4.1.b—Identify the scale of a bar graph and/or the key of a pictograph.

MAE 3.4.1.a—Identify a characteristic of a bar graph or a pictograph. (e.g., quantities, comparisons).

Key Terms

bar graph, category, compare, data point, title, value

Additional Resources or Links

https://www.insidemathematics.org/sites/default/files/materials/parking%20cars.pdf

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-5

MA 5.4.2.b

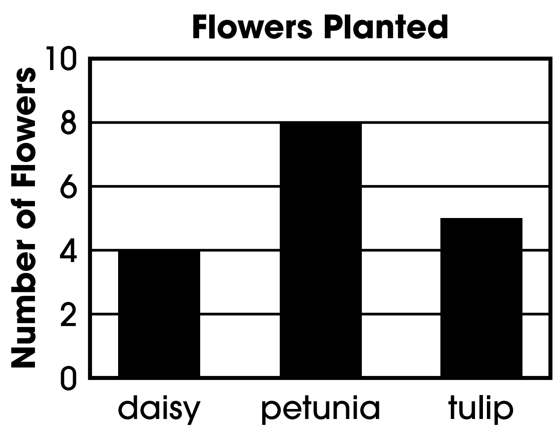
Formulate questions that can be addressed with data and make predictions about the data.

Extended: Solve a problem with addition or subtraction of whole numbers using information from a bar graph.

Scaffolding Activities for the Extended Indicator

❑ **Identify relevant information presented in a bar graph.**

- Identify information from a bar graph. For example, present the bar graph shown. Identify the title to determine what the graph is about. Identify each category represented by the bars. Identify the scale of 2, and explain its meaning in determining the value of the data in each category.



- Ask students to identify the relevant information presented in a bar graph and to use the scale. For example, present the bar graph shown above.

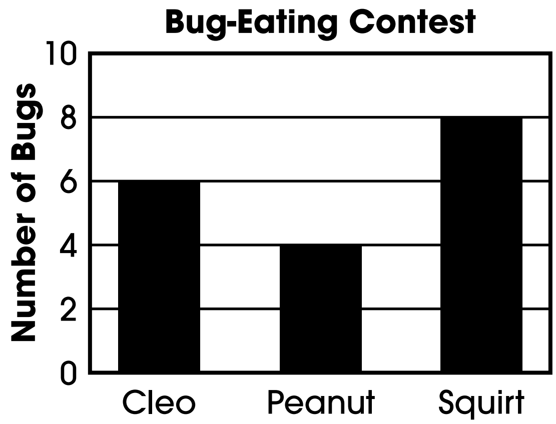
  Ask students to use the graph to answer the following questions.

  ► "What is this graph about?"
  ► "How many tulips were planted?"
  ► "Were more petunias or daisies planted?"
  ► "Does this graph include information about roses?"

❑ **Solve a problem with addition or subtraction of whole numbers using information from a bar graph.**

- Solve a word problem using addition and subtraction of values presented in a bar graph. For example, present the scenario and bar graph shown.

Three geckos decided to have a bug-eating contest to see which gecko could eat the most bugs in one minute. The results from the contest are shown in the bar graph.



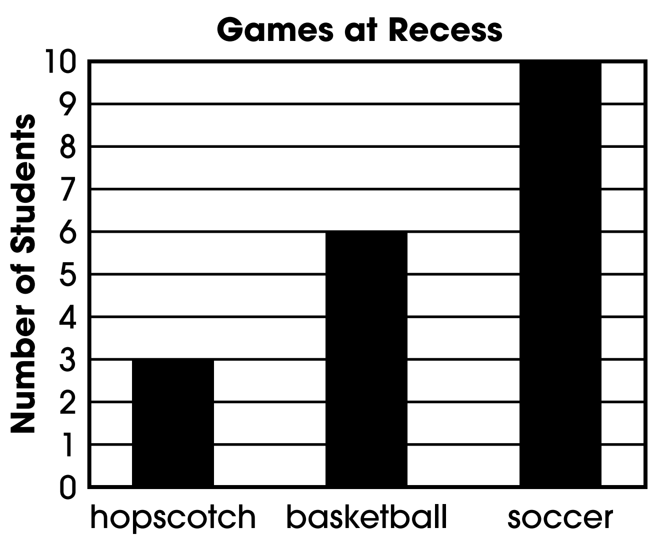
Use the bar graph to answer the following questions.

- ► How many more bugs did Cleo eat than Peanut ate?
- ► Squirt ate more bugs than Cleo and Peanut put together. Is this statement true or false?
- ► Which two geckos ate exactly 10 bugs altogether?

Demonstrate following a sequence of steps to answer the questions: (1) determine what information is needed to answer the question, (2) find the needed information on the bar graph, (3) determine the arithmetic operation to use (add or subtract), and (4) carry out the operation to get the answer.

- Ask students to solve problems using addition and subtraction of values presented in a bar graph with a scale of 1. For example, present the scenario and bar graph shown.

  Students were playing games at recess. Here is a bar graph showing the number of students playing each game.

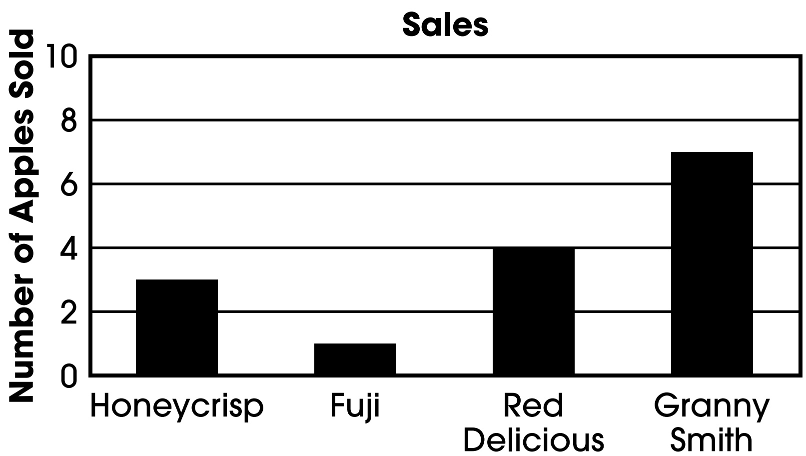


Use the bar graph to answer the following questions.

- ► How many more students are playing soccer than hopscotch?
- ► How many students are playing basketball and soccer?

- Ask students to solve problems using addition and subtraction of values presented in a bar graph with a scale other than 1. For example, present the scenario and bar graph shown.

  A farmer sold different varieties of apples at a farmer's market. The results of the farmer's sale are shown in the bar graph.



  Use the bar graph to answer the questions.

  ► How many more Granny Smith apples were sold than Red Delicious apples?
  ► How many total Honeycrisp and Fuji apples were sold?

Prerequisite Extended Indicators

MAE 5.4.2.a—Interpret information in a bar graph using at least two data points.

MAE 3.4.1.b—Identify the scale of a bar graph and/or the key of a pictograph.

MAE 3.1.2.a—Add and subtract through 20 without regrouping.

Key Terms

add, bar graph, category, data, scale, subtract, title, value

Additional Resources or Links

https://www.insidemathematics.org/classroom-videos/public-lessons/2nd-grade-math-word-problem-clues

https://www.engageny.org/resource/grade-2-mathematics-module-7-topic-lesson-5

Alternate Mathematics Instructional Supports for NSCAS Mathematics Extended Indicators Grade 5