Unfinished Learning Series
Math Community of Practice

Session 4:
Plan & Take Action Part II
Planning Intentional Small Group and Individual Supports

Planning & Taking Action to Address Unfinished Learning Reflection
• Since our last session, what have you done to learn more or support teachers/schools with planning for and delivering just in time acceleration supports? • What did you learn? What might you do the same or differently next time?

Practice Recommendations for Assisting Students Struggling with Mathematics

1. **Systematic Instruction:** Provide systematic instruction during intervention to develop student understanding of mathematical ideas.
2. **Mathematical Language:** Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.
3. **Representations**: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.
4. **Number Lines:** Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced Mathematics.
5. **Word Problems**: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.
6. ***Build Fluency:** Regularly include quality fluency practice to build fluency in mathematics.

*Adapted from Fuchs, L.S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K.S.., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades (WWC 2021006). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from http://whatworks.ed.gov/.

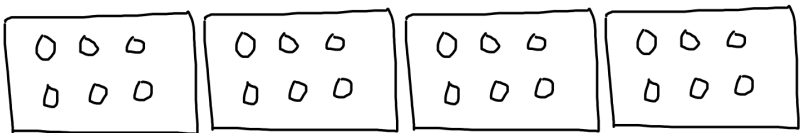
Systematic Instruction In Action
Lesson Excerpt

Mia is a fifth grade student at Brightwood Academy. Based on the school's universal screening and curriculum embedded diagnostics, her teachers identified Mia had unfinished learning with grade 3 multiplication and division standards. To ensure Mia is ready to access grade 5 multi-digit multiplication and division concepts, they created a plan to develop Mia's conceptual understanding and procedural fluency with the prerequisite standards. She receives individual intervention instruction three times a week in addition to the grade 5 core math block instruction as part of this plan.

Mrs. Teal: We're going to start off our math workshop time with a "Tell Me All You Can" to review what you already know about multiplication Remember, I'll show you an expression, or problem and you tell me all you can without giving away the answer. You can use words, pictures, and numbers to tell me what you know about the expression Ready? Tell me all you can about 4 x 6.

Mia: It means you have 4 groups with 6 in each group. Like the donuts.

Draws 4 rectangles with 6 circles in each rectangle.



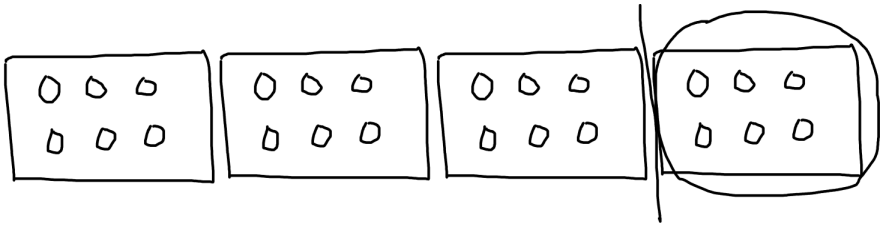
Mrs. Teal: What is a story that might go along with your picture?

Mia: It could be an equal group story like the donut task we did. Tamia had 4 boxes and there were 6 donuts in each of the boxes. You can count by sixes 4 times to find out how many donuts altogether.

Mrs. Teal: That is one way we can think about 4 x 6. Can you think of any other ways to think about 4 x 6?

Mia: It's one more group of 6 than 3 boxes of 6. Like Tamia had 3 boxes of donuts at first and then bought one more box.

Draws a line after the third box and circles the last box.



Mrs. Teal: That's an interesting way of thinking about it. I like how you said there is one more group of 6 than 3 boxes of 6. That helps me understand the size of each box is 6 donuts. What else can you tell me about 4 x 6?

Mia: It's kinda like 6 x 4, but the numbers are just switched around.

Mrs. Teal: Take a look at our math word chart. What do we call the 6 and the 4?

Mia: Oh, factors. The factors are switched around because it's the Commutative Property.

Mrs. Teal: Does switching the order of the factors change the story you put with 4 x 6?

Mia: It's still an equal group problem, but it's not exactly the same because 6 x 4 would look like 6 boxes with 4 donuts in each box. It would still be the same total.

Mrs. Teal: Say more about what you mean by it would still be the same total.

Mia: Both would have the same amount of donuts. If you get 4 boxes with 6 donuts each, it's the same total amount of donuts as 6 boxes with 4 donuts in each box.

Mrs. Teal: What does that tell you about 4 x 6 and 6 x 4?

Mia: That they have the same answer.

Mrs. Teal: What math word have we been using to describe the total amount in all the groups?

Mia: Product.

Mrs. Teal: So 4 x 6 and 6 x 4 have the same….

Mia: Product.

Mrs. Teal: You've told me quite a bit about what 4 x 6 means. What can you tell me about the product?

Mia: Can I tell you the answer now?

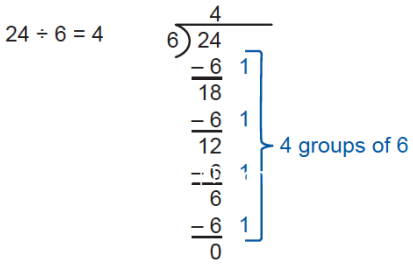
Mrs. Teal: Yes, but also tell me how you thought about it.

Mia: It's 24 because you could just see 2 boxes of 6 are 12 and 12 + 12 = 24.

Mrs. Teal: You used a two fact to think about a 4 fact. Nice strategy. How could we use your equal-sized group picture of 4 x 6 to think about 24 ÷ 6?

Mia: If we know there are 24 donuts and we put 6 donuts in each box, we can keep subtracting 6 until we get to 0. Like this. *Counts out 24 counters and removes groups of 6, and explains how she subtracted each group from 24 until all the donuts were in boxes.*

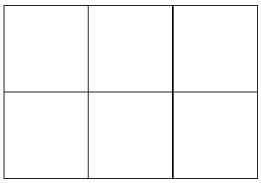
Mrs. Teal: Remember how we kept track of the number of groups we were subtracting?
Models a written record of the division and has the student explain how the written record matches the action with the counters.



Mrs. Teal presents an equal group task where the unknown is the number of groups for Mia to practice division with one-digit divisors which is a skill they have been working on for the past 2 weeks.

Truffles Task

Patricio makes different kinds of truffles. He puts the truffles in boxes that look like this.



Truffle Box

Use the information in the table to find how many boxes he will need for each of his top-selling truffle flavors.

Patricio's Truffle Flavors
96 raspberry truffles
132 dark chocolate truffles
336 butterscotch crunch

Mia: 10 boxes would hold 60 truffles because 10 x 6 = 60. Then I just kept adding 6 more until I got to 96. I counted how many sixes I added and it was 6. So I knew 10 + 6 = 16. He needs 16 boxes for the raspberry.

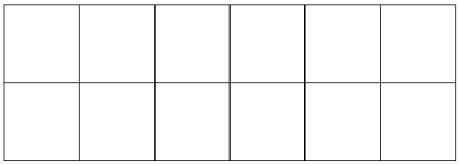
Mrs. Teal: Neat strategy! I noticed you started with a tens fact which made it quicker to find the number of boxes. Let's keep in mind what you knew about 10 boxes of 6 and see if our repeated subtraction division strategy can also work to find the number of boxes.

Mrs. Teal has Mia use the repeated subtraction strategy to divide the number of raspberry truffles by 6, and discusses how the strategy was similar and different from Mia's first approach. She has Mia practice the repeated subtraction division strategy to find the number of boxes needed for the other two flavors.

Mrs. Teal: You noticed the unknown was the number of groups. Using an equal-sized group model, you solved division problems with single digit divisors. Now we are going to think about how we can use equal-sized groups to solve division problems with 2-digit divisors.

*Mrs. Teal presents the next part of the task that includes **worked-out examples** of a multi-digit division problem that have been solved using the equal-sized group model using repeated subtraction.*

Mrs. Teal: Patricio decides to try selling truffles in larger boxes that look like this.



Large Truffle Box

Mrs. Teal: How do you think this will affect the number of boxes he uses for truffles?

Mia: The boxes will have 12 truffles instead of 6 so it's like he's putting two boxes together.

Mrs. Teal: Yes, that is a helpful way of thinking about it. So if he puts 12 truffles in a box instead of 6, will he need a fewer number of boxes or a greater number of boxes?

Mia: More because he's putting two boxes together.

Mrs. Teal: Let's test that out. Let's say Patricio decides to try out a new strawberry flavor truffle. Since he's not sure how it will sell, he only makes 24 of them. If he puts them in boxes of 6, we know he will need 4 boxes because that matches the model you made with the counters earlier. Use the counters to figure out how many boxes he would need if he puts them in boxes of 12.

Mia: Two.

Mrs. Teal: I notice you pushed two groups together to make the larger box. Did that create a fewer number of boxes or a greater number of boxes?

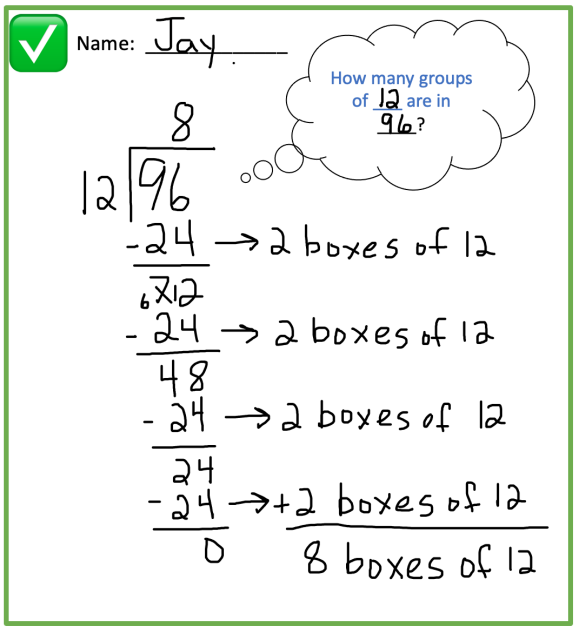
Mia: It made fewer boxes because you put two boxes together so it's half as many boxes.

Mrs. Teal: Why does it make sense he will need fewer numbers of boxes for 12 truffles than 6 truffles?

Mia: Because there is more in a box so you don't need as many boxes.

Mrs. Teal: Yes, I agree with your reasoning and let's keep that in mind as we take a look at Jay's work.

Mrs. Teal: Explain what Jay did here to find the number of larger boxes needed for the raspberry truffles.



Mia: He made 2 boxes of 12 which was 24 and minused 24 from 96 which got him 72.

Mrs. Teal: Instead of "minused" let's use subtract. Why did he subtract 24 from 96?

Mia: He subtracted 24 from 96 to find how many were left.

Mrs. Teal: He was finding how many of what were left?

Mia: How many truffles were left.

Mrs. Teal: Oh, okay I got it. Thank you for clarifying that for me. So there are 72 truffles left and what did he do next?

Mia: He made 2 more boxes of 12 and did 72 - 24 = 48.

Mrs. Teal: So how did he get a quotient of 8?

Mia: He added up all the boxes he used on the side and the answer was 8.

Mrs. Teal: Why did he add all the groups of 12 he had recorded on the side?

Mia: Because that is what you do to get the total.

Mrs. Teal: The total what? What does the quotient of 8 tell in this situation?

Mia: It tells us how many larger boxes he needs for the raspberry.

Mrs. Teal: So he needed 8 large boxes for the raspberry truffles. How does that compare to the number of smaller boxes you found he needed for the raspberry truffles?

Mia: He needed 8 large boxes and 16 small boxes.

Mrs. Teal: Hmmm. What do you know about the numbers 16 and 8?

Mia: 8 is half of 16.

After having Mia explain Jay's next worked example to find the number of larger boxes needed for 120 truffles, Mrs. Teal has Mia complete Jay's work to find the number of larger boxes needed for 336 truffles. In subsequent lessons Mrs. Teal gradually moves Mia from explaining and completing worked examples to having her solve multi-digit division problems using accessible numbers on her own.

Let's Discuss
• What do you notice about the questions and tasks in the lesson? • How does the sequence of the questions and tasks in the lesson support the student with making sense of division?

Tips to Carry Out Systematic Instruction
1. Review and integrate previously learned content throughout intervention to ensure that students maintain understanding of concepts and procedures. 2. When introducing new concepts and procedures, use accessible numbers to support learning. 3. Sequence instruction so that the mathematics students are learning builds incrementally. 4. Provide visual and verbal supports. 5. Provide immediate, supportive feedback to students to address any misunderstandings.

Mathematical Language in Action Lesson Vignette
Teacher prompts students to use mathematical terminology in their explanations.   **327**   **– 148**   **Kerry:** I noticed that 8 is bigger than 7.   **Teacher:** You first ***compared*** the ***digits*** in the ones place? Did you mean 8 is ***greater than*** 7? Remember, when a number is "bigger" or "larger" than another number we say ***greater than.***   **Kerry:** Yes, I started with the ***ones place***, 7 and 8. 8 was larger, so I needed to cross out the 2 and make it 1. That made 17.   **Teacher:** Because 8 was ***greater than*** 7, you ***regrouped***. You took 10 from the 20 in 327 and added it to the 7 ones to get 17. Then you changed the 2 in the ***tens place*** to a 1 in the ***tens place***. The teacher points to the numeral 2 in the ***tens place***.   **Kerry:** Yes, because there were 2 ***tens***, I used one of them to make 17. Then I subtracted the 8 and got 9 ***ones***. Next, I looked over and the 4 was ***greater than*** the 1. So, I had to change it again.   **Teacher:** You are describing how you knew to ***regroup*** the ***ones*** and then needed to use that approach again for the ***tens place***. When you ***regrouped***, you used what you knew about ***place value***.   **Kerry:** So then, I ***regrouped*** from the ***hundreds*** because there were 3 ***hundreds*** in 327. I could break apart 300 into 200 and 100. I added 100 to the 1 ***ten*** in the ***tens*** column, and now I can ***subtract*** 40 from 110! That gave me a 7 in the ***tens place*** for the answer. That's 7 ***tens.***   **Teacher:** So, your answer for the ***difference*** is 79 so far?   **Kerry:** Yes, and then I just had to ***subtract*** the ***hundreds***. I did not need to ***regroup***. 200 minus 100 equals 100. The ***difference*** is 179

Tips to Carry Out Mathematical Language

1. Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.

   Strategies for teaching mathematical vocabulary:
   - Connect vocabulary to visual representations
   - Use hand gestures to provide context and meaning
   - Use graphic organizers

2. Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.

Example word list that can be used across grade K-8 by all teachers in the school.

Rather than using this term...	**Consider using this term...**
Borrowing or Carrying	Regrouping
Reduce	Simplify
Flat Shape or Fat Shape	Two-Dimensional or Three-Dimensional Shape
Bigger, Smaller	Greater Than, Less Than
Flip-Flop Property	Commutative Property

**For more examples and ideas for using clear, concise, and correct mathematical language see The Math Pact, Elementary: Achieving Instructional Coherence Within and Across Grades by Karen Karp, Barbara Daughtery and Sarah Bush*

3. Support students in using mathematically/precise language during their verbal and written explanations of their problem solving.

   Strategies for supporting students in using mathematically precise language:
   - Sentence Frames (i.e, The array has __ rows and ___columns.)
   - Guiding Questions
   - Word Walls with visual
   - Restate with correct language

Representations In Action Video
● How did the representation support the student in making sense of place value? ● What feedback and coaching support might you give this teacher?

Tips to Carry Out Representations
1. Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered. 2. When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations. 3. Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as “thinking tools.” 4. Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.

Component of Fluency	Explanation
Efficiency	Solving a procedure in a reasonable amount of time by selecting an appropriate strategy and readily implementing that strategy.
Flexibility	Knowing multiple procedures and applying or adapting strategies to solve procedural problems
Accuracy	Correctly solving a procedure

Access Example Computational Strategies
Seven Significant Computational Strategies

Building Fluency In Action Video
• How did the practice encourage the use of strategy? • How does this fluency practice compare to your school's approach to fluency practice?

Tips to Carry Out Building Fluency

1. Identify already-learned topics for activities to support fluency and create a timeline.

Examples of activities that can support fluency for various intervention topics.

Intervention Topic	Fluency Focus	Relevancy to the Intervention
Fractions intervention (grade 4 and up)	Multiplication basic facts Equivalencies for benchmark fractions one-half and 1	Relevant for finding equivalent fractions and for fraction addition and subtraction Relevant for using benchmark numbers as a strategy to compare or order fractions or to estimate fraction magnitude on a number line
Place value with multi-digit addition and subtraction (grade 2 and up)	Addition and subtraction basic facts Evaluate the problem to determine if regrouping is necessary	Relevant so that students can efficiently add or subtract each place value Relevant so that students can determine if regrouping is needed as a standard practice when adding or subtracting numbers with multiple digits

2. When using a timed activity, ensure that students have an efficient strategy to use as they complete the timed activity. Plan timed activities that focus on previously learned content. Include the strategies you want students to use during timed activities during other portions of the intervention lessons. **Timed activities are not the same as timed tests.*

3. Provide immediate feedback by asking students to correct errors using an efficient strategy.

4. Engage students in different types of fluency practice such as fluency routines, worked examples, games, centers, and independent practice.

Professional Learning Fluency Resources

- Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention by Jennifer Bay-Williams and Gina Kling
- Figuring Out Fluency in Mathematics Teaching and Learning, Grades K-8: Moving Beyond Basic Facts and Memorization (Corwin Mathematics Series) 1st Edition by Jennifer M. Bay-Williams and John J. SanGiovanni

Call to Action Examples

1. School-based leaders:
    a. Select one teacher to support in planning a small group or individual intervention based on evidence from student work and progress monitoring data.
    b. Observe a small group or individual intervention to look for how the practice recommendations for intervention shared today are incorporated.
2. District-based leaders:
    a. Observe an intervention co-planning session and/or observe the teacher's instruction.
3. Establish a collaborative team with expertise in math instruction and curriculum to interrogate your current approach to mathematics intervention and create a plan to improve your schools/districts math intervention.
4. Do a fluency walk-through to see how the teachers or schools you support are approaching fluency instruction and practice.
5. Convene a fluency focus group to establish a collective understanding and vision for fluency instruction at your school or district.

To WHICH of these do you commit in the coming days, weeks, and months?

Resources for Ongoing Learning: Quick Links

- 13 Rules that Expire
- 12 Rules that Expire in Middle School
- Research on Timed Tests
- Math Recovery Resources

Intervention Resources

Where can I learn more about the different types of measures used to gauge and monitor student performance within intensive intervention?

- Mathematics Progress Monitoring in Intensive Interventions

What are some examples of mathematics intervention progress monitoring tools?

- Academic Progress Monitoring Tools Chart

Where can I find information about evidence-based mathematics intervention curriculum and resources?

- Evidence for ESSA
- National Center on Intensive Intervention
- What Works Clearinghouse

Where can I find information about evidence-based mathematics intervention strategies?

- Teaching LD
- Evidence Based Intervention Network

How do you incorporate effective problem-solving strategies within intensive intervention?

- Improving Mathematical Problem Solving in Grades 4 Through 8
- Using word problem schema and attack strategies

What are some evidence-based practices for fraction instruction within intensive interventions?

- Developing Effective Fractions Instruction for Kindergarten Through 8th Grade