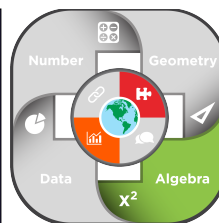


Teach students to utilize the structure of algebraic representations



Examining the underlying structure of an algebra problem (the algebraic representation), regardless of how the problem itself is communicated (for example, symbolic, numeric, verbal, or graphic), can help students see similarities among problems, connections, and solution paths. It also leads to development of understanding about algebraic expressions.

How to carry out the recommendation

1. Promote the use of language that reflects mathematical structure.
2. Encourage students to use reflective questioning to notice structure as they solve problems.
3. Teach students that different algebraic representations can convey different information about an algebra problem.

Potential roadblocks

1. I like to simplify mathematical language, and my students seem to respond positively to my use of simplified and informal language as well. Doesn't this approach make it easier for students than using complicated mathematical language?
2. My students race through problems. How do I get students to slow down, pause to ask themselves questions, and think about the problem?
3. Diagrams don't seem to be very useful to some of my students.

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References: Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., & Zbiek, R. M. (2019). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2015-4010). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance.

<https://ies.ed.gov/ncee/wwc/PracticeGuide/20>



Recommendation 2: Teach students to utilize the structure of algebraic representations

How to carry out the recommendation

1. Promote the use of language that reflects mathematical structure.

Using precise language is important for helping students understand algebraic structure. Teachers should use and model precise mathematical language that is related to the structure of the algebraic expression. (See Example 2.2 on page 18 in the practice guide referenced on the first page of this document for an example using the distributive property.) Doing so will not only help students develop an understanding of the structure but also lay a foundation for them to reflect, ask questions, and create appropriate representations. Teachers should restate students' responses, using the appropriate mathematical language, to help them grow in their ability to use precise language. Guiding students to use more precise mathematical language helps them focus on and build understanding of the mathematical validity of a problem.

Examples of imprecise language with the more precise restatements

Imprecise Language	Precise Mathematical Language
Take out the x .	Factor x from the expression. Divide both sides of the equation by x , with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.

Note. Taken from Example 2.3 on page 18 in the practice guide referenced on the first page of this document.

2. Encourage students to use reflective questioning to notice structure as they solve problems.

When students ask themselves questions about solving a problem, they are more likely to think about the structure of the problem and solution methods they might use. Teachers should model reflective questions using think-alouds focused on algebraic structure when demonstrating problem-solving. Additionally, providing lists of reflective questions can be helpful as students move from modeling to more independent work. Teachers should encourage students to work in pairs to develop and record their own reflective questions and then carry this practice to independent work.

Examples of reflective questions

- What am I being asked to do in this problem?
- How would I describe this problem using precise mathematical language?
- Is this problem structured similarly to another problem I've seen before?
- How many variables are there?
- What am I trying to solve for?
- What are the relationships between the quantities in this expression or equation?
- How will the placement of the quantities and the operations impact what I do first?

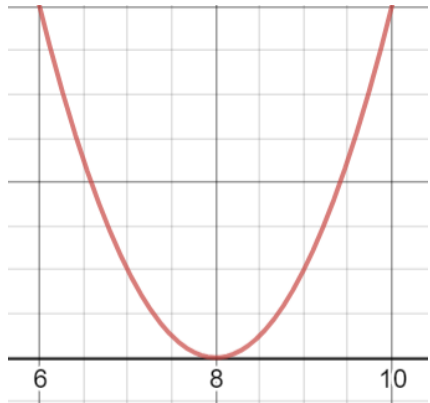
Note: Taken from Example 2.5 on page 20 in the practice guide referenced on the first page of this document.

Recommendation 2: Teach students to utilize the structure of algebraic representations

3. Teach students that different algebraic representations can convey different information about an algebra problem.

Teachers should encourage students to identify and explain different representations of the same problem in order to help them better understand the underlying mathematical structure. During whole-class instruction, teachers should provide a model of both the similarities and differences, showing how different representations of the same information might make solving problems easier. Teachers should also encourage students to see how different representations might better present information about the structure of the problem than others might. As needed, diagrams can help students visualize the problem structure, organize their thoughts about how to solve the problem, and transform the problem into another representation.

Using different representations to understand the structure of a problem

Representation	Example
Word problem	<i>The Browns want to purchase wall-to-wall carpeting for the two bedrooms in their house. One room is square with each side x feet long. The other room is adjoining but is rectangular. The short side shares a wall with the square bedroom. The long side is 16 feet long. The Browns will purchase 192 square feet to complete the job. What is the size of the square room?</i>
Diagram	x 16  x
Equation	<i>Area of a square s^2 Area of a rectangle $b \times h$ $x^2 + 16x = 194$ Find x and calculate x^2</i>
Graph	 <i>Find the intercept.</i>

Note: Adapted from Example 2.8 on pages 22–23 in the practice guide referenced on the first page of this document.

Recommendation 2: Teach students to utilize the structure of algebraic representations

Potential roadblocks and how to address them

Roadblock	Suggested Approach
<i>I like to simplify mathematical language, and my students seem to respond positively to my use of simplified and informal language as well. Doesn't this approach make it easier for students than using complicated mathematical language?.</i>	Using precise mathematical language ensures that students understand the algebraic concepts in the problem. Language can be simplified but should still clearly link to mathematical structure and ideas. When students use informal language, teachers should restate in correct mathematical language to build students' capacity to use precise language.
<i>My students race through problems. How do I get students to slow down, pause to ask themselves questions, and think about the problem?</i>	Using more challenging or less familiar problems may slow students down and require them to pay attention and notice structure. Another possibility is to have students answer reflective questions as they work or solve problems using multiple solution paths or representations.
<i>Diagrams don't seem to be very useful to some of my students.</i>	Students may not need diagrams to solve problems, but diagrams can help them notice and understand structural components. Make this explicit to students and continue to model the use of diagrams.



For more information on the research evidence and references to support this recommendation, or for more detailed explanation from the What Works Clearinghouse committee who developed this recommendation, please refer to the practice guide cited at the bottom of the first page of this document.