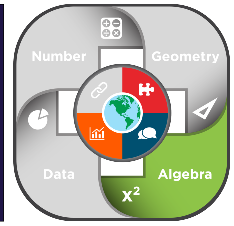
This document provides a summary of **Recommendation 1** from the WWC practice guide *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students*. Full reference at the bottom of this page.

Use solved problems to engage students in analyzing algebraic reasoning and strategies



Solving algebraic problems requires students to engage in abstract and critical thinking beyond the arithmetic work they experienced previously. In developing algebraic reasoning, students must analyze and process multiple pieces of information to find a solution to a problem. Examining and discussing possible sources of error and the multiple steps of solved problems will allow students to strengthen their algebraic reasoning skills.

How to carry out the recommendation

1. Have students discuss solved problem structures and solutions to make connections among strategies and reasoning.
2. Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors.
3. Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.

Potential roadblocks

1. I already use solved problems during whole-class instruction, but I'm not sure students are fully engaged with them.
2. I do not know where to find solved problems to use in my classroom and do not have time to make new examples for my lessons.
3. I'm worried that showing students incorrect solved problems will confuse them.

Reference: Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., & Zbiek, R. M. (2019). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2015-4010). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/20

How to carry out the recommendation

1. Have students discuss solved problem structures and solutions to make connections among strategies and reasoning.

Teachers should provide opportunities for students to examine solved problems through guiding questions. Teachers can have students explain the reasoning and discuss strategies used. They should keep students engaged and adjust guidance to meet the students' needs and the curricular goals. Guiding questions can be verbal or written. Examples of questions to facilitate student discussions of solved problems include the following:

- What were the steps to solve the problem?
- Could fewer steps have been used?
- Is this a strategy that would work in all cases? Why or why not?
- Is there another way to solve the problem?
- Is there a way to make the solution path more clear?
- What are the mathematical ideas connected to the solution path?

Note. Adapted from Example 1.1 on page 5 in the practice guide referenced on the first page of this document.

Teachers can deepen students' analysis and discussion by asking them to focus on the structure of the solved problem. Thinking about structure includes having students examine the mathematical features of a given problem as well as any mathematical relationships that might be present in an expression, representation, or equation. Questions to guide analysis and discussion of structure include the following:

- What quantities are present in this problem? Are they discrete or continuous?
- What operations and relationships among the quantities are shown in the problem? Is the problem expressing an equality or inequality?
- This problem uses parentheses. What do they indicate about the problem's structure?

Note. Adapted from Example 1.2 on page 6 in the practice guide referenced on the first page of this document.

2. Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors.

A variety of learning goals can be achieved through discussion of solved problems, so teachers should align solved problems with their lesson objectives. Sources of solved problems include previous student work, publisher-supplied examples, and those teachers create on their own. Options for including multiple solved problems in a lesson can include:

- Selecting solved problems that apply the same concept, but with varying degree of difficult, the presenting them from simplest to most complex application.
- Displaying multiple examples side by side to encourage identifying patterns in the solution steps across problems.
- Showing problems individually to encourage deeper discussion of each problem.

Note. Adapted from page 6 in the practice guide referenced on the first page of this document.

When presenting solved problems, teachers should include different solution paths as well as examples that contain errors. Once students examine several correctly solved problems, teachers can use incorrectly solved problems to help them identify and build understanding of concepts and solution processes. The following is a sample procedure for introducing incorrectly solved problems:

- Give students correct solved problems to study and discuss.
- Once students have an understanding of correct strategies and problems, present an incorrect solved problem to students.
- Display the incorrect solved problem by itself or side-by-side with a correct version of the same problem.
- Clearly label that the problem is solved incorrectly.
- Engage in discussion of the error and what steps led to the incorrect answer.

Note. Taken from Example 1.5 on page 9 in the practice guide referenced on the first page of this document.

For examples of ways to present and discuss solved problems, as well as how to align with various learning objectives, see pages 7–11 in the practice guide referenced on the first page of this document.

Parallel correct and incorrect solved problems, completing the square

Show students the correct and incorrect solved problems together. Ask students to describe the error (shown in bold text below), and guide students' discussion of why the error occurred.

	Correct solved problem	Incorrect solved problem: Strategic and reasoning error	Incorrect solved problem: Procedural error
Equation $x^2 + 6x = 27$	$x^2 + 6x = 27$ $x^2 + 6x + 9 = 27 + 9$ $(x + 3)^2 = 36$ $x + 3 = \pm 6$ $x + 3 = 6 \quad x + 3 = -6$ $x = 6 - 3 \quad x = -6 - 3$ $x = 3 \quad x = -9$	$x^2 + 6x = 27$ $x^2 + 6x + 9 = 27 + 9$ $(x + 3)^2 = 36$ $\mathbf{x + 3 = 6}$ $x = 6 - 3$ $x = 3$	$x^2 + 6x = 27$ $\mathbf{x^2 + 6x + 9 = 27}$ $(x + 3)^2 = 27$ $x + 3 = \pm 3\sqrt{3}$ $x = -3 + 3\sqrt{3} \quad x = -3 - 3\sqrt{3}$
Description of error	N/A	The student did not include the negative square root as a solution.	The student did not add 9 to both sides when completing the square. This means the new equation is not equivalent to the previous equation.

Questions to guide discussion of error	N/A	If a number squared is 36, what could the number be equal to? What properties of numbers and operations can we use to justify each step in the example?	If you add something to one side of the equation, what else do you need to do? Why? What property is this? The original equation tells us how $x^2 + 6x$ and 27 are related. What is that relationship? If 27 and $x^2 + 6x$ equal each other, then what should be the relationship between 27 and $x^2 + 6x + 9$?

Note. *Taken from Example 1.7 on page 11 in the practice guide referenced on the first page of this document.*

3. **Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.**

Using solved problems in a variety of contexts may lead to improved use of solution strategies. Teachers can use whole-group instruction to provide an overview of the solution strategy in a solved problem. Next, teachers can allow students to engage with the solved problem in pairs or small groups, including incorrectly solved problems to push students toward deeper, more critical analysis of the problem solution. Teachers can follow this pair or small-group work with whole-group discussion to correct misconceptions and ensure that all components of the problem have been scrutinized. Teachers should move from solved problems to incomplete solved problems, and then to independent practice.

Incomplete solved problems

$-x + 7 \geq 9$ $-x \geq 2$ __________	$3(x + 2) + 12 \leq 4(1 - x)$ __________ $3x + 18 \leq 4 - 4x$ $7x \leq -14$ $x \leq -2$	$2(x + 7) - 5(3 - 2x) \geq 7x - 4$ $2x + 14 - 15 + 10x \geq 7x - 4$ __________ $5x \geq -3$ $x \geq -\frac{3}{5}$

Note. *Taken from Example 1.10 on page 14 in the practice guide referenced on the first page of this document.*

Potential roadblocks and how to address them

Roadblock	Suggested Approach
I already use solved problems during whole-class instruction, but I'm not sure students are fully engaged with them.	Ask questions and be sure to include all students in the discussion to motivate them to think critically. Model think-aloud questions (for example, "Will the strategy work for every problem like this?" "Why or why not?" "How would you modify the solution, if you can, to make it clearer to other students?"). See Examples 1.1 and 1.2 in the practice guide referenced on the first page of this document. Additionally, use solved problems beyond whole-group settings to be sure that they are scrutinized in more meaningful ways. Include solved problems in class assessments to make whole-class work relevant to students. See Examples 1.9, 1.10, and 1.11 in the practice guide referenced on the first page of this document.
I do not know where to find solved problems to use in my classroom and do not have time to make new examples for my lessons	Find sample or worked problems in published curricular materials. Use past or current de-identified student work (such as homework, projects, assessments) as other examples, particularly for unique solution paths or incorrectly solved problems. Share across classrooms to increase your access.
I'm worried that showing students incorrect solved problems will confuse them.	Although students may not be familiar with examining incorrectly worked problems, doing so can help them build important critical-thinking skills. Be sure that students are clearly aware that a problem contains an error, then focus on the steps to understand the process and where it went wrong. Fully discuss each step to prevent confusion and build recognition and understanding of how the error occurred. See Examples 1.5, 1.6, and 1.7 in the practice guide referenced on the first page of this document.

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For more information on the research evidence and references to support this recommendation, or for more detailed explanation from the What Works Clearinghouse committee who developed this recommendation, please refer to the practice guide cited at the bottom of the first page of this document.