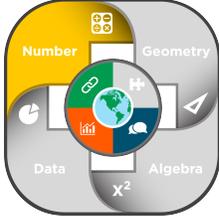


Teach number and operations using a developmental progression



Understanding what skills and knowledge children already possess is the starting place for instruction. A developmental progression can provide a road map of next steps. When teaching numbers and operations, for example, teachers should ensure that children progress through each level of the developmental progression. Within each level, children should begin by practicing with small sets of objects and progress to larger sets until they master the skills and knowledge at that level. Teachers unfamiliar with a developmental progression can view an example of one for number knowledge on page 13 of the practice guide referenced below.



How to carry out the recommendation
<ol style="list-style-type: none">1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.4. Encourage children to label collections with number words and numerals.5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Potential roadblocks
<ol style="list-style-type: none">1. I want to provide strong math foundations for my children, but I am not really comfortable with math myself.2. Each child in the class is at a different level in the developmental progression I am using to guide instruction.3. A child is stuck at a particular point in the developmental progression.

Reference: Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children* (NCEE 2014-4005). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. <https://ies.ed.gov/ncee/wwc/PracticeGuide/18>



How to carry out the recommendation

1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.

Children should be able to determine the number of objects in a small set without counting. This is known as *subitizing*. During classroom transitions, teachers may find small sets of objects (five or less) in the classroom and ask, “How many (object name) do you see?” After children can successfully identify three objects, they should be able to understand related sets that have the same number of objects. For example, three pencils and three erasers are two sets with the same number of objects. Once children have experience in recognizing sets of objects containing similar items, teachers can progress to sets with dissimilar items (for example, a set of three containing a pencil, a crayon, and an eraser). When developing subitizing, children may overgeneralize the term “three” or “four” to mean “many,” so teachers should identify a set of three objects as “three” and a set of four objects as “not three” to help children recognize the difference. Teachers then can challenge children to find sets of objects around the classroom that are “three” or “not three.” See Example 1 on page 16 of the practice guide referenced on the first page of this document.

2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.

One-to-one counting occurs when children count with number words in a consecutive order to determine the number of objects in a set, using only one number name for each unique item. For example, a child counting a set of pens points at a pen and says “one,” then points at the next pen and says “two,” and finally points at the last pen and says “three.” Children should begin with small sets (1 to 4 objects). Teachers should help them in realizing that the last number they count is the total number in a set. Then, children can progress to larger sets (4 to 10 objects). Teachers should also demonstrate that order does not affect the result by using objects around the classroom and counting them in different ways. For example, when counting pens, no matter which pen children start with, they will reach the same result. For another sample activity, see Example 2 on page 18 of the practice guide referenced on the first page of this document.

Errors are to be expected when children learn how to count. Common counting errors include sequence errors, coordination errors, and keeping-track errors, as well as skimming and not recognizing cardinality. In sequence errors, children confuse the sequence of numbers. In this case, teachers might ask the children to sing the number sequence. If children are skipping specific numbers, teachers should focus on practicing that part of the sequence. Coordination errors involve children labeling an object with more than one word or pointing to an object without counting it. Teachers might correct these errors by encouraging children to slow down and count each object once. When children make keeping-track errors, they may count an object twice. Teachers can help children differentiate counted objects from uncounted objects by having them pick up the counted objects and place them to the side. For more information on common counting errors and recommendations on how to remedy them, see Table 4 on page 19 of the practice guide referenced on the first page of this document.

3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.

Children can progress in making meaningful comparisons of sets by, for example, identifying “more” and “fewer.” First, teachers should present children with two sets of objects, one of which is obviously larger, and ask them which set has more or fewer. Next, teachers can demonstrate that, the further in the counting sequence children count, the larger the numbers become. The sample chart below provides a visual of this increase in size or quantity and can be helpful. Real-world examples, such as counting to determine who has more points in a game, can also help children increase their ability to compare quantities. Also helpful is knowing what number comes next without counting (number-after relations). Teachers can reinforce this knowledge by asking children such questions as “This is the 6th pen, so the next pen will be how many?”

Sample cardinality chart

									●
								●	●
							●	●	●
						●	●	●	●
					●	●	●	●	●
				●	●	●	●	●	●
			●	●	●	●	●	●	●
		●	●	●	●	●	●	●	●
	●	●	●	●	●	●	●	●	●
●	●	●	●	●	●	●	●	●	●
1	2	3	4	5	6	7	8	9	10

Note. Taken from page 20 of the practice guide referenced on the first page of this document.

4. Encourage children to label collections with number words and numerals.

Numerals are a way to represent quantity. To help children understand this, teachers should pair number words with numerals. For example, “3,” “three,” and “three objects” have the same meaning. Teachers can label sets of objects that children see in the classroom. For example, a set of three pens can be labeled “3, •••, three.” The three dots provide a scaffolding and represent the quantity for children who do not recognize numerals yet. See Example 3 on page 22 of the practice guide referenced on the first page of this document for an additional activity.

5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Children should proceed to developing an understanding of the effects of altering the number of objects in a set as they prepare for math problem-solving. To start, children can remove or add one object, recount, and review how the number of objects has changed. They can also use counting strategies in problem-solving activities in the classroom, such as counting the number of groups to determine how many whiteboards to hand out.

After children have had opportunities to explore adding and subtracting objects from sets, teachers can move to situations in which the final results are hidden from view. For example, teachers can show children a set of four pens, cover the pens with a cloth, take one pen from underneath the cloth, and then ask the children to determine how many pens are left under the cloth. Once the children decide how many objects are left, teachers can remove the cloth and have the children count to see if they solved the problem correctly. Snack time is also a great opportunity for children to apply counting skills. Teachers can ask, “How many will you have after you eat one snack item?” or, “How many will you have after your friend gives you one snack item?” to have children problem-solve without seeing the end set.

Recommendation 1: Teach number and operations using a developmental progression

Potential roadblocks and how to address them

Roadblock	Suggested Approach
<i>I want to provide strong math foundations for my children, but I am not really comfortable with math myself.</i>	Teachers who feel less comfortable with math should base classroom projects on real-world examples. Setting up a toy store in the classroom provides a more comfortable setting in which to integrate lessons. Any activity that is of interest and involves counting presents an opportunity to build children's math skills.
<i>Each child in the class is at a different level in the developmental progression I am using to guide instruction</i>	Teachers might split children into groups, using the developmental progression to create groups of children at a similar level. Dividing children into groups allows teachers to assign tasks based on each group's level of proficiency. Teachers can also create groups containing children at diverse levels of proficiency. This allows for children at a higher level to model a skill for others.
<i>A child is stuck at a particular point in the developmental progression</i>	If children are stuck, they have likely not yet mastered a skill from an earlier level in the developmental progression. Teachers should use the developmental progression to help identify the unmastered skill and provide opportunities for children to practice it further, before returning to the point at which they were stuck.



For more information on the research evidence and references to support this recommendation, or for more detailed explanation from the What Works Clearinghouse committee who developed this recommendation, please refer to the practice guide cited at the bottom of the first page of this document.