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| Alignment of the ACT plus writing to Nebraska Statewide Content StandardsAugust 2018 | MathematicsThis document summarizes the alignment evidence that the Nebraska Department of Education (NDE) has collected to support the connection between the ACT plus Writing and Nebraska’s College and Career Ready Standards in Mathematics. This evidence comes directly from Nebraska educators who participated in activities designed to conduct this evaluation and is provided to guide educators in their understanding how the content of the ACT samples from the knowledge, skills, and abilities included within the Nebraska Content Standards. |

**Alignment Evidence: Collecting, Organizing, and Evaluating**

With the adoption of the ACT as the statewide High School assessment for English Language Arts (ELA), Mathematics, and Science, the Nebraska Department of Education (NDE) sought direct information as to how the knowledge, skills, and abilities measured by the ACT aligned to those specified in the statewide content standards. To gather this information, Nebraska educators identified connections between the ACT and the statewide content standards in two ways:

* The first approach was a review of three forms of the ACT and an evaluation of the alignment of each item to the Nebraska standards. The purpose of this approach was to see how (a sample of) actual test items were targeting the same knowledge, skills, and abilities outlined in the statewide standards. Three forms of the ACT were reviewed as a sample of the type of items that could be expected on any given form of the test. This study was conducted in the summer of 2017.
* The second approach was an alignment of the ACT measurement targets (e.g., standards, reporting area descriptors) to the statewide content standards. The purpose of this approach was to evaluate the connection between the full range of knowledge, skills, and abilities (that could be measured by any given item on the ACT) to the statewide content standards. This study was conducted in early 2018.

The purpose of this document is to summarize the results of each study in a way that highlights how the ACT samples from across the full set of Nebraska’s College and Career Ready Standards for Mathematics. As noted in the results, some areas of the standards (indicators) are covered to a greater extent than others.

**How to Understand the Results**

* The results are presented in the tables below by Domain, grade level expectation (GLE), and curricular indicator.
* The results describe how each part of the Nebraska standards were connected to the areas of the ACT Mathematics Test (see pages 9-13 of this document).
	+ The results for the Standards alignment indicate how the indicators connected to the different areas of the ACT Mathematics test
	+ The results for the item alignment indicate how the ACT test content (multiple-choice items) connected to the indicators, GLEs, and Domains.

Overall, 79% of the ACT Mathematics Test content was linked to the 11th grade Mathematics Standards. However, some items were linked to multiple indicators. Therefore, the total percentages in the Item Alignment column may slightly overestimate the total percentage of content that was directly linked to these standards.

**ACT Content linked to Nebraska’s College and Career Ready Standards**

| **Academic Content Standards [Category/GLE/Curricular Indicator]** | **Measurement Target (Standards) Alignment** | **Item Alignment** |
| --- | --- | --- |
| **MA 11.1 NUMBER: Students will communicate number sense concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.** | **71% of the indicators were linked to specific areas of the ACT Mathematics Test** | **86% of the indicators were directly measured by the ACT Mathematics test (26% of all multiple-choice items).** |
|  | **MA.11.1.1 Numeric Relationships: Students will demonstrate, represent, and show relationships among the subsets of real numbers and the complex number system.** | **66% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **50% of the Indicators were covered by items on the ACT Mathematics test (5% of all multiple-choice items).** |
|  |  | MA 11.1.1.a Compare and contrast subsets of the complex number system, including imaginary, rational, irrational, integers, whole, and natural numbers. | Not covered | 2% of the multiple-choice |
|  |  | MA 11.1.1.b Recognize that closure properties apply to the subsets of the complex number system, under the standard operations. | Linked to the *Number and Quantity* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.1.1.c Use drawings, words, and symbols to explain the effects of operations such as multiplication and division on the magnitude of quantities in the real number system, including powers and roots (e.g., if you take the square root of a number, will the result always be smaller than the original number?). | Linked to the *Number and Quantity* and *Algebra* areas of the ACT Mathematics test | 3% of the multiple-choice |
|  | **MA 11.1.2 Operations: Students will compute with real and complex numbers.** | **75% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **100% of the Indicators were covered by items on the ACT Mathematics test (16% of all multiple-choice items).** |
|  |  | MA 11.1.2.a Compute with subsets of the complex number system, including imaginary, rational, irrational, integers, whole, and natural numbers. | Linked to the *Number and Quantity* area of the ACT Mathematics test | 5% of the multiple-choice |
|  |  | MA 11.1.2.b Simplify expressions with rational exponents. | Linked to the *Number and Quantity* and *Algebra* areas of the ACT Mathematics test | 2% of the multiple-choice |
|  |  | MA 11.1.2.c Select, apply, and explain the method of computation when problem solving using real numbers (e.g., models, mental computation, paper-pencil, or technology). | Not covered | 7% of the multiple-choice |
|  |  | MA 11.1.2.d Use estimation methods to check the reasonableness of real number computations and decide if the problem calls for an approximation (including appropriate rounding) or an exact number. | Linked to the *Number and Quantity* area of the ACT Mathematics test | 2% of the multiple-choice |
| **MA 11.2 ALGEBRA: Students will communicate algebraic concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.** | **91% of the indicators were linked to specific areas of the ACT Mathematics Test** | **78% of the indicators were directly measured by the ACT Mathematics test (42% of all multiple-choice items).** |
|  | **MA 11.2.1 Algebraic Relationships: Students will demonstrate, represent, and show relationships with functions.** | **88% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **62% of the Indicators were covered by items on the ACT Mathematics test (7% of all multiple-choice items).** |
|  |  | MA 11.2.1.a Define a function and use function notation. | Linked to the *Functions* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.1.b Analyze a relation to determine if it is a function given graphs, tables, or algebraic notation. | Linked to the *Functions* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.2.1.c Classify a function given graphs, tables, or algebraic notation, as linear, quadratic, or neither. | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.2.1.d Identify domain and range of functions represented in either algebraic or graphical form. | Linked to the *Functions* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.1.e Analyze and graph linear functions and inequalities (point-slope form, slope-intercept form, standard form, intercepts, rate of change, parallel and perpendicular lines, vertical and horizontal lines, and inequalities). | Linked to the *Algebra, Functions, and Geometry* areas of the ACT Mathematics test | 3% of the multiple-choice |
|  |  | MA 11.2.1.f Analyze and graph absolute value functions (finding the vertex, symmetry, transformations, determine intercepts, and minimums or maximums using the piecewise definition). | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.2.1.g Analyze and graph quadratic functions (standard form, vertex form, finding zeros, symmetry, transformations, determine intercepts, and minimums or maximums). | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.1.h Represent, interpret, and analyze inverses of functions algebraically and graphically. | Not covered | 1% of the multiple-choice |
|  | **MA 11.2.2 Algebraic Processes: Students will apply the operational properties when evaluating rational expressions, and solving linear and quadratic equations, and inequalities.** | **93% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **86% of the Indicators were covered by items on the ACT Mathematics test (25% of all multiple-choice items).** |
|  |  | MA 11.2.2.a Convert equivalent rates (e.g., miles per hour to feet per second). | Linked to the *Algebra* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.2.2.b Identify and explain the properties used in solving equations and inequalities. | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.2.c Simplify algebraic expressions involving integer and fractional exponents. | Linked to the *Algebra* area of the ACT Mathematics test | 2% of the multiple-choice |
|  |  | MA 11.2.2.d Perform operations on rational expressions (add, subtract, multiply, divide, and simplify). | Not covered | 2% of the multiple-choice |
|  |  | MA 11.2.2.e Evaluate expressions at specified values of their variables (polynomial, rational, radical, and absolute value). | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | 2% of the multiple-choice |
|  |  | MA 11.2.2.f Solve an equation involving several variables for one variable in terms of the others. | Linked to the *Algebra* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.2.g Solve linear and absolute value equations and inequalities. | Linked to the *Algebra* area of the ACT Mathematics test | 3% of the multiple-choice |
|  |  | MA 11.2.2.h Analyze and solve systems of two linear equations and inequalities in two variables algebraically and graphically. | Linked to the *Algebra* area of the ACT Mathematics test | 4% of the multiple-choice |
|  |  | MA 11.2.2.i Perform operations (addition subtraction, multiplication, and division) on polynomials. | Linked to the *Algebra* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.2.j Factor polynomials to include factoring out monomial terms and factoring quadratic expressions. | Linked to the *Algebra* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.2 k. Recognize polynomial multiplication patterns and their related factoring patterns (e.g., (a + b)2 = a2 + 2ab + b2, a2 – b2 = (a + b) (a - b)). | Linked to the *Algebra* area of the ACT Mathematics test | 2% of the multiple-choice |
|  |  | MA 11.2.2.l Make the connection between the factors of a polynomial and the zeros of a polynomial. | Linked to the *Algebra* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.2.2.m Combine functions by composition and perform operations (addition, subtraction, multiplication, division) on functions. | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.2.2.n Solve quadratic equations involving real coefficients and real or imaginary roots. | Linked to the *Algebra* area of the ACT Mathematics test | 5% of the multiple-choice |
|  | **MA 11.2.3 Applications: Students will solve real-world problems involving linear equations and inequalities, systems of linear equations, quadratic, exponential, square root, and absolute value functions.** | **100% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **100% of the Indicators were covered by items on the ACT Mathematics test (10% of all multiple-choice items).** |
|  |  | MA 11.2.3.a Analyze, model, and solve real-world problems using various representations (graphs, tables, linear equations and inequalities, systems of linear equations, quadratic, exponential, square root, and absolute value functions). | Linked to the *Algebra* and *Functions* areas of the ACT Mathematics test | 10% of the multiple-choice |
| **MA 11.3 GEOMETRY: Students will communicate geometric concepts and measurement concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.** | **33% of the indicators were linked to specific areas of the ACT Mathematics Test** | **54% of the indicators were directly measured by the ACT Mathematics test (28% of all multiple-choice items).** |
|  | **MA 11.3.1 Characteristics: Students will identify and describe geometric characteristics and create two- and three-dimensional shapes.** | **12% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **88% of the Indicators were covered by items on the ACT Mathematics test (21% of all multiple-choice items).** |
|  |  | MA 11.3.1.a Know and use precise definitions of ray, line segment, angle, perpendicular lines, parallel lines, and congruence based on the undefined terms of geometry: point, line and plane. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.1.b Prove geometric theorems about angles, triangles, congruent triangles, similar triangles, parallel lines with transversals, and quadrilaterals using deductive reasoning. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.1.c Apply geometric properties to solve problems involving similar triangles, congruent triangles, quadrilaterals, and other polygons. | Not covered | 10% of the multiple-choice |
|  |  | MA 11.3.1.d Identify and apply right triangle relationships including sine, cosine, tangent, special right triangles, and the converse of the Pythagorean Theorem. | Linked to the *Functions* and *Geometry* areas of the ACT Mathematics test | 5% of the multiple-choice |
|  |  | MA 11.3.1.e Create geometric models to visualize, describe, and solve problems using similar triangles, right triangles, and trigonometry | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.1.f Know and use precise definitions and terminology of circles, including central angle, inscribed angle, arc, intercepted arc, chord, secant, and tangent. | Not covered | 2% of the multiple-choice |
|  |  | MA 11.3.1.g Apply the properties of central angles, inscribed angles, angles formed by intersecting chords, and angles formed by secants and/or tangents to find the measures of angles related to the circle. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.1.h Sketch, draw, and construct appropriate representations of geometric objects using a variety of tools and methods which may include ruler/straight edge, protractor, compass, reflective devices, paper folding, or dynamic geometric software. | Not covered | No items directly linked |
|  | **MA 11.3.2 Coordinate Geometry: Students will determine location, orientation, and relationships on the coordinate plane.** | **45% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **36% of the Indicators were covered by items on the ACT Mathematics test (4% of all multiple-choice items).** |
|  |  | MA 11.3.2.a Derive and apply the midpoint formula. | Linked to the *Geometry* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.3.2.b Use coordinate geometry to analyze linear relationships to determine if lines are parallel or perpendicular. | Linked to the *Geometry* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.3.2.c Given a line, write the equation of a line that is parallel or perpendicular to it. | Linked to the *Geometry* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.3.2.d Derive and apply the distance formula. | Linked to the *Geometry* and *Numbers & Quantity* areas of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.3.2.e Use coordinate geometry to prove triangles are right, acute, obtuse, isosceles, equilateral, or scalene. | Not covered | No items directly linked |
|  |  | MA 11.3.2.f Use coordinate geometry to prove quadrilaterals are trapezoids, isosceles trapezoids, parallelograms, rectangles, rhombi, kites, or squares | Not covered | No items directly linked |
|  |  | MA 11.3.2.g Perform and describe positions and orientation of shapes under a single translation using algebraic notation on a coordinate plane. | Linked to the *Geometry*, *Algebra,* and *Functions* areas of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.3.2.h Perform and describe positions and orientation of shapes under a rotation about the origin in multiples of 90 degrees using algebraic notation on a coordinate plane. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.2.i Perform and describe positions and orientation of shapes under a reflection across a line using algebraic notation on a coordinate plane. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.3.2.j Perform and describe positions and orientation of shapes under a single dilation on a coordinate plane. | Not covered | No items directly linked |
|  |  | MA 11.3.2.k Derive the equation of a circle given the radius and the center. | Not Covered | No items directly linked |
|  | **MA 11.3.3 Measurement: Students will perform and compare measurements and apply formulas.** | **60% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **40% of the Indicators were covered by items on the ACT Mathematics test (3% of all multiple-choice items).** |
|  |  | MA 11.3.3.a Convert between various units of length, area, and volume (e.g., such as square feet to square yards). | Not covered | No items directly linked |
|  |  | MA 11.3.3.b Convert between metric and standard units of measurement. | Not covered | No items directly linked |
|  |  | MA 11.3.3.c Apply the effect of a scale factor to determine the length, area, and volume of similar two- and three- dimensional shapes and solids. | Linked to the *Geometry* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.3.3.d Find arc length and area of sectors of a circle. | Linked to the *Geometry* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.3.3.e Determine surface area and volume of spheres, cones, pyramids, and prisms using formulas and appropriate units. | Linked to the *Geometry* area of the ACT Mathematics test | 2% of the multiple-choice |
| **MA 11.4 DATA: Students will communicate data analysis/probability concepts using multiple representations to reason, solve problems, and make connections within mathematics and across disciplines.** | **57% of the indicators were linked to specific areas of the ACT Mathematics Test** | **50% of the indicators were directly measured by the ACT Mathematics test (17% of all multiple-choice items).** |
|  | **MA 11.4.2 Analysis & Applications: Students will analyze data to address the situation.** | **45% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **36% of the Indicators were covered by items on the ACT Mathematics test (6% of all multiple-choice items).** |
|  |  | MA 11.4.2.a Identify and compute measures of central tendency (mean, median, mode) when provided data both with and without technology. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 3% of the multiple-choice |
|  |  | MA 11.4.2.b Explain how transformations of data, including outliers, affect measures of central tendency. | Not covered | 1% of the multiple-choice |
|  |  | MA 11.4.2.c Compare data sets and formulate conclusions. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.4.2.d Support conclusions with valid arguments. | Not covered | No items directly linked |
|  |  | MA 11.4.2.e Develop linear equations for linear models to predict unobserved outcomes using the regression line and correlation coefficient with technology. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 1% of the multiple-choice |
|  |  | MA 11.4.2.f Describe the shape, identify any outliers, and determine the spread of a data set. | Not covered | No items directly linked |
|  |  | MA 11.4.2.g Explain the impact of sampling methods, bias, and the phrasing of questions asked during data collection, and the conclusions that can rightfully be made. | Not covered | No items directly linked |
|  |  | MA 11.4.2.h Explain the differences between a randomized experiment and observational studies. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | No items directly linked |
|  |  | MA 11.4.2.i Using scatter plots, analyze patterns and describe relationships in paired data. | Linked to the *Statistics & Probability* area of the ACT Mathematics test1 | No items directly linked |
|  |  | MA 11.4.2.j Recognize when arguments based on data confuse correlation with causation. | Not covered | No items directly linked |
|  |  | MA 11.4.2.k Interpret data represented by the normal distribution, formulate conclusions, and recognize that some data sets are not normally distributed. | Not covered | No items directly linked |
|  | **MA 11.4.3 Probability: Students will interpret and apply concepts of probability.** | **100% of the Indicators were linked to specific areas of the ACT Mathematics test.** | **100% of the Indicators were covered by items on the ACT Mathematics test (11% of all multiple-choice items).** |
|  |  | MA 11.4.3.a Construct sample spaces and probability distributions. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 2% of the multiple-choice |
|  |  | MA 11.4.3.b Use appropriate counting techniques to determine the probability of an event. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 7% of the multiple-choice |
|  |  | MA 11.4.3.c Determine if events are mutually exclusive and calculate their probabilities in either case. | Linked to the *Statistics & Probability* area of the ACT Mathematics test | 2% of the multiple-choice |

**ACT Mathematics Test Information**

The text included here is taken directly from the ACT Technical Supplement[[1]](#footnote-1) and describes the reporting categories for the ACT Mathematics Test. Each reporting category is accompanied by a description of the knowledge, skills, and abilities measured as well as the approximate percentage of test items that make up this section.

Nine scores are reported for the ACT mathematics test: a total test score based on all 60 questions and eight reporting category scores. The eight reporting categories addressed in the mathematics test are Preparing for Higher Math, which includes separate scores for Number & Quantity, Algebra, Functions, Geometry, and Statistics & Probability, Integrating Essential Skills, and Modeling.

**Preparing for Higher Math** [57-60% of the test]

This reporting category captures the more recent mathematics that students are learning, starting when students begin using algebra as a general way of expressing and solving equations. This category is divided into the following five subcategories.

* **Number & Quantity** [7-10% of the test]

Coming into high school, students have some knowledge of the real number system. Because they have an understanding of and fluency with rational numbers and the four basic operations, they can work with irrational numbers by manipulating rational numbers that are close. Students are ready to move from integer exponents to rational exponents and are also ready to probe deeper into properties of the real number system. Students extend their knowledge to include complex numbers, which offer the solutions to some simple equations that have no real-number solutions, and students learn to compute in this system. Students go further, exploring properties of complex numbers – again, learning more about real numbers. Students explore vectors and matrices and view them as number systems with properties, operations, and applications. Throughout high school, students are maturing in their understanding of quantity and its connections to measurement. Attending to the types of quantities and units can guide solution strategies and help avoid errors. Students work with derived quantities, and when modeling, they choose appropriate quantities to model.

* **Algebra** [12-15% of the test]

Students coming into high school build on their understanding of linear equations to make sense of other kinds of equations and inequalities: what their graphs look like, how to solve them, and what kinds of applications they have for modeling. They continue to make sense of expressions in terms of their parts in order to use their fluency strategically and to solve problems. Through repeated reasoning, students develop a general understanding of solving equations as a process that provides justification that all the solutions will be found. Students extend their proficiency to equations such as quadratic, polynomial, rational, radical, and systems, integrating an understanding of solutions in terms of graphs. Families of equations have properties that make them useful for modeling. Polynomials form a system analogous to adding, subtracting, and multiplying integers; solutions of polynomial equations are related to factors of a polynomial. Students recognize these relationships in applications and create expressions, equations, and inequalities to represent problems and constraints. Students see rational expressions as a system analogous to rational numbers, apply the binomial theorem, and solve simple matrix equations that represent systems of linear equations.

* **Functions** [12-15% of the test]

Functions have been with students since their early years: consider the counting function that takes an input of “seven” and gives “eight” and an input of “twelve” to give “thirteen.” Understanding general properties of functions will equip students for problem solving with new functions they create over their continued studies and careers. Functions provide a framework for modeling real-world phenomena, and students become adept at interpreting the characteristics of functions in the context of a problem and become attuned to differences between a model and reality. Some functions accept all numbers as inputs, but many accept only some numbers. Function notation gives another way to express functions that highlights properties and behaviors. Students work with functions that have no equation, functions that follow the pattern of an equation, and functions based on sequences, which can even be recursive. Students investigate particular families of functions – like linear, quadratic, and exponential – in terms of the general function framework: looking at rates of change, algebraic properties, and connections to graphs and tables, and applying these functions in modeling situations. Students also examine a range of functions like those defined in terms of square roots, cube roots, polynomials, exponentials, logarithms, and trigonometric relationships, and also piecewise-defined functions.

Students see solving an equation in terms of an inverse function. Students have seen shifts in graphs due to parameter changes, but now they develop a unified understanding of translations and scaling through forms such as f(x-c), f(x) + c, a f(x) and f(-ax). Students connect the trigonometry of right triangles to the unit circle to make trigonometric functions, and they explore algebraic relationships among these functions. They use these functions to model periodic behavior.

Students graph rational functions and learn about asymptotes. They compose functions in other ways besides translation and scaling, going deeper into how inverse functions apply to solving equations with more than one solution, in particular for trigonometric functions. They explore algebraic properties of trigonometric functions such as angle addition properties.

* **Geometry** [12-15% of the test]

In high school, students add depth to what they know about transformations, reflections, rotations, and dilations and add precision to their understanding of congruence, similarity, and symmetry. Students justify by using definitions and theorems, tying in calculations and diagrams, considering cases, understanding general versus specific statements, applying counterexamples, and putting statements together into coherent arguments. Students make constructions, solve problems, and model with geometric objects. Informal arguments give a chain of reasoning that leads to formulas for the area of a circle and then on to volume of cylinders, pyramids, and cones. Students understand trigonometric ratios as functions of the angle through the lens of similar triangles, and they solve right-triangle problems. All these results transfer to the coordinate plane, where analytic treatment of distance allows students to derive conditions for parallel and perpendicular lines, to split a line segment into pieces with a given ratio of lengths, to find areas, and to develop equations for circles and for parabolas that have a directrix parallel to an axis.

Students go further into trigonometry, deriving a formula for the area of a general triangle in terms of side lengths and the sine of an angle, moving on to the law of sines and law of cosines, which give straightforward answers to questions about nonright triangles. Students derive equations for ellipses and hyperbolas. Students use Cavalieri’s principle to justify formulas, such as the formula for volume of a sphere.

* **Statistics & Probability** [8-12% of the test]

In high school, students add to their understanding of distributions of a single quantity, describing center and spread with statistics and interpreting these in the context of the data. Students describe distance from the mean in standard-deviation units, and for distributions that look approximately normal, they approximate the probability of events using probabilities from the normal distribution.

Before high school, students have used two-way tables and scatter plots to look at relationships between different quantities and have used linear functions to model relationships that look linear. Now students pay more attention to informal model fit and use other functions to model relationships; they use models for prediction, interpreting characteristics of the model in the context of the data, and interpreting the correlation coefficient for linear models. From two-way tables, students interpret relative probabilities (including joint, marginal, and conditional relative frequencies but not tied to these terms) and relate these to probabilities. Students look for association and distinguish correlation and causation.

Students learn about the role of randomness in sample surveys, experiments, and observational studies. Students use data to estimate population mean or proportion and make informal inferences based on their maturing judgment of likelihood. They can compare qualities of research reports based on data and can use simulation data to make estimates and inform judgment.

Before high school, students have tacitly used independence, but now the idea is developed with a precise definition. Students relate the sample space to evens defined in terms of “and,” “or,” and “not,” and calculate probabilities, first using empirical results or independence assumptions, and later using the ideas of conditional probability. Students understand the multiplicative rule for conditional probability and study permutations and combinations as a tool for counting. Students model a sample space with a “random variable” by giving a numerical value to each event. Students apply expected value and probability to help inform decisions.

**Integrating Essential Skills** [40-43% of the test]

Students learn some of the most useful mathematics before eighth grade: rates and percentages; proportional relationships; area, surface area, and volume; average and median; expressing numbers in different ways; using expressions to repeat quantities and equations to capture relationships; and other topics. Each year, students should grow in what they can accomplish using learning from prior years. Students should be able to solve problems of increasing complexity, combine skills in longer chains of steps, apply skills in more varied contexts, understand more connections, and increase fluency. In order to assess whether student shave had appropriate growth, questions in this reporting category are at a cognitive level of at least depth of knowledge 2 for high school students, with a significant portion at depth of knowledge level 3.

**Modeling** [ > 27% of the test]

Modeling uses mathematics to represent with a model an analysis of an actual, empirical, situation. Models often help us predict or understand the actual. However, sometimes knowledge of the actual helps us understand the model, such as when addition is introduced to students as a model of combining two groups. The Modeling reporting category represents all questions that involve producing, interpreting, understanding, evaluating, and improving models. Each modeling question is also counted in the other appropriate reporting categories above. Thus, the Modeling reporting category is an overall measure of how well a student uses modeling skills across mathematical topics.

1. https://www.act.org/content/dam/act/unsecured/documents/ACT-Technical-Manual-Supplement.pdf [↑](#footnote-ref-1)