



Unfinished Learning Series Math Community of Practice

Session 2: Assessing and Diagnosing Unfinished Learning in Math

7.EE.B.4.a (7.2.3.c)

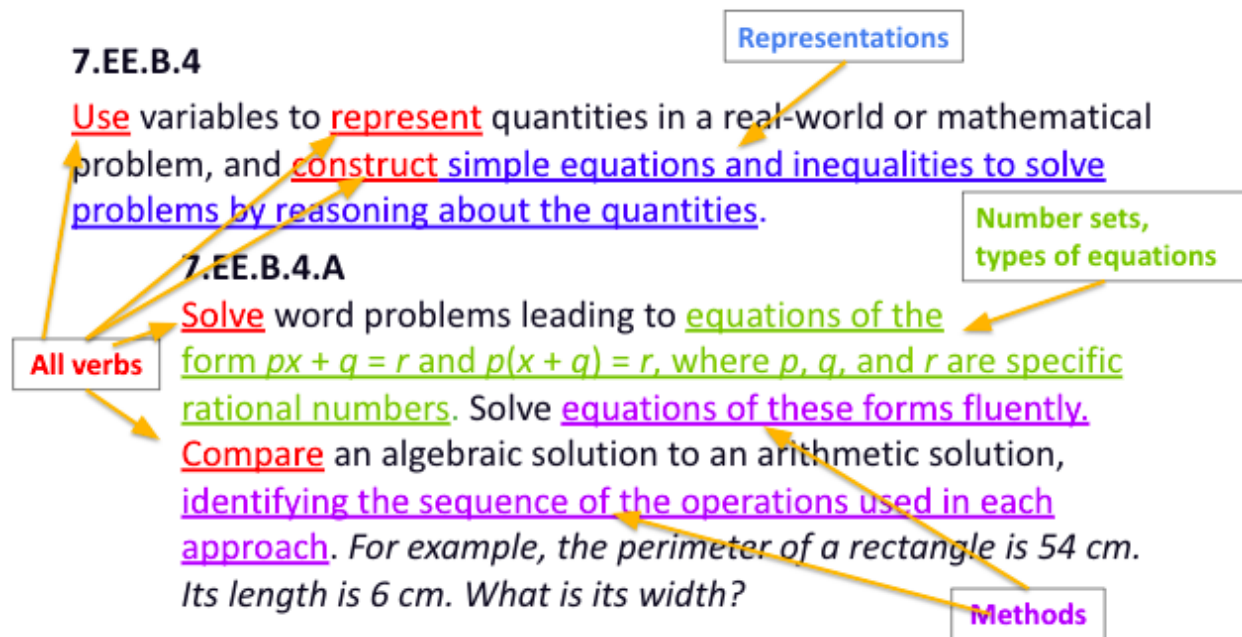
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Standard Analysis Case Study



This is Ms. Hutchins first year teaching seventh grade math at Brightwood Academy. Prior to teaching seventh grade, she taught ninth grade social studies for one year at another school. Ms. Franklin, the grade 7 content lead, is facilitating a planning meeting with the grade 7 team. The grade 7 team is preparing to teach a topic on fraction comparison. Before the meeting Ms. Franklin has requested the teachers review and annotate the grade level standard, 7.EE.B.4.a addressed in their upcoming topic.

Ms. Hutchins comes to the meeting prepared with her standard annotations:



Ms. Franklin: What did you notice this standard was targeting?

Ms. Hutchins: It's targeting constructing and solving equations.

Mr. Leonard: It's comparing an arithmetic solution to an algebraic solution after solving the equations.

Ms. Hutchins: I also noticed the specific types of word problems should be solved. These word problems must have rational numbers and involve the distributive property. If we had not unpacked this standard, I might have used only integers.



Ms. Franklin: That's a really important observation. That's why this is an important step in planning our lessons. We must attend to the language of the standards. Did anyone notice anything else as they unpacked this standard?

Mr. Leonard: I noticed that students are expected to be able to solve equations fluently.

Ms. Hutchins: Oh...I did not notice that. I'm so glad you brought that up. That means we need to need to spend some time practicing solving equations.

Ms. Franklin: That is true but we can support students with solving equations fluently by leveraging the sprints that are available to us. We can also be strategic about the selection of problems we are assigning to ensure students have adequate practice solving equations.

Ms. Hutchins: Okay, that makes sense. How are they expecting students to compare an arithmetic solution to an algebraic solution. I am not sure what that even means.

Ms. Franklin: Let's think about that as a group. What would students need to be able to do to compare an arithmetic solution to an algebraic solution?

Mrs. Williams: Well they would need to know how to show the necessary mathematics needed to arrive at the solution without using algebra. Arithmetic just means to solve with numbers.

Mr. Leonard: Yeah, it's more than that. Depending on the problem they need to show all the operations needed to arrive at the solution arithmetically but then they also have to show the same steps to solving the problem algebraically. This is a good idea to help students make the connection between these two approaches.

Ms. Hutchins: So if I understand what you are all saying, then students are essentially comparing two approaches where they arrive at the same solution. The difference is the algebraic method contains a variable representing the unknown quantity.

Ms. Franklin: Yes, that is correct. What else do we notice the arc of learning for this topic?

Mrs. Williams: We incorporate some geometry with solving equations. We use the various angle relationships like linear pairs, complementary and supplementary angles to enforce the skill of solving equations.

Ms. Hutchins: I really like how they transition to using algebra and incorporating it within the geometry.

Ms. Franklin: It's so great to see everyone has come prepared for this meeting. You all are doing great. Now let's see how deeply you analyzed this topic? What is happening after students use the angle relationships to solve equations?

Ms. Hutchins: They then explore solving inequalities.

Mrs. Williams: Very true. They also explore graphing the solutions to inequalities on a number line. The great thing I like about those later lessons is that students also have to interpret the solutions of the inequalities within the context of the problems.

Mr. Leonard: It will be important for us to have students include at least one number to the left and the right of the initial value on the number line. This will help them better interpret the solutions in order to answer the real world application problem.

Ms. Franklin: That's a great point and a perfect segway to my next question. You mentioned real-world applications, so we know one aspect of rigor we should expect to see play out in this topic. Are there any other aspects of rigor evident from the language of the standard?

Mrs. Williams: I would say procedural since we saw the word fluently within the standard. I'm thinking conceptual as well but I am not sure.

Ms. Hutchins: That's where I landed as well.

Ms. Franklin: Take a look at this standard in the LDOE Companion Document for Teachers to see the aspects of rigor the standard is referring to. Let's also look for the pre-requisite standards aligned to our grade level standard.

Ms. Hutchins: So this standard contains all three aspects of rigor. Conceptual, Procedural and Application. I do not think I would have figured this all out on my own. I'm so glad we are able to do this work as a team.

Ms. Franklin: Yes, the aspect of rigor this standard is targeting is conceptual understanding, procedural skill and fluency, and application. Conceptually, students will use variables to represent quantities to construct equations. When solving the equations, they will compare the arithmetic solution to the algebraic solution to make the connection between both approaches. We will give them the time to become fluent with solving these equations. Then they will apply these skills by constructing and solving equations that represent real world scenarios.

Mr. Leonard: This may be a little off topic at this point but I was wondering how tape diagrams fit into this module?

Mrs. Williams: They are important models students have used for years now. Tape diagrams could be a great scaffold for students since it is something they are familiar with. We should use tape diagrams.

Ms. Hutchins: I never learned to use tape diagrams. I was just taught to perform the inverse operation when solving equations. It was just a procedure I memorized.

Mr. Leonard: Same here but tape diagrams are models the standards expect students to use to model real world scenarios. They used tape diagrams often in 6th grade when discussing ratios and proportions. They used it less this year but it was still present at times to help students make that connection to prior learning.

Mrs. Williams models a problem using a tape diagram then solves the same problem algebraically.

Ms. Hutchins: Now I see how tape diagrams can be used to solve equations. I would have never thought to take that approach.

Ms. Franklin: Those are helpful reflections on the aspects of rigor and how tape diagrams can be used as a scaffold to support student understanding of the grade level standard. Let's complete an assessment task aligned to 7.EE.B.4.a so we have a more clear picture of the standard and what students are expected to know and show.

Team Know/Show Chart for 7.EE.B.4.a

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

| Aspect of Rigor | Mathematical Objects |
|--|---|
| <p>What <u>type</u> of knowledge are students required to demonstrate?</p> <ul style="list-style-type: none"> <input type="checkbox"/> Conceptual understanding <input type="checkbox"/> Procedural fluency <input type="checkbox"/> Application | <p>What are the mathematical <u>objects</u> involved? (i.e types of equations, numbers sets, models etc.)</p> <p>Variables to represent quantities Equations must include specific rational numbers Equations must be include more than one operation</p> <p>What actions are students expected to <u>do</u> with the mathematical objects? (add, subtract, solve, determine, compare, write, etc.)</p> <p>Students are expected to use variables, construct equations and solve equations that represent real world scenarios and compare the arithmetic solution to the algebraic solution of an equation</p> |
| Know | Show |
| <p>What are the important <i>concepts</i> that frame the standard, and what are students expected to <i>understand about the concepts</i>?</p> <ul style="list-style-type: none"> ● Use variables to represent unknown quantities ● Perform operations with rational numbers ● Simplify equations involving the distributive property ● Solve multi-step equations ● Create an equation from a real world scenario ● Solve an equation arithmetically ● Solve an equation algebraically ● Explain the steps taken to solve an equation | <p>What are students expected to <i>do or show</i>, and with <i>what</i> to demonstrate the understanding of the standard?</p> <ul style="list-style-type: none"> ● Create equations using variables ● Simplify equations by isolating the variable on one side of the equation ● Simplify rational numbers within an equation ● Explain the connection between the arithmetic and algebraic solution of an equation ● Translate a real world scenario into an equation ● Solve equations using the properties of equality ● Interpret the solution to an equation |

Learning Progression Analysis Protocol

Step 1: Identify the pre-requisite standards connected to the grade level standard in the [Nebraska Essential Instructional Guide](#), and/or using the Achieve the Core [Coherence Map](#).

Step 2: Read the prerequisite standards. Annotate the following...

- Any unfamiliar language or questions you have about the standard
- Aspect of rigor the standard is targeting (conceptual understanding, procedural fluency, application)
- Concept(s) students are expected to understand or know
- What students are expected to do or show
- Strategies and models students are expected to use
- Specifics or limits specified in the standard
- Connections to the grade level standard

Step 3: Read about the prerequisite standards in the [LDOE Companion Document for Teachers](#).

Step 4: Complete assessment tasks aligned to the prerequisite standard.

Step 5: Compare your work with a colleague or the exemplar response. Discuss the following:

- What do you students need to understand and be able to do to be successful on these tasks?
- How do the concepts and skills students need to be successful prepare them for grade level instruction?
- What misconceptions or incomplete understandings may this task reveal?

Ms. Hutchins' Prerequisite Standard Annotations & Example Assessment Tasks

| 7.EE.B.4.a Progression Analysis Example | |
|--|--|
| <p>6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>7.EE.B.4.a - This is foundational for students to use variables to represent unknown quantities and write the expression to represent a real-world or mathematical problem.</p> | <p>6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.</p> <p>7.EE.B.4.a - This is foundational for students to write and solve equations that are one step or two step equations involving rational numbers.</p> |

Part C: 6.EE.B.7

7.Allison is buying movie tickets for her and her friends. The price of each ticket is \$8.50 and she spent a total of \$42.50. Write and solve an equation to determine how many movie tickets Allison purchased.

Let t = the number of tickets purchased

$$\begin{array}{r} 8.50t = 42.50 \\ 8.50 \quad 8.50 \end{array}$$

$$t = 5$$

Allison brought a total of 5 tickets

8.Lilla's water bottle has some water in it already, but she wants to completely fill it up before going to practice. She adds 21 ounces of water to completely fill the 40-ounce bottle. Write and solve an equation to determine the amount of water that was in Lilla's bottle before she completely filled the bottle.

Let n = number of ounces of water

$$\begin{array}{r} n + 21 = 40 \\ -21 \quad -21 \\ \hline n = 19 \end{array}$$

Lilla had 19 ounces of water in her water bottle

9.Brandon's mother will allow him to spend no more than \$90 on video games. Each video game costs \$40. Write and solve an inequality that determines how many video games Brandon is allowed to purchase.

Let v = the number of video games Brandon can purchase

$$\begin{array}{r} 40v \leq 90 \\ 40 \quad 40 \end{array}$$

$$v \leq 2.25$$

Brandon will be able to purchase 2 video games.

Eureka Acceleration Tool

Grade 7 Module 3 Topic B

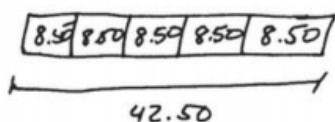
Diagnostic Assessment: Grade 7

Eureka Module 3, Topic B

Student One

Part C: 6.EE.B.7

7. Allison is buying movie tickets for her and her friends. The price of each ticket is \$8.50 and she spent a total of \$42.50. Write and solve an equation to determine how many movie tickets Allison purchased.



$$8.50 + 8.50 + 8.50 + 8.50 + 8.50 = 42.50$$

8. Lilla's water bottle has some water in it already, but she wants to completely fill it up before going to practice. She adds 21 ounces of water to completely fill the 40-ounce bottle. Write and solve an equation to determine the amount of water that was in Lilla's bottle before she completely filled the bottle.

$$\begin{array}{r} x - 21 = 40 \\ + 21 \quad + 21 \\ \hline \end{array}$$

9. Brandon's mother will allow him to spend no more than \$90 on video games. Each video game costs \$40. Write and solve an inequality that determines how many video games Brandon is allowed to purchase.

$$\begin{array}{r} 40x = 90 \\ \hline 40 \quad 40 \end{array}$$

Brandon can purchase 2 games.

$$x = 2.25$$

| Strengths | Unfinished Learning |
|--|--|
| <p>The student interprets the scenario and uses a tape diagram, equation and inequality to model the situations.</p> <p>The student performs the inverse operation to isolate the variable on side on the equation/inequality.</p> | <p>The student does not create an equation for the first problem(arithmetic solution).</p> <p>The student used the incorrect operation to represent one scenario but correctly solved their created equation.</p> <p>The student work does not use an inequality symbol to describe the relationship in one scenario</p> |

Diagnostic Assessment: Grade 7
Eureka Module 3, Topic B

Student Two

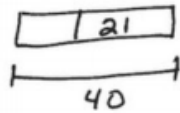
Part C: 6.EE.B.7

7. Allison is buying movie tickets for her and her friends. The price of each ticket is \$8.50 and she spent a total of \$42.50. Write and solve an equation to determine how many movie tickets Allison purchased.

$$\frac{8.50x}{8.50} = \frac{42.50}{8.50} \quad 5 \text{ tickets}$$

$$x = 5$$

8. Lilla's water bottle has some water in it already, but she wants to completely fill it up before going to practice. She adds 21 ounces of water to completely fill the 40-ounce bottle. Write and solve an equation to determine the amount of water that was in Lilla's bottle before she completely filled the bottle.



$$n + 21 = 40$$

$$\begin{array}{r} -21 \quad -21 \\ \hline n = 19 \end{array}$$

9. Brandon's mother will allow him to spend no more than \$90 on video games. Each video game costs \$40. Write and solve an inequality that determines how many video games Brandon is allowed to purchase.

$$\frac{40g}{40} < \frac{90}{40}$$

$$g < 2.25 \quad 2 \text{ games}$$

| Strengths | Unfinished Learning |
|--|---|
| <p>The student interpreted each scenario and correctly created an equation/inequality</p> <p>The student used the inverse operation to solve each equation/inequality</p> <p>The student drew tape diagrams to represent one of the scenarios.</p> | <p>The student work does not use the correct inequality symbol to represent the scenario.</p> |

Diagnostic Assessment: Grade 7
Eureka Module 3, Topic B

Student Three

Part C: 6.EE.B.7

7. Allison is buying movie tickets for her and her friends. The price of each ticket is \$8.50 and she spent a total of \$42.50. Write and solve an equation to determine how many movie tickets Allison purchased.

$$\begin{array}{r} 8.50 \\ \times 3 \\ \hline 25.50 \end{array} \quad \begin{array}{r} 8.50 \\ \times 4 \\ \hline 34.00 \end{array} \quad \begin{array}{r} 8.50 \\ \times 5 \\ \hline 42.50 \end{array} \quad \text{Allison can purchase 5 tickets.}$$

8. Lilla's water bottle has some water in it already, but she wants to completely fill it up before going to practice. She adds 21 ounces of water to completely fill the 40-ounce bottle. Write and solve an equation to determine the amount of water that was in Lilla's bottle before she completely filled the bottle.

$$\begin{array}{r} W - 40 = 21 \\ + 40 \quad + 40 \\ \hline W = 61 \end{array}$$

9. Brandon's mother will allow him to spend no more than \$90 on video games. Each video game costs \$40. Write and solve an inequality that determines how many video games Brandon is allowed to purchase.

$$\begin{array}{r} 40V \geq 90 \\ \hline \frac{40}{40} \quad \frac{90}{40} \\ V \geq 2.25 \end{array} \quad \text{Brandon must buy more than 2 games.}$$

| Strengths | Unfinished Learning |
|---|---|
| <p>The student used an arithmetic approach to find the solution in a scenario</p> <p>The student created an equation/inequality to represent the scenario(s).</p> | <p>The student did not create the correct equation but solved their created equation correctly</p> <p>The student used the incorrect inequality symbol to represent the scenario.</p> <p>The student incorrectly describes the solution to the scenario based on their inequality</p> |

Ms. Hutchins Data Snapshot

| Assessment Task | Got It | Almost Got It | Not Yet |
|-----------------|---|--|---|
| #7 | <i>Evidence of understanding in models and explanation</i> Dakari | <i>Evidence of understanding in models drawn, no explanation</i> Janelle, Ivette, Kapone | <i>No Evidence</i> Sydney, Rochelle, Nyla, Byrce, Isaiah, Neveah, Anniyah, Edwin, Joseph, Elijah, Kamal, Malayah, Richard, Jeremiah, Andre, Zion |
| #8 | <i>Correct comparison and complete reasoning</i> Dakari, Janelle, Ivette, Kapone, Rochelle, Nyla | <i>Correct Comparison, Incomplete Reasoning and/or Inaccurate Model</i> Sydney, Isaiah, Neveah, Anniyah, Richard, Zion, Edwin, Elijah | <i>Incorrect Comparison, and/or Faulty Reasoning</i> Byrce, Joseph, Kamal, Malayah, Jeremiah, Andre |
| #9 | <i>Correct comparison and complete reasoning</i> Dakari, Janelle, Ivette, Kapone, Nyla, Elijah, Isaiah | <i>Correct Comparison, Incomplete Reasoning</i> Sydney, Rochelle, Zion | <i>Incorrect Comparison, and/or Faulty Reasoning</i> Byrce, Neveah, Anniyah, Edwin, Joseph, Kamal, Malayah, Richard, Jeremiah, Andre |

| Strengths | Misconceptions/Unfinished Learning |
|---|--|
| <ul style="list-style-type: none"> ● Solve equations/inequalities by performing inverse operation ● Creating an equation/inequality based on real world scenario ● Use of tape diagrams to create and solve equation ● Use the correct operation to describe the real world scenario ● Correctly translating the words into an inequality symbol | <ul style="list-style-type: none"> ● Solving equations using only arithmetic approach ● Matching the correct operation to the real world scenario ● Translating words to inequality symbols ● Determining the real world context and the mathematical implications ● Checking the solution to equation/inequality ● Incorporate visuals to support with understanding of equation/inequality |

Let's Reflect

To what extent is this work currently happening at your school/in your classroom?

What implications might this learning have on how you support schools or teachers with assessing and diagnosing unfinished learning in your role?

Example Data Meeting Agenda

| | |
|---|--|
| 0. PREWORK for Content Lead or to be completed by team at start of meeting | <p>Collect Student Work from Teachers</p> <p>Review Task and Determine What Constitutes Evidence:</p> <ul style="list-style-type: none"> ● What would an exemplar response require? <ul style="list-style-type: none"> ○ Be specific and pull from the standard, Eureka Acceleration Tool, or curriculum exemplars related to this task to identify <i>criteria for success</i> <p>Create Exemplar</p> <ul style="list-style-type: none"> ● What would an exemplar response look like? ● Is this what you would expect from students? |
| 2. NORM ON WHAT CONSTITUTES EVIDENCE | <p>NORM ON WHAT CONSTITUTES EVIDENCE</p> <ul style="list-style-type: none"> ● Review criteria for success and norm as a group (make any necessary adjustments) ● Review 1 piece of student work that meets the criteria for success and discuss why it meets the criteria <ul style="list-style-type: none"> ○ What makes this response exemplary? ○ How is it similar to your exemplar? ○ How is it different from your exemplar? ○ Does it require any adjustments to the <i>criteria for success</i> you identified? |
| 3. INTERPRET EVIDENCE | <p>INDEPENDENT SORT & ANALYSIS</p> <ul style="list-style-type: none"> ● Sort student work by your criteria for success into three categories (Got it, Almost Got it, Not Yet) ● Review student work: <ul style="list-style-type: none"> ○ What trends do you see in the student work (successes and misconceptions)? ○ Look at Got It student work: <ul style="list-style-type: none"> ■ What do they understand? ■ What supported them in being successful? ○ Look at Almost Got It student work: <ul style="list-style-type: none"> ■ What do they understand? What do they not yet understand? What is their key misconception? What adjustment or support could move them to mastery? ○ Look at Not Yet student work: <ul style="list-style-type: none"> ■ What do they understand? ■ What do they not yet understand? What adjustment or support could better help them access this task and move toward mastery? |

| | |
|------------------------|---|
| | |
| 4. DIAGNOSE | <p>GROUP DISCUSSION AND CALIBRATION</p> <ul style="list-style-type: none"> ● Review list of student strengths and what students do not yet understand. ● Discuss what is needed for students to move to mastery (think about trends) ● Identify 1-2 highest leverage instructional points that would support whole group (or subgroup) advancing understanding of foundational standards ● Work toward agreement on the 1-2 highest impact analysis statements using the sentence starter: <ul style="list-style-type: none"> ○ Student understanding would improve the most if..... |
| 5. ACTION STEPS | <p>DETERMINE GOALS</p> <ul style="list-style-type: none"> ● Is there a teaching point that needs to be mastered by the whole class or a sub-group? ● What additional supports are needed for students not yet approaching mastery to help them access the content? ● What small group or individual student goals might you set? <p>WRAP-UP AND KEY TAKEAWAYS</p> <ul style="list-style-type: none"> ● Whip-around to share the most important thing to implement in order to address unfinished learning. |