

## TIME VALUE OF MONEY

**Return of vs. Return on Investment:** We **EXPECT** to get more than we invest!

Invest \$1,000 it becomes \$1,050 \$1,000 return of \$50 return on

Factors to consider when assessing “Return on” investment

1. Time - \$50 return in one year \$60 return in two years.

2. Common Size Measurement

Which is better \$1,000 or \$10,000 return on investment?

Rate of Return – assumes one year

$$\frac{\$ \text{ Amount of Return}}{\$ \text{ Amount of Investment}}$$
  
(Negative Rate of Return)

### **Uncertainty: RISK**

**Expected Rate of Return:** Takes into consideration chance of an outcome occurring. It is a weighted average of possible rates of return.

Risk Factors: Inflation + Business Risk + Liquidity Risk

Risk Preferences: Risk Taker vs. Risk Averse

### **Simple vs. Compound Interest**

Principal x Rate x Time = Interest

The interest rate used takes into consideration the risk factors above.

Simple: Interest on Principal Only

$$2008 \quad \$10,000 \times .1 \times 1 = \$1,000$$

$$2009 \quad \$10,000 \times .1 \times 1 = \$1,000$$

$$2010 \quad \$10,000 \times .1 \times 1 = \$1,000$$

$$\text{Total Interest} \quad \$ 3,000$$

$$\text{Return of Principal} \quad \underline{10,000}$$

$$\$13,000$$

Compound Interest: Interest added to Principal at Specified Points in Time  
Compounding Period: Point where interest is added to Principal.

Annual Compounding – Interest added to principal once a year.

$$2008 \quad \$10,000 \times .1 \times 1 = \$1,000$$

$$2009 \quad \$11,000 \times .1 \times 1 = \$1,100$$

$$2010 \quad \$12,100 \times .1 \times 1 = \$1,210$$

$$\text{Total Interest} \quad \$ 3,310$$

$$\text{Return of Principal} \quad \underline{10,000}$$

$$\$13,310 \quad \text{Where did the extra \$310 come from?}$$

What if Compounded Semi- Annually?

$$2008 \quad \$10,000 \quad \times .1 \times 1/2 = \$500$$

$$\quad \$10,500 \quad \times .1 \times 1/2 = \$525$$

$$2009 \quad \$11,025 \quad \times .1 \times 1/2 = \$551.25$$

$$\quad \$11,576.25 \times .1 \times 1/2 = \$578.81$$

$$2010 \quad \$12,155.06 \times .1 \times 1/2 = \$607.75$$

$$\quad \$12,762.81 \times .1 \times 1/2 = \$638.14$$

|                     |                  |
|---------------------|------------------|
| Total Interest      | \$ 3,400.95      |
| Return of Principal | <u>10,000.00</u> |
|                     | \$13,400.95      |

What if compounded Monthly for 3 years? NO!!!!!!!

1. How many compounding?
2. How will interest differ from above?

The desired NOT to do 36 compounds leads us to:

$$(1 + r/c)^n \quad \text{Where} \quad \begin{array}{l} r = \text{Annual Rate} \\ c = \text{Number of Compounds per year} \\ n = \text{Total Number of Compounding} \end{array}$$

$$(1 + .10/2)^6 = 1.340095$$

Therefore: If invest \$10,000 3 years compounded semiannually

$$\$10,000 \times 1.340095 = \$13,400.95$$

## **FUTURE VALUE OF AMOUNT OF \$1**

- PV** = Lump sum of money today
- FV** = Lump sum of money at some point in the future
- r** = Annual Rate of Return
- c** = Number of compoundings in a year
- n** = Total number of compounding over time period

$$\mathbf{FV = PV \times (1 + r/c)^n}$$

\$10,000 invested 3 years compounded monthly will become ?

Use calculator to solve:

$$FV = \$10,000 \times (1 + .10/12)^{36} = \$13,481.82 \quad \text{Why Higher?}$$

## PRESENT VALUE OF \$1

You want to have \$13,400.95 in 3 years how much would you have to invest today to achieve this goal assuming you can earn 10% compounded semiannually?

$$FV = PV \times (1 + r/c)^n$$

$$\$13,400.95 = PV \times (1 + .10/2)^6$$

$$FV \times \frac{1}{(1 + r/c)^n} = PV$$

$$\$13,400.95 \times \frac{1}{(1 + .10/2)^6} = PV$$

$$\$10,000 = PV$$

Another View: \$10,000 today is the equivalent of \$13,400.95 3 years from today ASSUMING the interest rate is 10% and is compounded semiannually.

Do monthly compounding using calculator.

You want to have \$10,000 in 5 years how much must you invest today if you can earn 8% that is compounded quarterly?

$$PV = FV \times \frac{1}{(1 + r/c)^n}$$

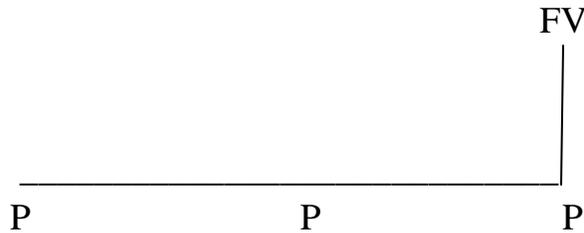
$$FV = \$10,000 \quad r = 8\% \quad c = 4 \quad n = 20 \quad PV = ?$$

$$PV \quad \$6,729.71$$

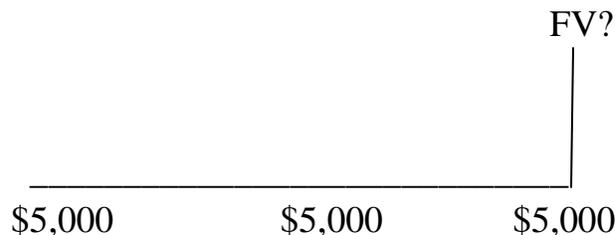
Ask for example from Class:

**ANNUITIES:** Equal cash payments at equal time intervals with the same interest rate of the entire time period. Interest is compounded at each payment date.

**Future Value of an Annuity:**



3 annual payments of \$5,000 starting 1/1/08 how much will you have when the last payment is made if you can earn 10% interest?



| Date   | Payment | Interest | Future Value |
|--------|---------|----------|--------------|
| 1/1/08 | \$5,000 | \$ 0     | \$ 5,000     |
| 1/1/09 | \$5,000 | 500      | 10,500       |
| 1/1/10 | \$5,000 | 1,050    | 16,550       |

Math formula 
$$FV = \text{Annuity} \times \frac{[(1 + r/c)^n - 1]}{r/c}$$

Calculator does math for us:

Ann = Annuity

**c = Number of Payments/ Compoundings per year**

r = Annual rate of interest

**n = Number of payments**

FV = Future Value

FV = ? Ann = \$5,000 r = 10% c = 1 n = 3 FV = \$16,550

What if made 15 annual payment with 10% annual interest?

FV = \$31,772.48 NOTE THAT NUMBER COME OUT NEGATIVE!

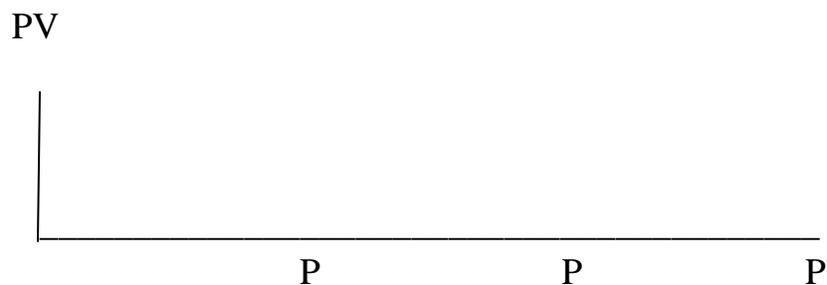
What if you wanted to have \$31,772.48 after making 15 payments how much would each payment have to be?

FV = \$31,772.48 Ann = ? c = 1 n = 15

What if you wanted to have \$1,000,000 after making monthly payments for 40 years and you could earn an 8% annual interest rate? How much must the monthly payments be?

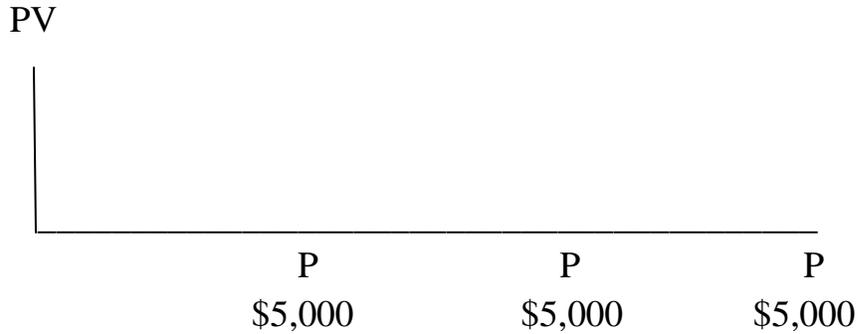
How much you would you have if you made monthly payments of \$300 per month for 40 years and could earn 10%? 8%? 5%?

**Present Value of an Annuity:** What is an annuity worth today



Formula:  $PV = \text{Annuity} \times \frac{[1 - 1/(1 + r/c)^n]}{r/c}$

What are three annual \$5,000 payments worth today if the interest rate is 10% and the first payment is one year from today. Stated another way – How much could you borrow if you agreed to pay \$5,000/ year for 3 years at 10% annual interest rate?



$PV = ?$      $Ann = \$5,000$      $r = 10\%$      $c = 1$      $n =$      $PV = \$12,434.26$

Proof:

| Date   | Payment | Interest    | Principal  | Present Value |
|--------|---------|-------------|------------|---------------|
| 1/1/08 |         |             |            | \$12,434.26   |
| 1/1/09 | \$5,000 | \$ 1,243.43 | \$3,756.57 | 8,677.87      |
| 1/1/10 | \$5,000 | 867.77      | 4,132.23   | 4,545.64      |
| 1/1/11 | \$5,000 | 454.56      | 4,545.64   | 0             |

Doesn't this look like a car payment?

You borrowed \$2,000 to buy a new stereo. The interest rate is 8% and the payments are semi-annual for 2 years. How much are your payments?

$PV = \$2,000$      $Ann = ?$      $r = 8\%$      $c = 2$      $n = 4$

$Ann = \$550.98$

## HOW DO YOU KNOW WHICH METHOD TO USE?

You want to have \$40,000 in 5 year, how much do you need to invest annually starting today to achieve this goal if the interest rate is 6%?

Is this a FV of \$1?

PV of \$1?

FV of Annuity?

PV of Annuity?

Use Steps:

- #1 Is this an annuity?
- #2 Is it a present or future value?
- #3 Identify the missing element
- #4 Solve for the missing element.