# Warm Up Problem

A candy jar contains 5 Jolly Ranchers (squares) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.
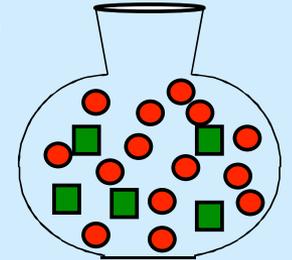
- **Please try to do this problem in as many ways as you can, both correct and incorrect**

- **If done, share your work with a neighbor.**

Productive Discussions of Cognitively Challenging Mathematics Tasks

PEG SMITH
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Overview

- Discuss cognitively demanding mathematical tasks (CDMT)

- Discuss the importance and challenge of facilitating a discussion around CDMT

- Describe 5+ practices that teachers can learn in order to facilitate discussions of CDMT more effectively

- Discuss how the 5 practices could help improve teaching with CDMT

- Plan a discussion around a CDMT

Overview

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Comparing Two Mathematical Tasks

- Think privately about how you would go about solving tasks A and B on page 2 of your handout (solve them if you have time)

- Talk with you neighbor about how you did or could solve each task

- Discuss how are the two versions of the task the same and how are they different.

Comparing Two Mathematical Tasks

Task A
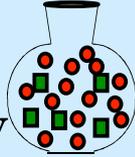
A candy jar contains 5 Jolly Ranchers (squares) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers.

How many Jawbreakers would you have? Explain how you know.

Task B

Find the value of the unknown in each of the proportions shown below.

1. $\frac{5}{2} = \frac{y}{10}$

2. $\frac{n}{8} = \frac{3}{12}$

3. $\frac{5}{20} = \frac{3}{d}$

4. $\frac{a}{24} = \frac{7}{8}$

5. $\frac{30}{6} = \frac{b}{7}$

6. $\frac{3}{x} = \frac{4}{28}$

The Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
Memorization • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	**Procedures With Connections** • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or ` placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	**Doing Mathematics** • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein, Smith, Henningsen, & Silver, 2000, p.16

Mathematical Tasks:
A Critical Starting Point for Instruction

Not all tasks are created equal, and *different tasks will provoke different levels and kinds of student thinking*.

Stein, Smith, Henningsen, & Silver, 2000

Mathematical Tasks:
A Critical Starting Point for Instruction

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Mathematical Tasks:
A Critical Starting Point for Instruction

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

Mathematical Tasks:
A Critical Starting Point for Instruction

If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.

Stein & Lane, 1996

Mathematical Tasks:
A Critical Starting Point for Instruction

If we want students to develop the capacity to think, reason, and problem solve then we need to *start* with high-level, cognitively complex tasks.

Stein & Lane, 1996

Overview

- Discuss cognitively demanding mathematical tasks (CDMT)
- Discuss the importance and challenge of facilitating a discussion around CDMT
- Describe 5+ practices that teachers can learn in order to facilitate discussions of CDMT more effectively
- Discuss how the 5 practices could help improve teaching with CDMT
- Plan a discussion around a CDMT

The Importance of Discussion

Provides opportunities for students to:

- Share ideas and clarify understandings
- Develop convincing arguments regarding why and how things work
- Develop a language for expressing mathematical ideas
- Learn to see things for other people's perspective

Jawbreakers and Jolly Ranchers Vignette

- What aspects of Mrs. Carlson's instruction would you identify as promising?

- What aspects of Mrs. Carlson's instruction would want to assist her in working on?

Jawbreakers and Jolly Ranchers Vignette: *What is Promising*

- Students are working on a mathematical task that appears to be both appropriate and worthwhile

- Students are encouraged to provide explanations and use strategies that make sense to them

- Students are working with partners and publicly sharing their solutions and strategies with peers

- Students' ideas appear to be respected

Jawbreakers and Jolly Ranchers Vignette: *What Can Be Improved*

- Beyond having students use different strategies, Mrs. Carlson's goal for the lesson is not clear

- Mrs. Carlson observes students as they work, but does not use this time to ***assess what students seem to understand*** or ***identify which aspects of students' work to feature*** in the discussion in order to make a mathematical point

- There is a "show and tell" feel to the presentations
  - not clear what each strategy adds to the discussion
  - different strategies are not related
  - key mathematical ideas are not discussed
  - no evaluation of strategies for accuracy, efficiency, etc.

How Expert Discussion Facilitation is Characterized

- Skillful improvisation
  - Diagnose students' thinking on the fly
  - Fashion responses that guide students to evaluate each others' thinking, and promote building of mathematical content over time

- Requires deep knowledge of:
  - Relevant mathematical content
  - Student thinking about it and how to diagnose it
  - Subtle pedagogical moves
  - How to rapidly apply all of this in specific circumstances

Some Sources of the Challenge in Facilitating Discussions

- Lack of familiarity

- Reduces teachers' perceived level of control

- Requires complex, split-second decisions

- Requires flexible, deep, and interconnected knowledge of content, pedagogy, and students

Purpose of the Five Practices

To make student-centered instruction more manageable by moderating the degree of improvisation required by the teachers and during a discussion.

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- Discuss how the 5 practices could help improve teaching with CDMT

- Plan a discussion around a CDMT

The Five Practices (+)

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)

2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)

3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)

4. **Sequencing** (e.g., Schoenfeld, 1998)

5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

The Five Practices (+)

0. **Setting Goals and Selecting Tasks**

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)

2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)

3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)

4. **Sequencing** (e.g., Schoenfeld, 1998)

5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

0$_1$. Setting Goals

- **It involves:**
  - Identifying what students are to know and understand about mathematics as a result of their engagement in a particular lesson
  - Being as specific as possible so as to establish a clear target for instruction that can guide the selection of instructional activities and the use of the five practices
- **It is supported by:**
  - Thinking about what students will come to know and understand rather than only on what they will do
  - Consulting resources that can help in unpacking big ideas in mathematics
  - Working in collaboration with other teachers

Mrs. Carlson's Class

Implied Goal

Students will be able to solve the task correctly using one of a number of viable strategies and realize that there are several different and correct ways to solve the task.

Possible Goals

- Quantities that are in a proportional (multiplicative) relationship grow at a constant rate.

- The unit rate specifies the relationship between the two quantities; it answers the question how many times larger one quantity is in comparison to the other.

- The constant of proportionality (unit rate) can be identified in a table, graph, equation, and verbal descriptions of proportional relationships.

02. Selecting a Task

- **It involves:**
  - Identifying a mathematical task that is aligned with the lesson goals
  - Making sure the task is rich enough to support a discussion (i.e., a cognitively challenging/high level task)
- **It is supported by:**
  - Setting a clear and explicit goal for learning
  - Using the **Task Analysis Guide** which provides a list of characteristics of tasks at different levels of cognitive demand
  - Working in collaboration with colleagues

Sandra Carlson's Task

A candy jar contains 5 Jolly Ranchers (squares) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers.

How many Jawbreakers would you have? Explain how you know.

1. Anticipating

- **It involves considering:**
  - The array of strategies that students might use to approach or solve a challenging mathematical task
  - How to respond to what students produce
  - Which strategies will be most useful in addressing the mathematics to be learned
- **It is supported by:**
  - Doing the problem in as many ways as possible
  - Discussing the problem with other teachers
  - Drawing on relevant research
  - Documenting student responses year to year

Jawbreakers and Jolly Ranchers: Anticipated Solutions

- **Unit Rate**--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.

- **Scale Factor**--Find that the number of JB (100) is 20 times the original amount (5) so the number of JR must be 20 times the original amount (13).

- **Scaling Up**--Increasing the number of JR and JB by continuing to add 5 to the JR and 13 to the JB until you reach the desired number of JR (100).

- **Incorrect Additive**--Find that the number of JB has increased by 95 (5 + 95 = 100) so the number of JR must also increase by 95 (13 + 95 = 108)

Jawbreakers and Jolly Ranchers: Incorrect Additive Strategy

Jordan and Kate's Solution

5 Jolly Ranchers —————→ *100 Jolly Ranchers*

+95

13 Jawbreakers —————→ *108 Jawbreakers*

+95

2. Monitoring

- **It involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking questions to get students back "on track" or to advance their understanding
- **It is supported by:**
  - Anticipating student responses beforehand
  - Carefully listening and asking probing questions
  - Using recording tools

Monitoring Tool

Strategy	Who and What	Order

List the different solution paths you anticipated

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.		
Scale Factor--Find that the number of JB (100) is 20 times the original amount (5) so the number of JR must be 20 times the original amount (13).		
Scaling Up--Increasing the number of JR and JB by continuing to add 5 to the JR and 13 to the JB until you reach the desired number of JR (100).		
Incorrect Additive--Find that the number of JB has increased by 95 (5 + 95 = 100) so the number of JR must also increase by 95 (13 + 95 = 108)		
OTHER		

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.		
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OTHER		

Make note of which students produced which solutions and what you might want to highlight

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.	Jerry, Nicole, Jon, David- *Said that they found 1 JR: 2.6 JB so now they could find easily find any number of JB*	
Scale Factor--Find that the number of JB (100) is 20 times the original amount (5) so the number of JR must be 20 times the original amount (13).	Owen, Joshua, Danielle, Marcus, Tracy, Gina- *Mentioned that the ratio had to be the same*	
Scaling Up--Increasing the number of JR and JB by continuing to add 5 to the JR and 13 to the JB until you reach the desired number of JR (100).	Kamiko, Mike, Jerilyn, Amanda- *Focused on adding 5 and 13; did not mention a multiplicative relationship (Table)*	
Incorrect Additive--Find that the number of JB has increased by 95 (5 + 95 = 100) so the number of JR must also increase by 95 (13 + 95 = 108)	Jordan, Kate, Mike, Sandra- *Focused on doing the "same thing" to both quantities, missed the fact that the ratio was not the same*	
OTHER – Ratio Table; Ricardo and Melissa- *scaled up but did not increment by the same amount; Use multiplication and addition*	Ellen, Adam *(drew candy jars- focused on addition)* Alicia, Max *(used green and red cubes- -moved from adding to multiplying)*	

Monitoring Tool

How might the data collected on the monitoring tool be useful to teachers?

3. Selecting

- **It involves:**
  - Choosing particular students to present because of the mathematics available in their responses
  - Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
  - Gaining some control over the content of the discussion (no more "who wants to present next?")
- **It is supported by:**
  - Anticipating and monitoring
  - Planning in advance which types of responses to select

Mrs. Carlson's Goals

- Quantities that are in a proportional (multiplicative) relationship grow at a constant rate.

- The unit rate specifies the relationship between the two quantities; it answers the question how many times larger one quantity is in comparison to the other.

- The constant of proportionality (unit rate) can be identified in a table, graph, equation, and verbal descriptions of proportional relationships.

Mrs. Carlson's Goals

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What students do you think Mrs. Carlson should select in order to accomplish the goals she has set?

4. Sequencing

- **It involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line

- **It is supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.	Jerry, Nicole, Jon, David- *Said that they found 1 JR: 2.6 JB so now they could find easily find any number of JB*	
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OTHER – Ratio Table; Ricardo and Melissa- *scaled up but did not increment by the same amount; Use multiplication and addition*	Ellen, Adam *(drew candy jars- focused on addition)* Alicia, Max *(used green and red cubes- - moved from adding to multiplying)*	

Indicate the order in which students will share

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.	Jerry, Nicole, Jon, David- *Said that they found 1 JR: 2.6 JB so now they could find easily find any number of JB*	
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In what order would you sequence the responses?

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of Jawbreakers for each Jolly Rancher (1 JR: 2.6 JB) then multiply both by 100.	Jerry, Nicole, Jon, David- *Said that they found 1 JR: 2.6 JB so now they could find easily find any number of JB*	4-Jerry
Scale Factor--Find that the number of JB (100) is 20 times the original amount (5) so the number of JR must be 20 times the original amount (13).	Owen, Joshua, Danielle, Marcus, Tracy, Gina- *Mentioned that the ratio had to be the same*	3-Owen
Scaling Up--Increasing the number of JR and JB by continuing to add 5 to the JR and 13 to the JB until you reach the desired number of JR (100).	Kamiko, Mike, Jerilyn, Amanda- *Focused on adding 5 and 13; did not mention a multiplicative relationship (Table)*	2-Kamiko
Incorrect Additive--Find that the number of JB has increased by 95 (5 + 95 = 100) so the number of JR must also increase by 95 (13 + 95 = 108)	Jordan, Kate, Mike, Sandra- *Focused on doing the "same thing" to both quantities, missed the fact that the ratio was not the same*	1 - Jordan
OTHER – Ratio Table; Ricardo and Melissa- *scaled up but did not increment by the same amount; Use multiplication and addition*	Ellen, Adam *(drew candy jars- focused on addition)* Alicia, Max *(used green and red cubes- -moved from adding to multiplying)*	

Jawbreakers and Jolly Ranchers Vignette

Possible Sequencing:

1. Jordan – incorrect additive approach *(raises the issue about how the quantities are related)*

2. Kamiko – used a table to scale up *(although she used repeated addition, it can be related to multiplication and to the other strategies)*

3. Owen – scale factor *(clearly shows that the ratio is preserved and that the new jar is 20 times the original)*

4. Jerry – unit rate *(shows the multiplicative relationship between the two quantities)*

5. Connecting

- **It involves:**
  - Encouraging students to make mathematical connections between different student responses
  - Making the key mathematical ideas that are the focus of the lesson salient

- **It is supported by:**
  - Anticipating, monitoring, selecting, and sequencing
  - During planning, considering how students might be prompted to recognize mathematical relationships between responses

Jawbreakers and Jolly Rancher Vignette

1. **Jordan**

100 JR is 95 more than the 5 we started with. So we will need 95 more JB than the 13 we started with.

5 JR + 95 JR = 100 JR

13 JB + 95 JB = 108 JB
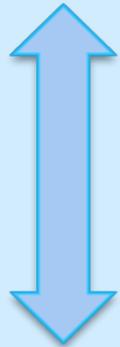
2. Kamiko

We just kept adding 5 to the JR column and 13 to the JB column. We stopped when we got to 100 JR. So it has to be 260 JB.

3. **Owen**

You ha *20 t* *mu* *So*

Jordan and Kamiko both used addition but got different answers. How is that possible? Who got the correct answer? Why?

4. Jerry

Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; that would use up 10 JB. So you have three JB left over. So we had to distribute the three JB to the 5 JR. 3 ÷ 5 = .6 so that would give the ratio of 1 JR to 2.6 JB. So then you just multiply 1 and 2.6 each by 100.

Jawbreakers and Jolly Rancher Vignette

1. Jordan

100 JR is 95 more than the 5 we started with. So we will need 95 more JB than the 13 we started with.

5 JR + 95 JR = 100 JR

13 JB + 95 JB = 108 JB

2. Kamiko

We just kept adding 5 to the JR column and 13 to the JB column. We stopped when we got to 100 JR. So it has to be 260 JB.

3. Owen

You have to multiply the five JR by 20 to get 100, so you'd also have to multiply the 13 JB by 20 to get 260. So it has to be 260.

4. Jerry

Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; th... ...ld use up 10 JB. So you h... ...we had ...e 5 ...e ...en ...ch by...

How is Owen's strategy related to Kamiko's table?

Jawbreakers and Jolly Rancher Vignette

1. Jordan

100 JR is 95 more than the 5 we started with. So we will need 95 more JB than the 13 we started with.

5 JR + 95 JR = 100 JR

13 JB

3. Owen

You have to multiply the five JR by 20 to get 100, so you'd also have to multiply the 13 JB by 20 to get 260. So it has to be 260.

4.

Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; that would use up 10 JB. So you have three JB left over. So we had to distribute the three JB to the 5 JR. $3 \div 5 = .6$ so that would give the ratio of 1 JR to 2.6 JB. So then you just multiply 1 and 2.6 each by 100.

2.

We ju____ column and ___ to ___ column. We stopped when we got to 100 JR. So it has to be 260 JB.

How are Jerry and Owen's strategies the same and how are they different?

Jawbreakers and Jolly Rancher Vignette

Where is the unit rate in Kamiko's table?

m... with.

5 JR + 95 JR = 100 JR

13 JB + 95 JB = 108 JB

2. Kamiko

We just kept adding 5 to the JR column and 13 to the JB column. We stopped when we got to 100 JR. So it has to be 260 JB.

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Jawbreakers and Jolly Rancher Vignette

1. [obscured by speech bubble]

[...] mo[...] with.

5 JR + 95 JR = 100 JR

13 JB + 95 JB = 108 JB

2. Kamiko

We just kept adding 5 to the JR column and 13 to the JB column. We stopped when we got to 100 JR. So it has to be 260 JB.

How could you use Jerry's strategy to find the number of JB in *any* new jar?

3. Owen

You have to multiply the five JR by 20 to get 100, so you'd also have to multiply the 13 JB by 20 to get 260. So it has to be 260.

4. Jerry

Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; that would use up 10 JB. So you have three JB left over. So we had to distribute the three JB to the 5 JR. 3 ÷ 5 = .6 so that would give the ratio of 1 JR to 2.6 JB. So then you just multiply 1 and 2.6 each by 100.

Overview

- Discuss cognitively demanding mathematical tasks (CDMT)

- Discuss the importance and challenge of facilitating a discussion around high-level tasks

- Describe 5+ practices that teachers can learn in order to facilitate discussions of high-level tasks more effectively

- Discuss how the 5 practices could help improve teaching with high-level tasks

- Plan a discussion

Why These Five Practices Likely to Help

- **Provides teachers with more control**
  - Over the content that is discussed
  - Over teaching moves: not everything improvisation

- **Provides teachers with more time**
  - To diagnose students' thinking
  - To plan questions and other instructional moves

- **Provides a reliable process for teachers to gradually improve their lessons over time**

Why These Five Practices Likely to Help

- **Honors students' thinking while guiding it in productive, disciplinary directions** (Ball, 1993; Engle & Conant, 2002)

  - Key is to support students' disciplinary authority while simultaneously holding them accountable to discipline

  - Guidance done mostly 'under the radar' so doesn't impinge on students' growing mathematical authority

  - At same time, students led to identify problems with their approaches, better understand sophisticated ones, and make mathematical generalizations

  - This fosters students' accountability to the discipline

Reflection

How do you think the 5 practices will be helpful in your own work with teachers?

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Now You Try It!

BAG OF MARBLES TASK

Our Task: Bag of Marbles

Ms. Rhee's math class was studying statistics. She brought in three bags containing red and blue marbles. The three bags were labeled as shown below:
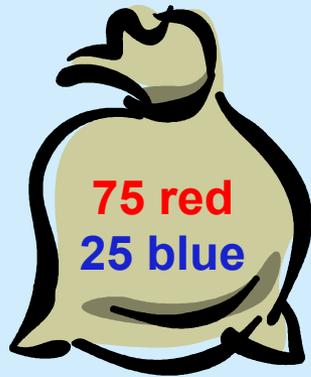
Bag X
75 red
25 blue
Total = 100 marbles
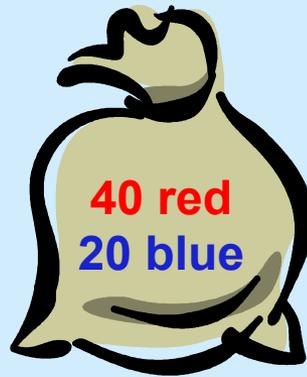
Bag Y
40 red
20 blue
Total = 60 marbles
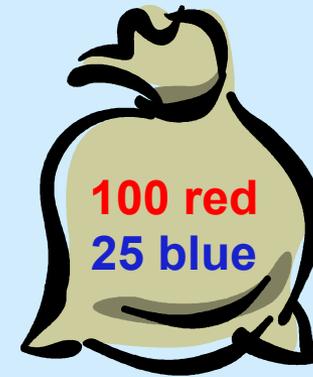
Bag Z
100 red
25 blue
Total = 125 marbles

Ms. Rhee shook each bag. She asked the class, "If you close your eyes, reach into a bag, and remove 1 marble, which bag would give you the best chance of picking a blue marble?"

Our Goal for the Lesson

- for students to understand that in order to compare different amounts and quantities you need a common basis for comparison

- for students to be able to distinguish different types of comparisons (i.e., part-to-part, part-to-whole, percents)

Anticipating Likely Responses

- Working individually, consider the correct and incorrect approaches that students might use to solve this task

- Working with a small group of peers, share the approaches you have anticipated so far and see what other approaches you can come up with together

- Make a list of the approaches you come up with

Monitoring Actual Responses

Which responses might you be on the look out for as you walk around the room and interact with students? Why?

Selecting and Sequencing Student Responses

Imagine that the students in your class produced the solutions A-H. Identify the solution paths that you would want to have shared during the group discussion and specify the order in which they would be shared.

Explain *why* you selected particular responses and *how* you determined the ordering of the presentations.

Selecting and Sequencing Student Responses

Review the groups' selection of responses to be shared and see if there are any patterns you notice.

Can we begin to generate any general "rules of thumb" for determining which responses to share?

Making Connections Between Student Responses

Suppose we agree to have students share solutions F, D, B, and G in that order.

What questions will you ask to prompt students:
- to make sense of the solution path presented?
- understand the key mathematical ideas?
- to make connections between different responses and ways of representing mathematical ideas?

Characteristics of Questions

- Press students to *explain what* they did and *justify why* it makes sense mathematically and in the context of the problem
- Encourage all students to *question and make sense of* the presented strategies
- Focus on the similarities and differences between different methods
- Help students make connections between strategies
- Encourage students to generalize

Reflection

How do you think using the 5 practices will help you as a professional developer, coach, or university professor?

Resources Related to the Five Practices

- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking. *Mathematical Thinking and Learning, 10,* 313-340.

- Smith, M.S., Hughes, E.K., & Engle, R.A., & Stein, M.K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14* (9), 549-556.

Resources Related to the Five Practices
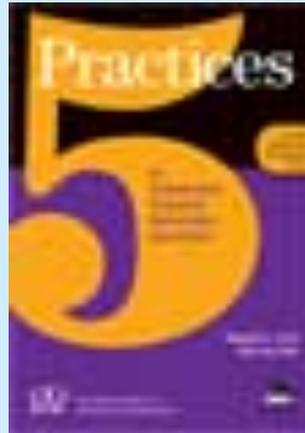
- Smith, M.S., & Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

For additional information, you can contact me at

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THANK YOU!

JR	JB
5	13
10	26
15	39
20	52
25	65
30	78
35	91
40	104
45	117
50	130
55	143
60	156
65	169
70	182
75	195
80	208
85	221
90	234
95	247
100	260

Kamiko's Table

20 rows in the table – she added 5 and 13 twenty times; same as multiplying both by 20 like Owen did.

JR	JB
5	13
10	26
15	39
20	52
25	65
30	78
35	91
40	104
45	117
50	130
55	143
60	156
65	169
70	182
75	195
80	208
85	221
90	234
95	247
100	260

Kamiko's Table

The number of JB in each row is 2.6 times the number of JR in the row. This is the unit rate that Jerry found.