WARM UP PROBLEM

A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?

Use drawings, words, or numbers to show how you got your answer.

- **Please try to do this problem in as many ways as you can, both correct and incorrect**

- **If done, share your work with a neighbor or look at the student work on page 6 of the handout.**

M4 Conference
March 8, 2014

Productive Discussions of Cognitively Challenging Mathematics Tasks

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Overview

- Discuss cognitively demanding mathematical tasks (CDMT)
- Discuss the importance and challenge of facilitating a discussion around (CDMT)
- Describe 5 practices that you can learn in order to facilitate discussions of CDMT more effectively
- Discuss how the 5 practices could help improve teaching with CDMT
- Plan a discussion

Overview

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Comparing Two Mathematical Tasks

- Think privately about how you would go about solving tasks A and B on page 2 of your handout (solve them if you have time)
- Talk with you neighbor about how you did or could solve each task
- Discuss how are the two versions of the task the same and how are they different.

Comparing Two Mathematical Tasks

TASK A

Mr. Harris' third grade class was responsible for setting up the chairs for their spring band concert. In preparation, they had to determine the total number of chairs needed and ask the school's engineer to retrieve that many chairs from the central storage area. Mr. Harris explained that they needed to set up 7 rows of chairs with 20 chairs in each row and to leave space for a center aisle. How many chairs would the school engineer need to retrieve? Use drawings, words, or numbers to explain how you got your answer.

TASK B

Solve:

1. 2 x 10 =_____
2. 7 x 20 =_____
3. 5 x 30 =_____
4. 4 x 60 =_____
5. 6 x 70 =_____
6. 3 x 50 =_____
7. 5 x 80=_____
8. 9 x 40=_____
9. 8 x 90=_____

The Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
<u>Memorization</u> • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	<u>Procedures With Connections</u> • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
<u>Procedures Without Connections</u> • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or` placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	<u>Doing Mathematics</u> • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

[redacted]

*These characteristics are derived from the work of Doyle on academic tasks (1988), Resnick on high-level thinking skills (1987), and from the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, & Henningsen, 1996; Stein, Lane, and Silver, 1996).

Stein, Smith, Henningsen, & Silver, 2000, p.16

Mathematical Tasks: A Critical Starting Point for Instruction

Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking***.

Stein, Smith, Henningsen, & Silver, 2000

Mathematical Tasks: A Critical Starting Point for Instruction

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Mathematical Tasks: A Critical Starting Point for Instruction

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

Mathematical Tasks: A Critical Starting Point for Instruction

If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.

Stein & Lane, 1996

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If we want students to develop the capacity to think, reason, and problem solve then we need to ***start*** with high-level, cognitively complex tasks.

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Overview

- Discuss cognitively demanding mathematical tasks (CDMT)
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- Plan a discussion

The Importance of Discussion

Provides opportunities for students to:

- Share ideas and clarify understandings
- Develop convincing arguments regarding why and how things work
- Develop a language for expressing mathematical ideas
- Learn to see things for other people's perspective

Leaves and Caterpillar Vignette

- What aspects of Mr. Crane's instruction do you see as promising?
- What aspects of Mr. Crane's instruction would you want to assist him in working on?

Leaves and Caterpillar Vignette *What is Promising*

- Students are working on a mathematical task that appears to be both appropriate and worthwhile
- Students are encouraged to provide explanations and use strategies that make sense to them
- Students are working with partners and publicly sharing their solutions and strategies with peers
- Students' ideas appear to be respected

Leaves and Caterpillar Vignette *What Can Be Improved*

- Beyond having students use different strategies, Mr. Crane's goal for the lesson is not clear
- Mr. Crane observes students as they work, but does not use this time to ***assess what students seem to understand*** or ***identify which aspects of students' work to feature*** in the discussion in order to make a mathematical point
- There is a "show and tell" feel to the presentations
  - not clear what each strategy adds to the discussion
  - different strategies are not related
  - key mathematical ideas are not discussed
  - no evaluation of strategies for accuracy, efficiency, etc.

How Expert Discussion Facilitation is Characterized

- **Skillful improvisation**
  - Diagnose students' thinking on the fly
  - Fashion responses that guide students to evaluate each others' thinking, and promote building of mathematical content over time
- **Requires deep knowledge of:**
  - Relevant mathematical content
  - Student thinking about it and how to diagnose it
  - Subtle pedagogical moves
  - How to rapidly apply all of this in specific circumstances

Some Sources of the Challenge in Facilitating Discussions

- Lack of familiarity
- Reduces teachers' perceived level of control
- Requires complex, split-second decisions
- Requires flexible, deep, and interconnected knowledge of content, pedagogy, and students

Purpose of the Five Practices

To make student-centered instruction more manageable by moderating the degree of improvisation required by the teachers and during a discussion.

Overview

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- Describe 5 practices that you can learn in order to facilitate discussions of CDMT more effectively
- Discuss how the 5 practices could help improve teaching with CDMT
- Plan a discussion

The Five Practices (+)

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

The Five Practices (+)

0. Setting Goals and Selecting Tasks
1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

01. Setting Goals

- **It involves:**
  - Identifying what students are to know and understand about mathematics as a result of their engagement in a particular lesson
  - Being as specific as possible so as to establish a clear target for instruction that can guide the selection of instructional activities and the use of the five practices
- **It is supported by:**
  - Thinking about what students will come to know and understand rather than only on what they will do
  - Consulting resources that can help in unpacking big ideas in mathematics
  - Working in collaboration with other teachers

Mr. Crane's Class

Implied Goal

Students will be able to solve the task correctly using one of a number of viable strategies and realize that there are several different and correct ways to solve the task.

Possible Goals

- Students will recognize that the relationship between leaves and caterpillars is multiplicative not additive – that the leaves and caterpillars need to grow at a constant rate (for every 2 caterpillars, there are 5 leaves; for each caterpillar, there are 2.5 leaves).
- Student will recognize that there are three related strategies for solving the task – unit rate, scale factor and scaling up.

0$_2$. Selecting a Task

- **It involves:**
  - Identifying a mathematical task that is aligned with the lesson goals
  - Making sure the task is rich enough to support a discussion (i.e., a cognitively challenging mathematical task)
- **It is supported by:**
  - Setting a clear and explicit goal for learning
  - Using the **Task Analysis Guide** which provides a list of characteristics of tasks at different levels of cognitive demand
  - Working in collaboration with colleagues

David Crane’s Task

A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?

Use drawings, words, or numbers to show how you got your answer.

1. Anticipating

likely student responses to mathematical problems

- **It involves considering:**
  - The array of strategies that students might use to approach or solve a challenging mathematical task
  - How to respond to what students produce
  - Which strategies will be most useful in addressing the mathematics to be learned

- **It is supported by:**
  - Doing the problem in as many ways as possible
  - Discussing the problem with other teachers
  - Drawing on relevant research
  - Documenting student responses year to year

Leaves and Caterpillar: Anticipated Solutions

- **Unit Rate**--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times
- **Scale Factor**--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)
- **Scaling Up**--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars
- **Additive**--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)

Leaves and Caterpillar: Incorrect Additive Strategy

Missy and Kate's Solution

They added 10 caterpillars, and so I added 10 leaves.

2 caterpillars → (+10) *12 caterpillars*

5 leaves → (+10) *15 leaves*

2. Monitoring

students' actual responses during independent work

- **It involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking questions to get students back "on track" or to advance their understanding

- **It is supported by:**
  - Anticipating student responses beforehand
  - Carefully listening and asking probing questions
  - Using recording tools

Monitoring Tool

Strategy	Who and What	Order

Monitoring Tool

Strategy	Who and What	Order

List the different solution paths you anticipated

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times		
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)		
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars		
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)		
OTHER		

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times		
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)		
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Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)		
OTHER		

Make note of which students produced which solutions and what you might want to highlight

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (number sentence) Kyra (picture)	
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars	Jamal (table) *Martin and Melissa did sets of leaves and caterpillars*	
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
OTHER—Multiplied leaves and caterpillars	Darnell and Marcus	

Monitoring Tool

How might the data collected on the monitoring tool be useful to teachers?

3. Selecting

student responses to feature during discussion

- **It involves:**
  - Choosing particular students to present because of the mathematics available in their responses
  - Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
  - Gaining some control over the content of the discussion (no more "who wants to present next?")
- **It is supported by:**
  - Anticipating and monitoring
  - Planning in advance which types of responses to select

Mr. Crane's Goals

- Students will recognize that the relationship between quantities is multiplicative not additive – that the 2 quantities (leaves and caterpillars) need to grow at a constant rate.
- Student will recognize that there are three related strategies for solving the task – unit rate, scale factor and scaling up.

Mr. Crane’s Goals

- Students will recognize that the relationship between quantities is multiplicative not additive – that the 2 quantities (leaves and caterpillars) need to grow at a constant rate.

- Student will recognize that there are three related strategies for solving the task – unit rate, scale factor and scaling up.

What students do you think Mr. Crane should select in order to accomplish the goals he has set?

4. Sequencing

student responses during the discussion

- **It involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line

- **It is supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (picture and number sentence) Kyra (picture)	
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	
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Indicate the order in which students will share

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (picture and number sentence) Kyra (picture)	
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	
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In what order would you sequence the responses?

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (picture and number sentence) Kyra (picture)	3 (Janine)
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	4 (Jason)
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars	Jamal (table) *Martin and Melissa did sets of leaves and caterpillars*	2 (Jamal) 1 (Martin)
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
OTHER—Multiplied leaves and caterpillars	Darnell and Marcus	

Leaves and Caterpillar Vignette

Possible Sequencing:

1. Martin – picture (scaling up)
2. Jamal – table (scaling up)
3. Janine -- picture/written explanation (unit rate)
4. Jason -- written explanation (scale factor)

5. Connecting

student responses during the discussion

- **It involves:**
  - Encouraging students to make mathematical connections between different student responses
  - Making the key mathematical ideas that are the focus of the lesson salient
- **It is supported by:**
  - Anticipating, monitoring, selecting, and sequencing
  - During planning, considering how students might be prompted to recognize mathematical relationships between responses

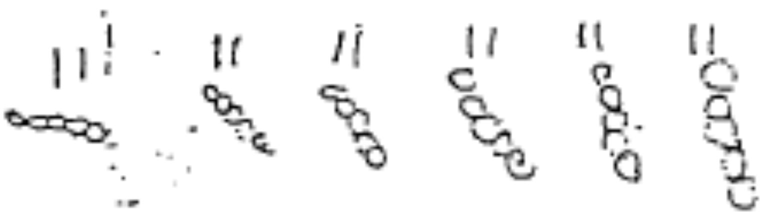
Leaves and Caterpillar Vignette

Possible Connections:

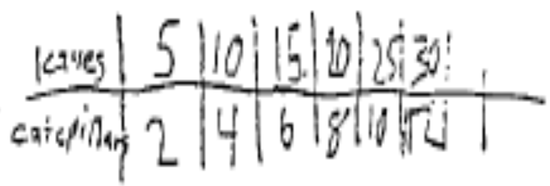
1. Martin – picture (scaling up)
2. Jamal – table (scaling up)
3. Janine -- picture/written explanation (unit rate)
4. Jason -- written explanation (scale factor)

Leaves and Caterpillar Vignette

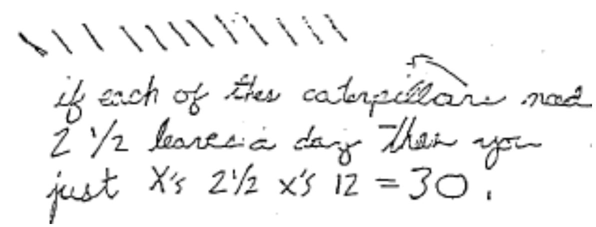
1. Martin – picture (scaling up)



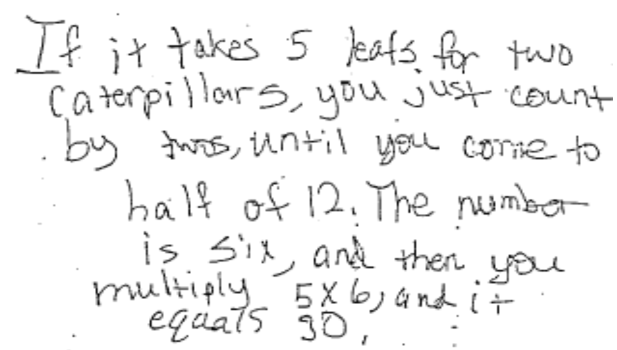
2. Jamal – table (scaling up)



3. Janine -- picture/written explanation (unit rate)

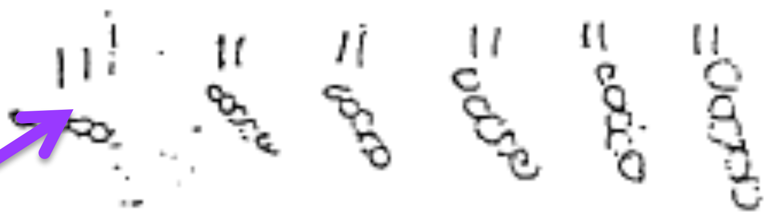


4. Jason -- written explanation (scale factor)



Leaves and Caterpillar Vignette

1. Martin – picture (scaling up)



2. Jamal – table (scaling up)

leaves	5	10	15	20	25	30
caterpillars	2	4	6	8	10	12

How is Martin's picture related to Jamal's table?

3. Janine -- picture/written explanation (unit rate)

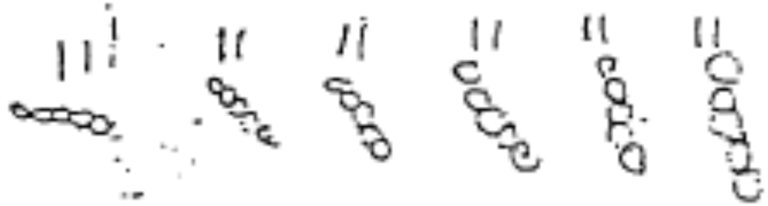
if each of the caterpillars need 2 ½ leaves a day then you just X's 2½ x's 12 = 30.

4. Jason -- written explanation (scale factor)

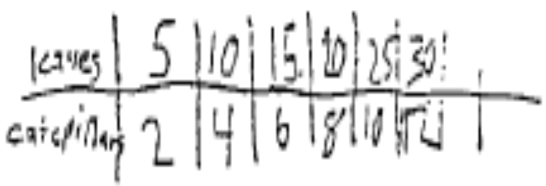
If it takes 5 leafs for two caterpillars, you just count by twos, until you come to half of 12. The number is six, and then you multiply 5 X 6, and it equals 30.

Leaves and Caterpillar Vignette

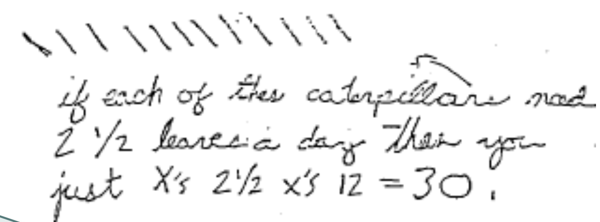
1. Martin – picture (scaling up)



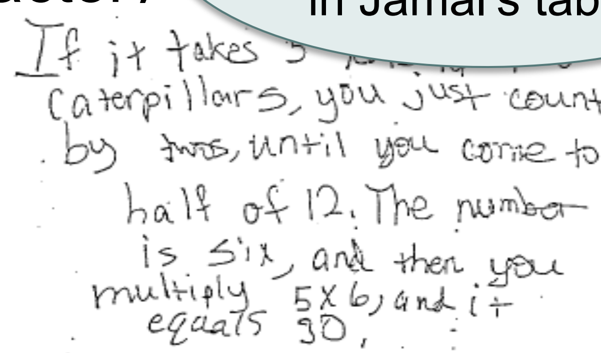
2. Jamal – table (scaling up)



3. Janine -- picture/written explanation (unit rate)



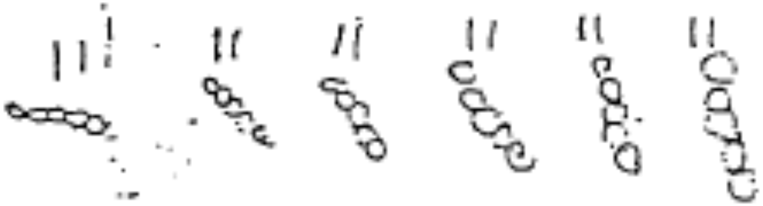
4. Jason – explana factor)



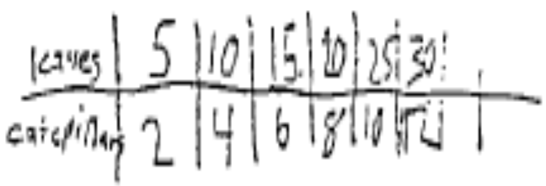
Where do you see the unit rate of 2 ½ in Jamal's table?

Leaves and Caterpillar Vignette

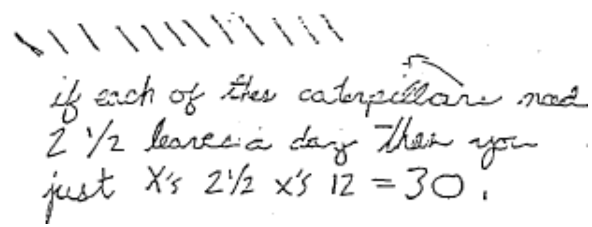
1. Martin – picture (scaling up)



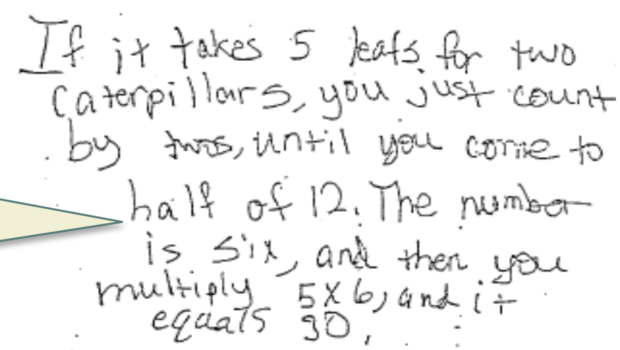
2. Jamal – table (scaling up)



3. Janine -- picture/written explanation (unit rate)



4. Jason -- written explanation (scale factor)



Where do you see the scale factor of 6 in the other solutions?

Overview

- Discuss cognitively demanding mathematical tasks (CDMT)
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- Describe 5 practices that you can learn in order to facilitate discussions of CDMT more effectively
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- Plan a discussion

Why These Five Practices Likely to Help

- **Provides teachers with more control**
  - Over the content that is discussed
  - Over teaching moves: not everything improvisation

- **Provides teachers with more time**
  - To diagnose students' thinking
  - To plan questions and other instructional moves

- **Provides a reliable process for teachers to gradually improve their lessons over time**

Why These Five Practices Likely to Help

- **Honors students' thinking while guiding it in productive, disciplinary directions** (Ball, 1993; Engle & Conant, 2002)
  - Key is to support students' disciplinary authority while simultaneously holding them accountable to discipline
  - Guidance done mostly 'under the radar' so doesn't impinge on students' growing mathematical authority
  - At same time, students led to identify problems with their approaches, better understand sophisticated ones, and make mathematical generalizations
  - This fosters students' accountability to the discipline

Reflection

How do you think the 5 practices will be helpful in your own work with teachers?

Overview

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Now You Try It!

Bag of Marbles Task

Setting Up Chairs

Mr. Harris' third grade class was responsible for setting up the chairs for their spring band concert. In preparation, they had to determine the total number of chairs needed and ask the school's engineer to retrieve that many chairs from the central storage area. Mr. Harris explained that they needed to set up 7 rows of chairs with 20 chairs in each row and to leave space for a center aisle. How many chairs would the school engineer need to retrieve? Use drawings, words, or numbers to explain how you got your answer.

Our Goal for the Lesson

- For students to understand that multiplication is the same as repeated addition
- For students to see how array models (area) can be used to show multiplication

Anticipating Likely Responses

- Working individually, consider the correct and incorrect approaches that students might use to solve this task
- Working with a small group of peers, share the approaches you have anticipated so far and see what other approaches you can come up with together
- Make a list of the approaches you come up with

Monitoring Actual Responses

Which responses might you be on the look out for as you walk around the room and interact with students? Why?

Selecting and Sequencing Student Responses

Imagine that the students in your class produced the solutions A-F. Identify the solution paths that you would want to have shared during the group discussion and specify the order in which they would be shared.

Explain *why* you selected particular responses and *how* you determined the ordering of the presentations.

Selecting and Sequencing Student Responses

Review the groups' selection of responses to be shared and see if there are any patterns you notice.

Can we begin to generate any general "rules of thumb" for determining which responses to share?

Making Connections Between Student Responses

Suppose we agree to have students share solutions E, B, F and D in that order.

What questions will you ask to prompt students:

- to make sense of the solution path presented?
- understand the key mathematical ideas?
- to make connections between different responses and ways of representing mathematical ideas?

Characteristics of Questions

- Press students to *explain what* they did and *justify why* it makes sense mathematically and in the context of the problem
- Encourage all students to *question and make sense of* the presented strategies
- Focus on the similarities and differences between different methods
- Help students make connections between strategies
- Encourage students to generalize

Reflections

How do you think using the 5 practices will help you as a professional developer, coach, or university professor?

Resources Related to the Five Practices

- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking*. Mathematical Thinking and Learning, 10*, 313-340.

- Smith, M.S., Hughes, E.K., & Engle, R.A., & Stein, M.K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14* (9), 549-556.

Resources Related to the Five Practices

- Smith, M.S., & Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

For additional information, you can contact me at

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