2*b* or Not 2*b*: Supporting Learners Who Struggle in Mathematics

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Overview of Session

This session focuses on ways to support students who struggle in Tiers 1 and 2 to become:

- Confident in mathematics
- Successful with <u>rigorous</u> mathematics
- Capable of working with complex mathematical ideas

Reflect

- When you think of learners who struggle, what types of instruction and content do you feel are important for them?

Making Cents

1. Take out some coins
2. Multiply the value of the coins in cents by 4
3. Add 10 to the product
4. Multiply your answer by 25
5. Add 115 to your answer
6. Add your age in years
7. Subtract the number of days in a normal year

Making Cents Debrief

- What do you notice about your answers?
- How can you describe what you notice to someone who is not present?

How do you describe what you see in terms of student learning?

Types of Understandings

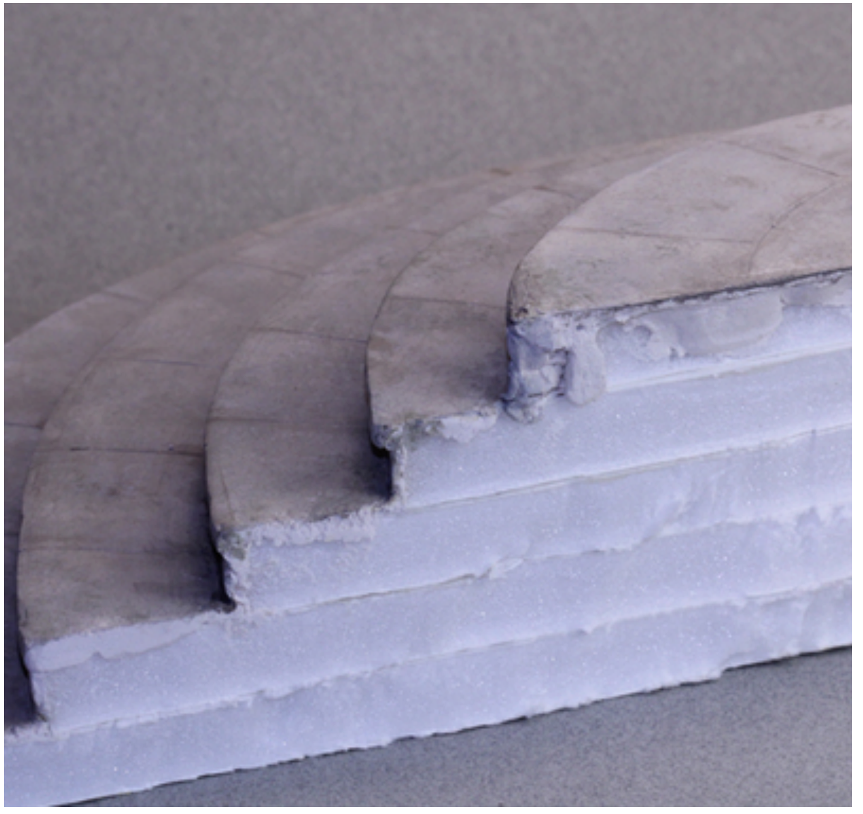
Procedural - Student can perform a computation or algorithm by following a series of prescribed steps

Conceptual - Student understands the basis of why a computation or algorithm works. They can apply it later without reteaching. Student can identify, describe, and explain a big idea related to a topic or a class of problems

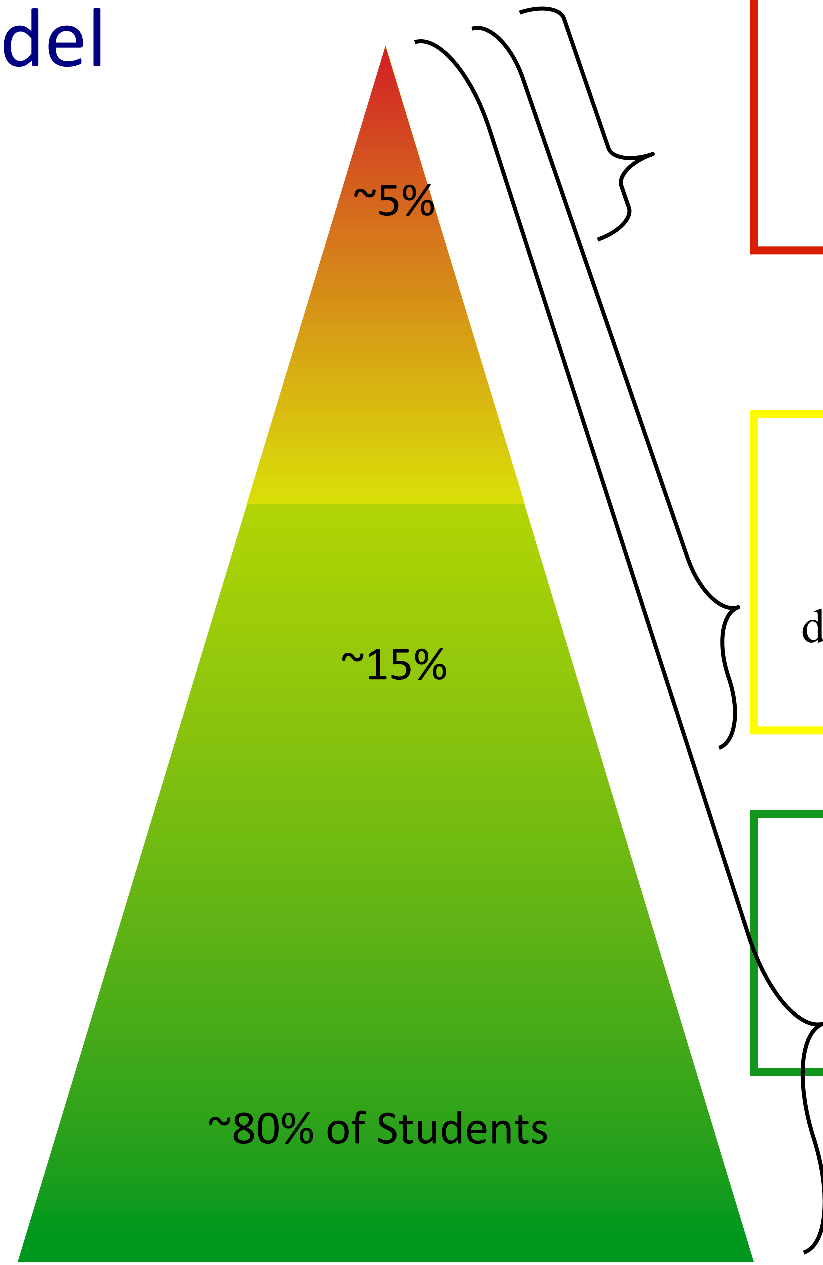
Problem solving - Student can solve a problem when there is no specific solution pathway or algorithm

Tiered Support Systems

- What types of tiered support do you offer in your school or district?



3-Tiered Support Model



<u>Tertiary Prevention</u>:
Specialized & individualized strategies for students who don't respond to Tier 2 supplementary support - INTENSIVE

<u>Secondary Prevention</u>:
Supplementary strategies for students who do not respond to core instruction - TARGETED

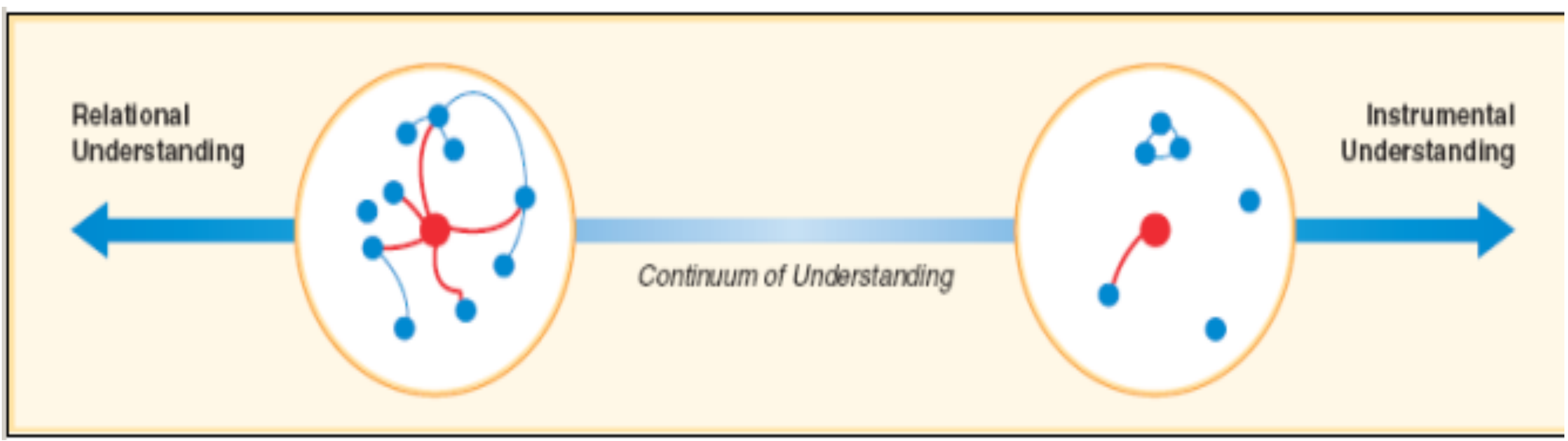
<u>Primary Prevention</u>:
High quality engaging core instruction - UNIVERSAL

Models within Tier 2

- Problem-solving model: School-based team uses multiple sources of data to determine interventions, often individualized
- Functional behavior assessment model: Students assessed before intervention, at intervals throughout intervention
- Standard protocol model: Intervention is a research-based, well-formulated curriculum

What are characteristics of students in Tier 2?

Compared to students who are not struggling, their brains might look very different!



Van de Walle, J., Karp, K., & Bay Williams, J. (2010). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

- What the problem is not—
  - difficulty reading, paying attention, or following directions
- What the problem is—
  - **Underdeveloped cognitive structures** (the mental processes necessary to connect new information with prior knowledge)

Geary, David C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37(*1), 4-15.

Components of A Strong Multi-Tiered System of Support Model

Uses a co-teaching approach as a collaboration between general education and special education

Includes research based teaching practices

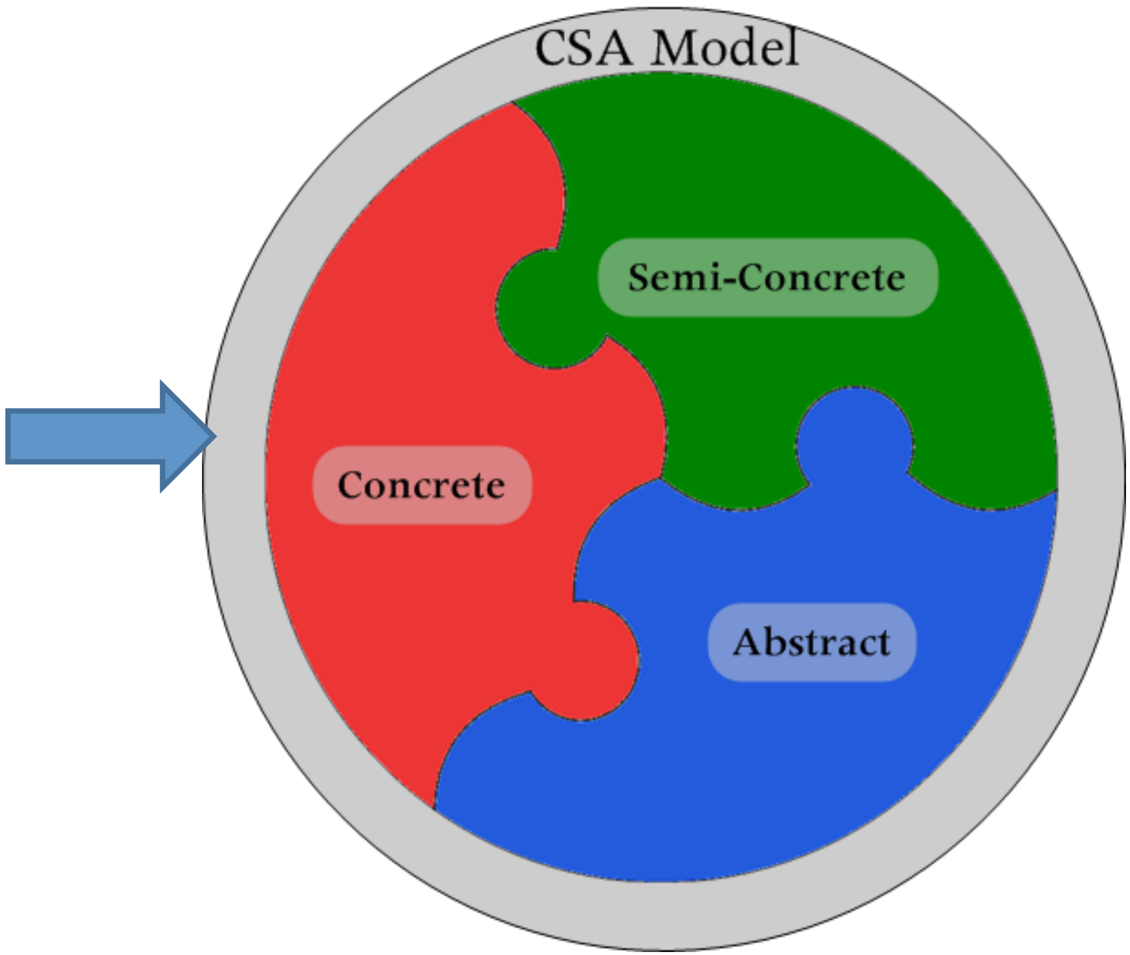
Uses screening and progress monitoring to Instruct with a preventative approach

Builds from students' strengths

Uses diagnostic assessments to align intervention

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies:Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.

CSA: Concrete—Semi-Concrete—Abstract



Intervention Recommendations from Research

- **C**oncrete—**S**emi-concrete—**A**bstract (CSA) representations
- Explicit instruction (not direct instruction)
- Underlying mathematical structures
- Examples (and counterexamples)
- Feedback – Not teacher to student but students' feedback to other students and teacher on what they know and don't know

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.
Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* New York: Routledge.

Create Mental Residues

- Establishes foundational understanding
- Models the physical action is the important
- Does not fade away or disappear
- Supports their thinking about the operation

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahweh, NJ: Erlbaum.

Characteristics of learning

- Introduce every topic with problem solving
- Every lesson includes five forms of communication
  - Reading
  - Speaking
  - Critical listening
  - Writing
  - Multiple representations
- Topics are connected
- Students have 8–15 days to move a concept to a skill
- Challenging problems for all students

Which comes first?

Concepts or Skills

What happens with procedural teaching

Focus on Skills

$$\frac{1}{12} + \frac{7}{8}$$

Focus on Skill

$$\frac{1}{12} + \frac{7}{8} = \frac{23}{24}$$

Focus on Sense Making

The sum of $\frac{12}{13}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. $\frac{1}{2}$
D. 1

The Two Worlds Collide: Sense Making Meets Skill

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. ½
D. 1

Explain your answer.

$$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$$ is closesst to 20.

Petit, Laird,& Marsden, 2010

Mathematical Outcomes

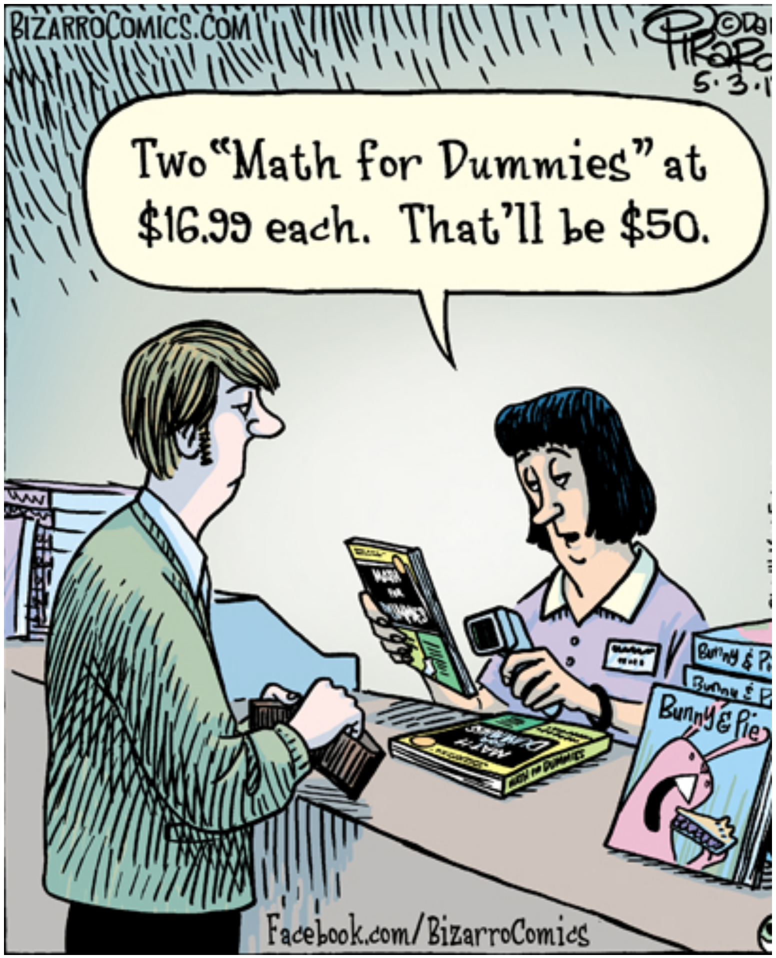
Small, fragmented, and isolated skills are not the desired outcome for students who struggle with math.

Quote from A Learner who Struggled

"As an adult, when I look back at math, I hated it. It was a mystery. Everything seemed to be hocus-pocus math. Do this, do that, and presto – you get the right answer. But I had too much stuff to memorize. I couldn't keep track of it. It was like being in a forest and seeing only the leaves on the trees and forgetting there's a forest!"

Personal communication, Mr. Burns (pseudonym)

Mathematical Outcomes



Goals for students who struggle:

- See patterns and generalize those patterns
- Apply the generalizations and patterns to other problems (see connections)
- Use reasoning and intuition when possible

Rules

- With the person sitting next to or around you, decide if the rules shown are always true.
- If it is not always true, find a counterexample.

- Addition and multiplication make larger.
- When you multiply by 10, add 0 to the end of the number.
- Two negatives make a positive.
- The longer the number, the larger the number.

Addition and multiplication make larger.

$32 + 67 = 99$

$15 \times 10 = 150$

$-3 + (-14) = -17$

$$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$$

When you multiply by 10, add a 0 at the end of the number.

15 x 10 = 150

4.5 x 10 = 45.0

Two negatives make a positive.

$$-8 \times (-3) = 24$$

$$-8 + (-3) = -11$$

The longer the number, the larger the number.

1,278,931 > 1,469

1.3 > 1.0118743

So---what do we do??

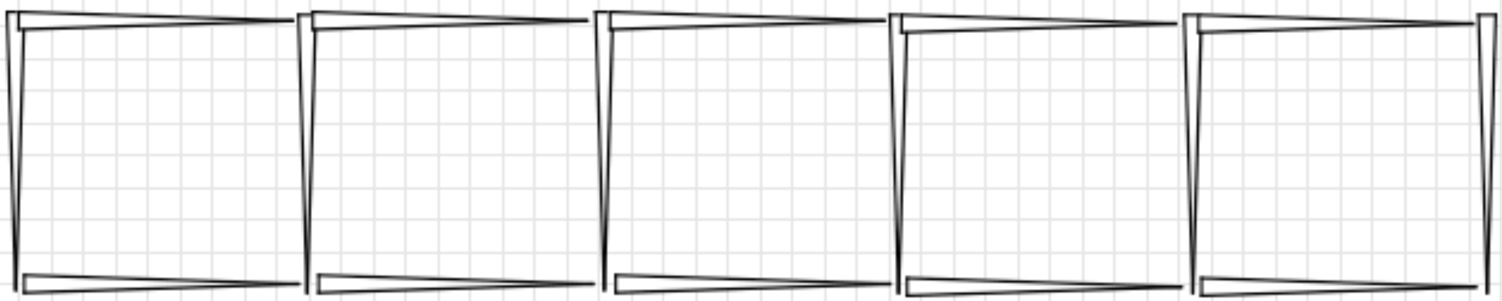
- Focus on BIG Ideas and generalizations

Think developmentally

- Link abstract symbols to geometric representations

Toothpick Lab

1. *Without counting*, find a way to determine how many toothpicks are needed to make this arrangement of five squares:



2. Draw a sketch that shows how you used the drawing in #1 to explain your counting technique.

Working with variables and equations

- Leaping to algebraic manipulations

$$14 - w = 9$$

Working with variables and equations

$$14 - w = 9$$

48% of students (1615) got it correct.

(2nd grade CCSSM standard)

A major misunderstanding

- Many students do not understand the equals sign.
- They believe it signifies that ***the answer*** comes next.

$$2x - 8 = 4x + 6$$

Developing understanding with CSA

- Consider multiple ways of solving an equation in a developmental sequence
  - Rather than starting with 'easy' equations and applying algebraic manipulations

$$5 + x = 12$$

$$5 - 5 + x = 12 - 5$$

$$x = 7$$

Developing understanding with CSA

- Logical reasoning and by inspection

$$5 + x = 12$$

What number added to 5 equals 12?

What basic fact do you know that could tell you the missing addend?

Developing understanding with CSA

- Working backwards and fact families

$$5 + x = 12$$

$5 + x = 12$

$x + 5 = 12$

$12 - 5 = x$

$12 - x = 5$

Developing understanding with CSA

- Making a table

$$3x + 2 = 4x - 3$$

x	$3x + 2$	$4x - 3$
2	8	5
4	14	13
5	17	17

Developing understanding with CSA

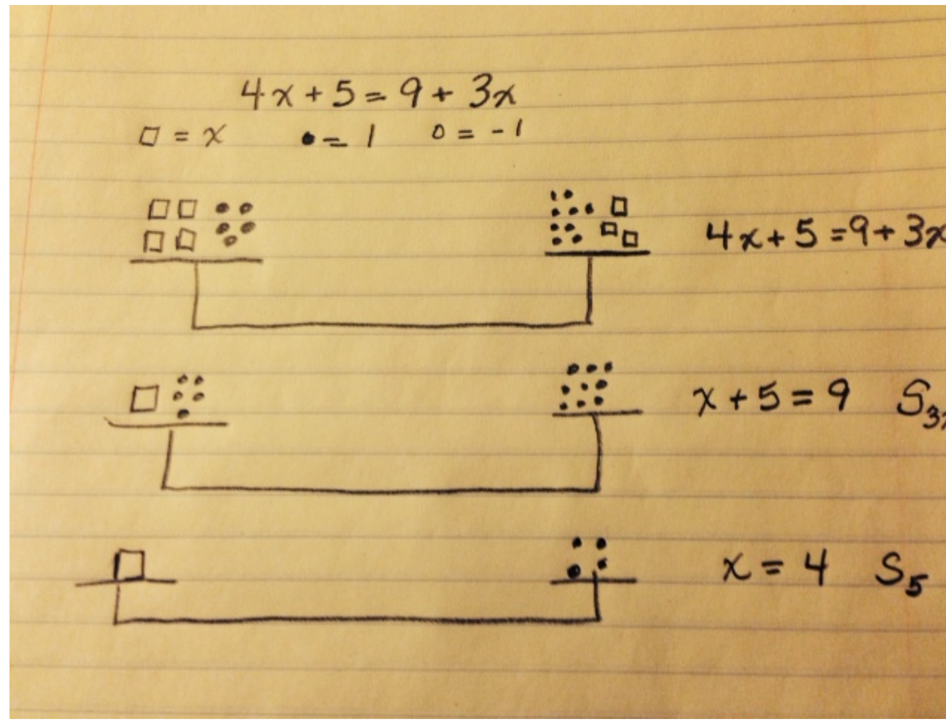
- Graphing

$$3x + 2 = 4x - 3$$

Developing understanding with CSA

Diagram

$3x + 2 = 4x - 3$



Developing understanding with CSA

- Algebraic manipulations

$$3x + 2 = 4x - 3$$

$$3x + 5 = 4x \qquad A_3$$

$$5 = x \qquad S_{3x}$$

Lessons Learned

- Forcing algebraic manipulations with showing steps in a particular may push students to forego logical thinking.
- Focus on procedural aspects lessen the likelihood of developing the bigger ideas.
- Vocabulary is important.

Sample of student work

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

a. $4 - x = 10 - 3x$

b. $3 + x = -(x + 3)$

c. $-2x = 6$

d. $x + 2 = 3$

Sample of student work

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

a. $4 - x = 10 - 3x$

b. $\mathbf{3 + \boldsymbol{x} = -(\boldsymbol{x} + 3)}$

c. $-2x = 6$

d. $\mathbf{\boldsymbol{x} + 2 = 3}$

Focus on Underlying Ideas Explicitly

Which represents the greater number or quantity?

1. $2x$ or $x + 2$

2. m or $-m$

3. $3x$ or x^3

Concept of Variable task

$2x$ or $x + 2$

It depends.

m or $-m$

It depends.

$3x$ or x^3

It depends.

Importance of Tasks

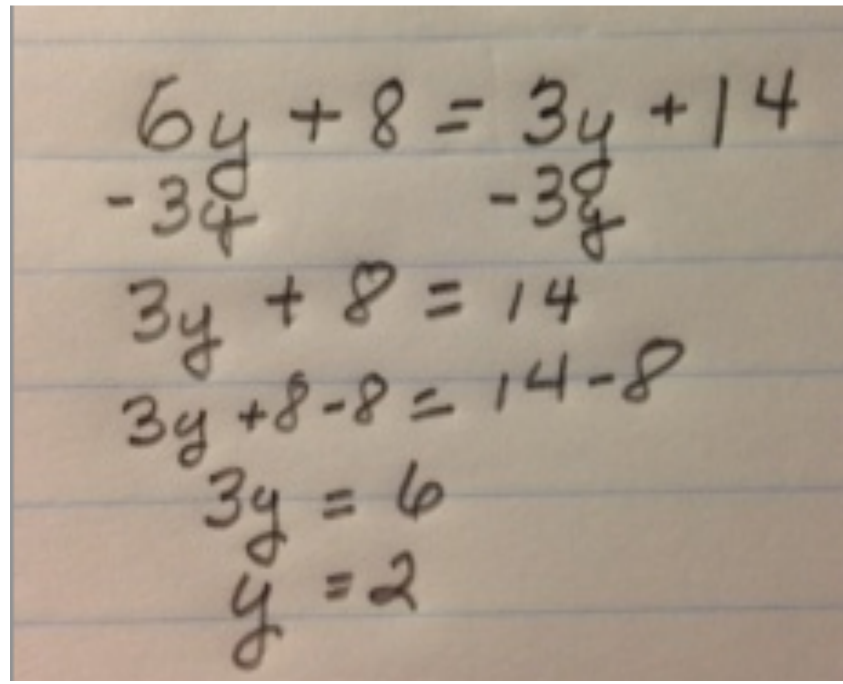
- Opportunities to extend the task to develop deeper
- Focus on relationships and reasoning

Importance of Tasks

- Opportunities to extend the task to develop deeper
- Focus on relationships and reasoning
- Should not be example based.

First, get the variable on one side (left!!).

Next, get the other number
on the other side.



Using example based teaching

- When skills are broken down into small pieces with very specific rules, it requires students to put the pieces together to form the whole.

Changing skill tasks to support deeper thinking

Solve for x:

a. $2x + 4 = 3x - 8$

Change the Task

- Reversibility question
  - Find an equation whose solution is 12.
  - Find another equation, with variables on both sides of the equal sign, whose solution is 12.

Change the Task

- Generalization questions
  - Find a linear equation whose solution is a whole number.
  - Is it possible to predict if the solution of an equation is a whole number? Why or why not?

Change the Task

- Flexibility question

  Solve:

  $2x - 8 = 3x + 4$

  Solve it another way.

Change the Task

- Flexibility question

  Solve:

  $2x - 8 = 12$

  $2(x + 2) - 8 = 12$

  $2(2x + 2) - 8 = 12$

Questions promote problem solving and generalizations

- Reversibility questions
  - Promotes the ability to think in different ways
  - Give answer, students create the problem

Questions to promote problem solving and generalizations

- Generalization questions
  - Asking students to find and describe patterns
  - What patterns do you notice?

Questions to promote problem solving and generalizations

- Flexibility questions
  - Asking students to solve a problem in multiple ways OR to use what they know about one problem to solve another one
  - Solve the problem in another way.
  - How are these problems alike? How are they different?

Perimeter and Area Problem

- Find the perimeter of a rectangle that measures 5.75 cm by 6.4 cm.
- Find the area of the rectangle.

Perimeter Problem and Algebra

- Giving students problems that have multiple solutions provides a context for them to generalize an idea.
  - Flexibility problems: Problems that can be solved in multiple ways or have multiple solutions

Perimeter Problem

Sketch a rectangle that has a perimeter of $6a + 4$ units.

Compare your sketch with those sitting around you.

What do you notice?

Perimeter Problem and Algebra

- Possible generalizations:
  - Multiple algebraic expressions can be simplified to $6a + 4$.
  - Other variables can be used but they must be 'subtracted out', such as sides of $1.5a + b - 2$ and $1.5a - b + 4$.
  - Expressions that have more than two terms can be simplified to a binomial.

Change the task

Brian said, “When I increase the perimeter of a rectangle, I also increase its area.” Do you agree with Brian? Why or why not?

Change the task

- Suppose that $a > 1$, $0 < b < 1$, and $0 < c < 2$. Complete each sentence by filling in the blank with <, =, >, or CT (for "can't tell").

$$a \cdot b \_\_\_ a$$

$$b \cdot c \_\_\_ b$$

$$a \cdot b \cdot c \_\_\_\_ b$$

$$b^2 \_\_\_\_ b$$

Change the task: Your Turn

Using the ideas of reversibility, flexibility, and generalization OR focusing on a Big Idea, what 'new' question could you ask about:

Subtract: $-8 - (-12)$

OR

Solve the system of equations:

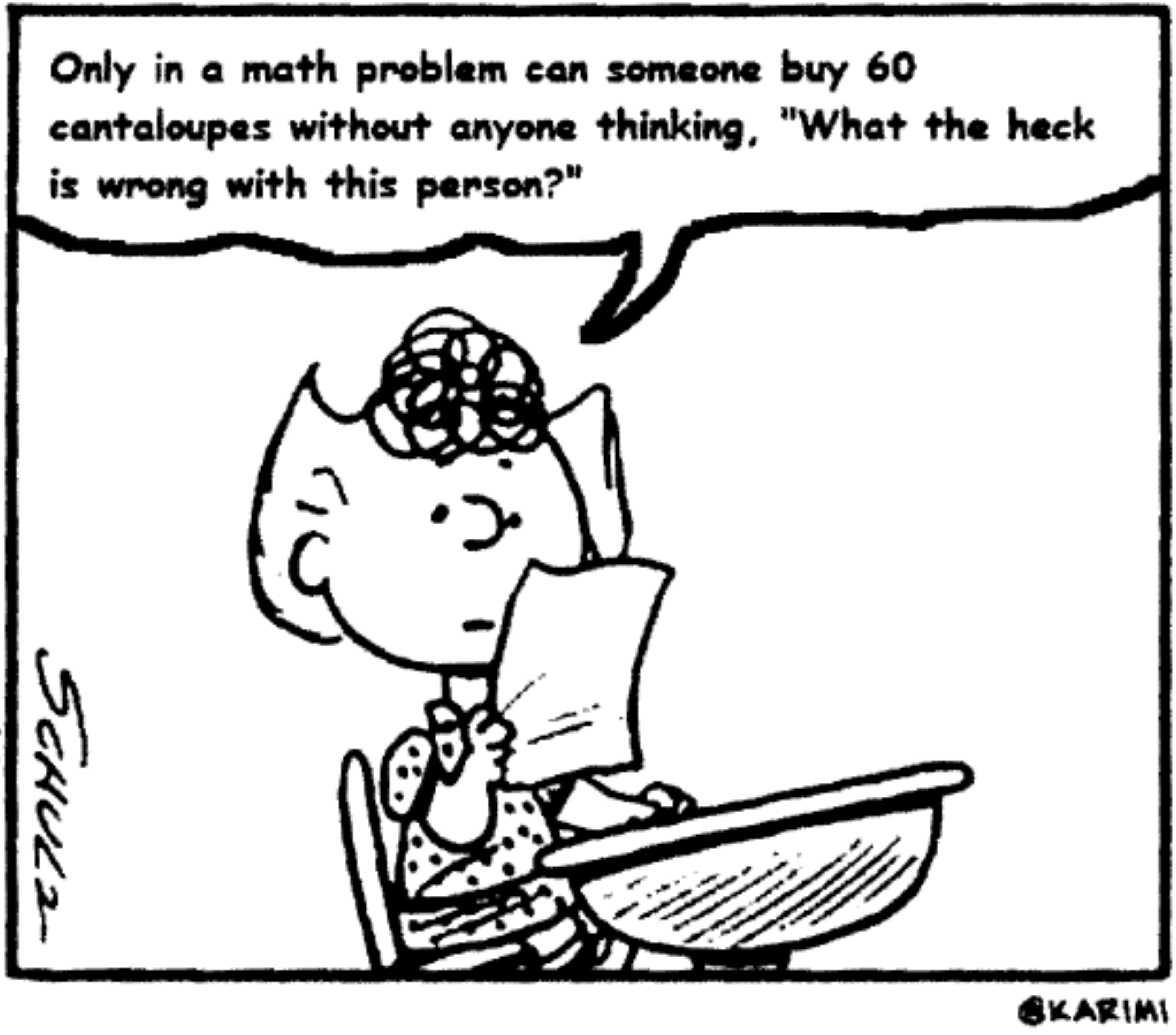
$$x + y = 6$$
$$y = -3$$

Reversibility, flexibility, Generalization

Jo solved a system of equations. She found a solution of (9, –3).

Find two equations that could have been the system of equations she solved.

Is there more than one solution? Why or why not?


Only in a math problem can someone buy 60 cantaloupes without anyone thinking, "What the heck is wrong with this person?"
SCHULZ
©KARIMI

The Myth of Keywords

- Keywords do not—
  - Develop of sense making or support making meaning
  - Build structures future applications
  - Appear in many problems

- Students use key words inappropriately

- Multi-step problems are impossible to solve with key words

Danger: Key Words Ahead

Mark has 33 packages of pencils. There are 6 pencils in each package. How many pencils does he have <u>in all</u>?

39 because it says in all.

The Infamous Shepherd Problem

:

There are 25 sheep and 5 dogs in a flock. How old is the shepherd?

How do you think students solved the problem?

Caldwell, Kobett & Karp (forthcoming) Addition and Subtraction in Practice. NCTM.

Results from 214 Students

	Added the numbers	Subtracted the numbers	Multiplied the numbers	Created a ratio	Other Incorrect procedure	Suggested no solution is possible
Third grade (n = 58)	76%	8%	0%	0%	14%	2 %
Sixth grade (n = 71)	48%	9%	21%	8%	6%	8 %
Seventh grade (n = 85)	48%	2%	17%	14%	9%	10 %

Caldwell, Kobett & Karp (forthcoming) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Really?

Caldwell, Kobett & Karp (forthcoming) *Putting essential understandings into practice in grades preK–2: Addition and subtraction.* A volume in the series edited by B. J. Dougherty, National Council of Teachers of Mathematics..

Not All Problem Solving is Created Equal

- Application/routine problems: solved by an algorithm
- Non-routine problems: a creative solution approach is needed

Do you think differently when you solve these problems?

Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Do you think differently when you solve these problems?

Word/Application

Brett wanted to put carpet on his bedroom floor. His room measures 10’ X 15’. How many square feet of carpet does he need?

Non-routine

You have 8 coins and a balance scale. The coins look alike, but one is counterfeit and lighter than the other 7. Find the counterfeit coin using 2 weighings on the balance scale.

Instructional Techniques: Classroom Routines

- Routine of classroom
  - Knowing what the responsibilities are for the students and the teachers
  - Sets expectations so that students are prepared

Instructional Techniques: Classroom Routines

- At the beginning of the school year, name the routines:
  - Collaborative groups
  - Poster sessions
  - Carousel
  - Expert groups
  - Cooperative groups
  - Teacher led

Questioning Routines

- Wait time
  - Indicate when asking a question what you expect
  - Consider using a ‘think time’
  - Think-pair-share is one possibility
  - Types of questions asked

Questioning Routines

- How do students respond?
  - Unison
  - Random
  - Pair think/response
  - Whiteboard or other display of answer

Vocabulary

- Use mathematical vocabulary with students so that they don't have to learn parallel sets of vocabulary words
  - Distributive property rather than rainbow property
  - Equals sign rather than the river to cross over
  - Substitute rather than plug in

Avoid generalizations or rule that expire

- Key words
  - Multiply when you see the word ‘of’
- Multiplication is the opposite of division
- You always put the variable first in an expression (y + 3 rather than 3 + y)
- The variable always goes on the left of the equation when you are solving it.

Explicit discussions

- Ask questions that focus students on specific features, characteristics, or structure:

How would you describe the difference between $3s$ and s^3?

Given the expression $3x – 2$, what is the effect on the value of the expression as x increases by 4?

Is $5t$ always greater than t?

Data Resources

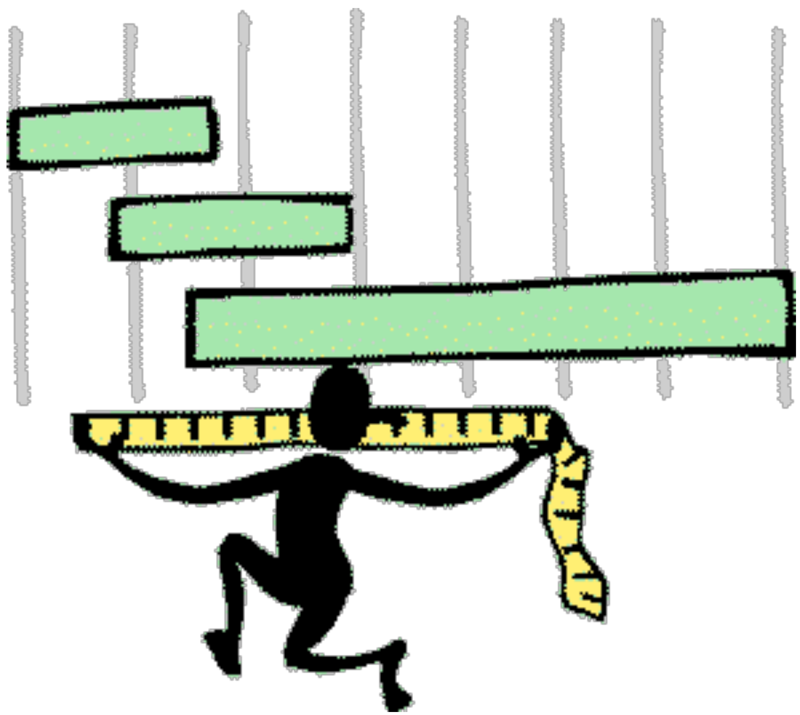
- Universal screener
- Progress monitoring

Universal Screener

- Middle and secondary students have a large amount of data.
- What new information can a universal screener offer?

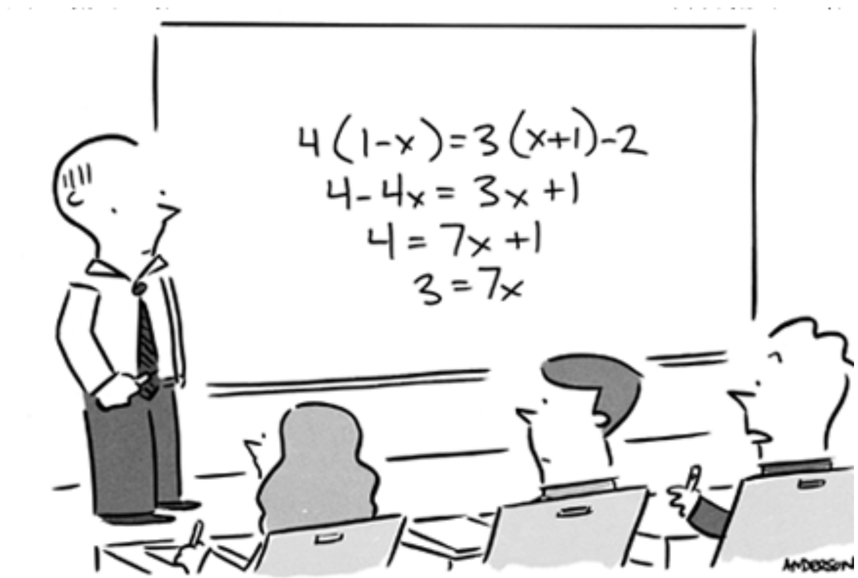
Progress Monitoring

How is progress monitoring different from classroom instruction?



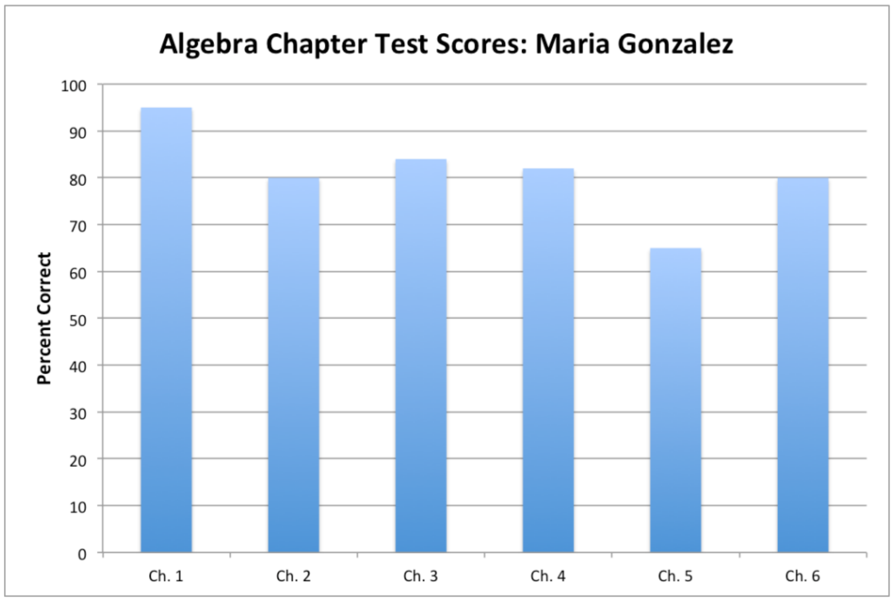
What are Big Ideas in Algebra for you?

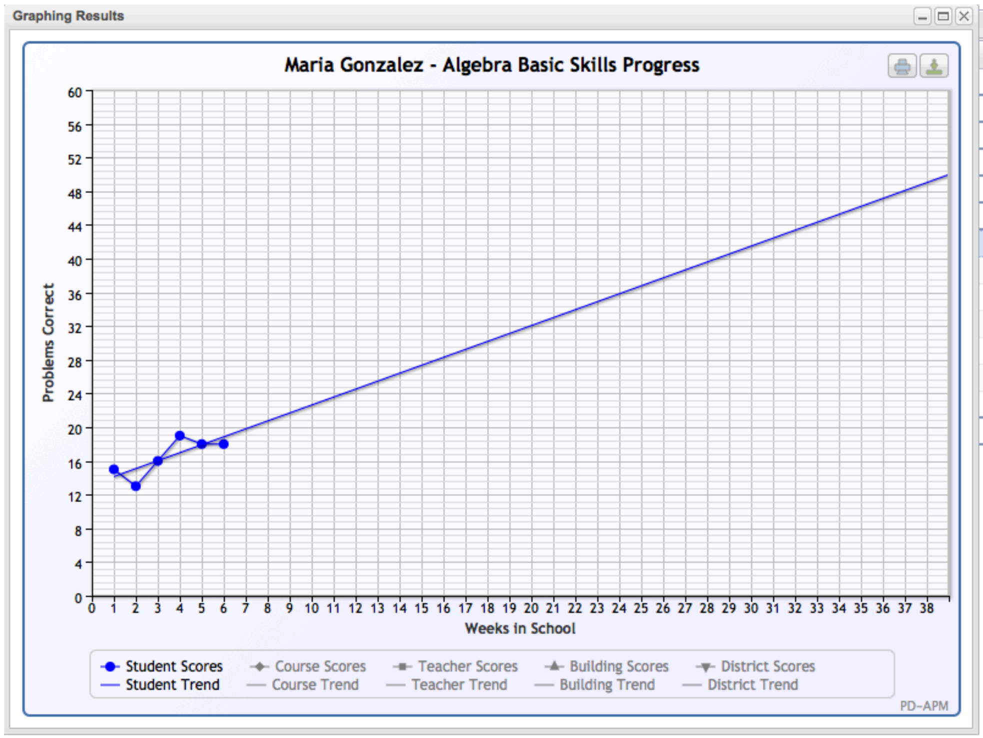
- What skills would be important to assess?
- What concepts would be important to assess?



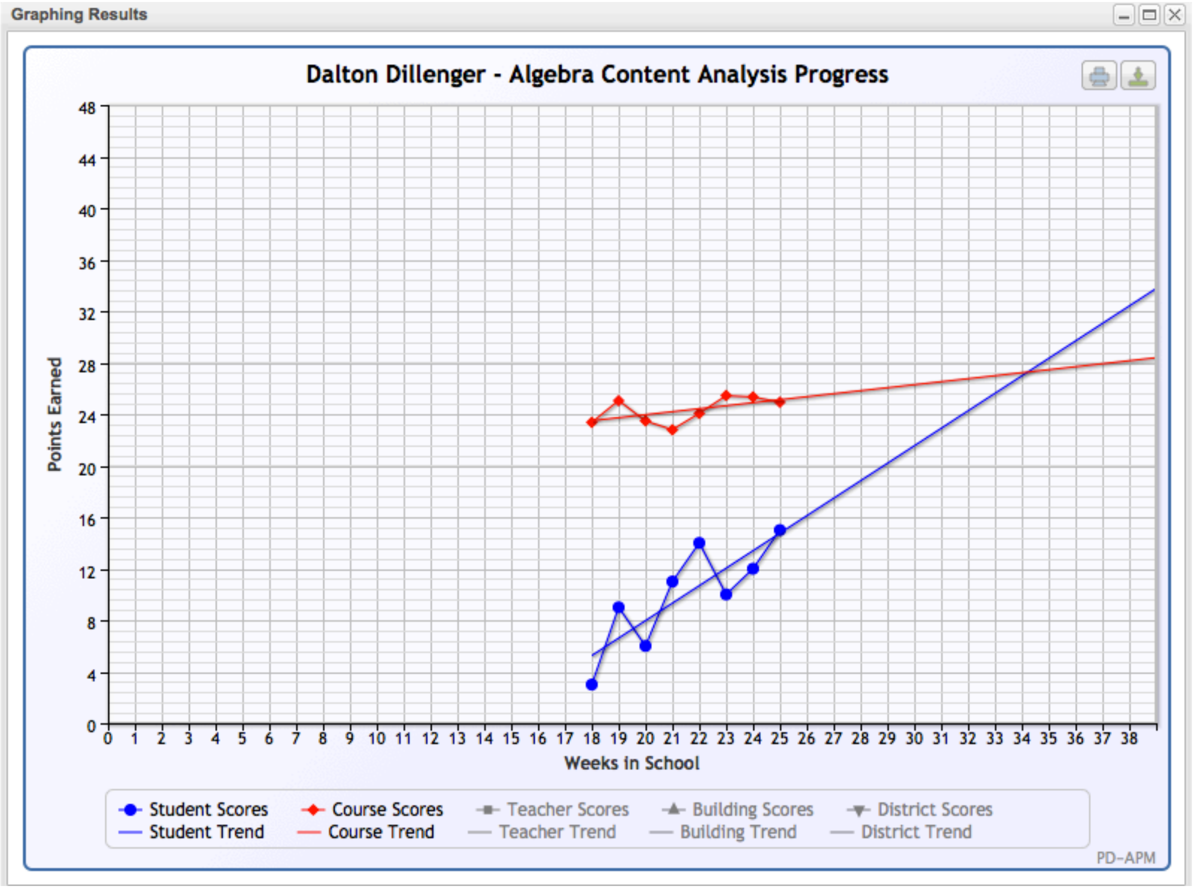
"Wouldn't it be more efficient to just find who's complicating equations and ask them to stop?"

Progress Monitoring Unique Features





Graph for Progress Monitoring



Data Resources

- How will you determine who is placed in Tier 2 instruction?
- How will you determine who leaves Tier 2 instruction?

Progress Monitoring Assessment

- Procedural measures: skills, algorithms
- Typically multiple-choice format

Progress Monitoring

- Conceptual measures
  - May not require computation
  - Focuses on reasoning
  - Emphasizes big ideas
  - May be multiple-choice but the multiple-choice options are linked to thinking, not just the answer
  - May be open-response items, scored with a metric/rubric

Concept of Variable

Jon said, "$m - 1$ is always greater than $1 - m$." Do you agree with Jon?

A. Yes, Jon is correct because m is a positive number.
B. Yes, Jon is correct because you cannot substitute a negative number for m.
C. No, Jon is not correct because $1 - m$ is greater than $m - 1$ when m is negative integer.
D. No, Jon is not correct because these expressions are equivalent.

Answer ___________

If $x = d + 2$ and $d + 2 + x = y$, then which of the following statements is true?

A. $y = 2d + 4 + x$
B. $4d + 2 = y$
C. $y = 2x$
D. $6d = y$

Answer ___________

Proportional Reasoning

For every foot of fence built (t), a carpenter needs a consistent number of nails (n). What does the equation tell you?

$$n = 12t + 24$$

A. You need 36 nails and boards.
B. The number of nails increases by 12 for every foot of fence.
C. The number of nails increases 12 times for every foot of fence.
D. The number of nails increases by 24 for every foot of fence.

Answer ___________

Do these ratios represent the same relationship? 3:4 and $\frac{18}{24}$

A. No, because one is written like a fraction.
B. No, because they are different numbers.
C. Yes, because they are both in the ratio of 3 to 4.
D. Yes, because they are written like a ratio.

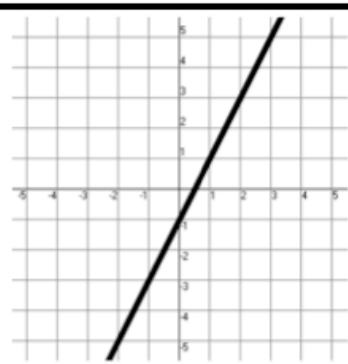
Answer ___________

Translations, Functions, and Graphing

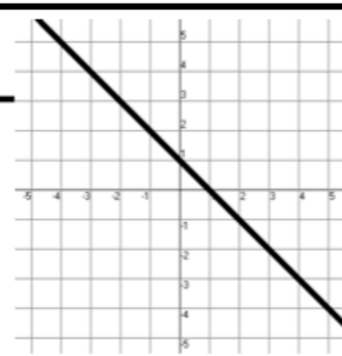
Match each equation to a corresponding graph or data table.

A	B	C	D
$y = x$	$2x = y + 1$	$y = 1.5$	$y = -x + 1$

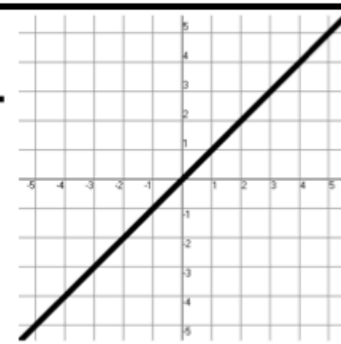
1. ___



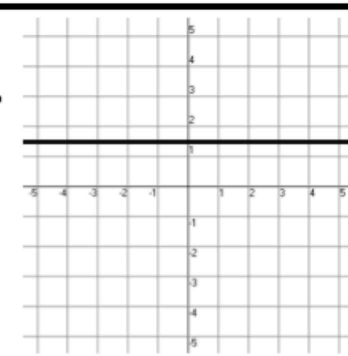
2. ___



3. ___



4. ___



5. ___

x	y
2	1.5
1	1.5
0	1.5
–1	1.5
–2	1.5

6. ___

x	y
2	–1
1	0
0	1
–1	2
–2	3

7. ___

x	y
2	3
1	1
0	–1
–1	–3
–2	–5

8. ___

x	y
4	4
2	2
0	0
–2	–2
–4	–4

9. ___

x	y
4	–3
2	–1
0	1
–2	3
–4	5

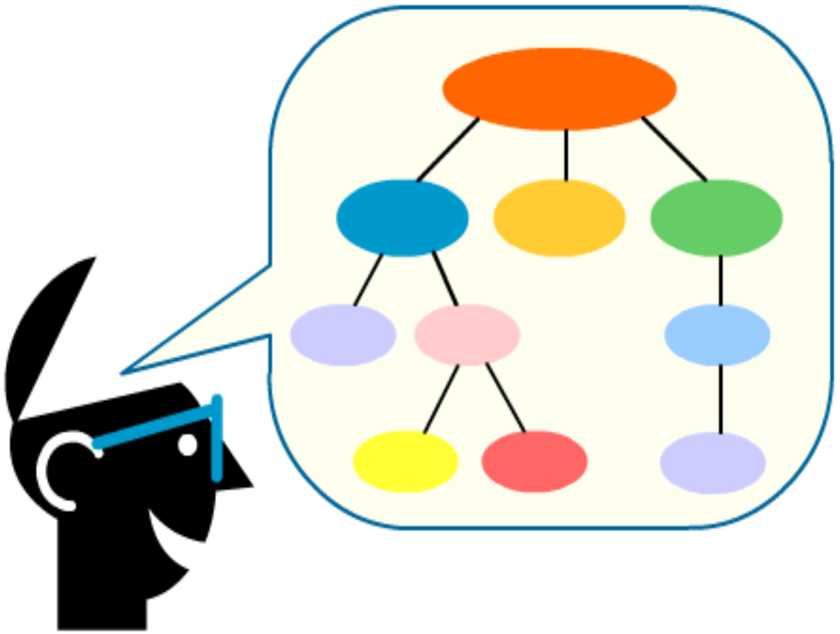
10. Which graph represents: The sum of a variable and one is equal to two multiplied by a second variable?

12. Jake graphed equation B. Jamie graphed the equation $y = 4x - 3$. How does Jamie's graph compare to Jake's graph?

A) Jamie's graph is translated down 3 units from Jake's graph.
B) Jamie's graph is translated up 3 units from Jake's graph.
C) Jamie's graph is steeper than Jake's graph.
D) Jamie's graph is less steep than Jake's graph.

Conceptual Focus

If $h + m = 9$, what does $h + m + 9$ equal?



Results from 2nd Semester Algebra I Students

If $h + m = 9$, what does $h + m + 9$ equal?

Difficulty = .52 (all students)
.72 (responses)
N = 243 total, 176 responses

Number of students	Selected Responses
126	18
16	$9hm$
5	0
2	-9
2	$hm9$

Conceptual Focus

Chelsea wrote the equation $c = 5h - 1$. If the value of h is increased by 2, the value of c will . . .

a. Decrease by 1

b. Decrease by 2

c. Increase by 5

d. Increase by 10

Results from 2nd Semester Algebra I Students

Chelsea wrote the equation $c = 5h - 1$. If the value of h is increased by 2, the value of c will . . .

a. Decrease by 1.
b. Decrease by 2.
c. Increase by 5.
d. Increase by 10.

Difficulty = .35 (all students)
.48 (responses)
N = 243 total, 178 responses
Number of students choosing each response:

a. 21	c. 35
b. 36	d. 86

Conceptual Focus

If $d - 243 = 542$, what does $d - 239$ equal?

Results from 2nd Semester Algebra I Students

If $d - 243 = 542$, what does $d - 239$ equal?

Difficulty = .44 (any non-zero score)
.27 (full credit score)

N = 243 total, 122 responses

Number of students	Selected Responses
45	546
17	538
9	785
5	542
4	539
3	60

Teacher Resources

We expect that the very best doctors will treat the most grievously ill patients. It should be no different in education. Great teachers have the skills to help the students who struggle the most.

Larson, M. (2011).. Supporting students' achievement of the Common Core. NCTM Conference presentation. Indianapolis, IN.

Closing Discussion

- Questions?
- Lingering issues?

"I'm afraid I still have more questions than answers."

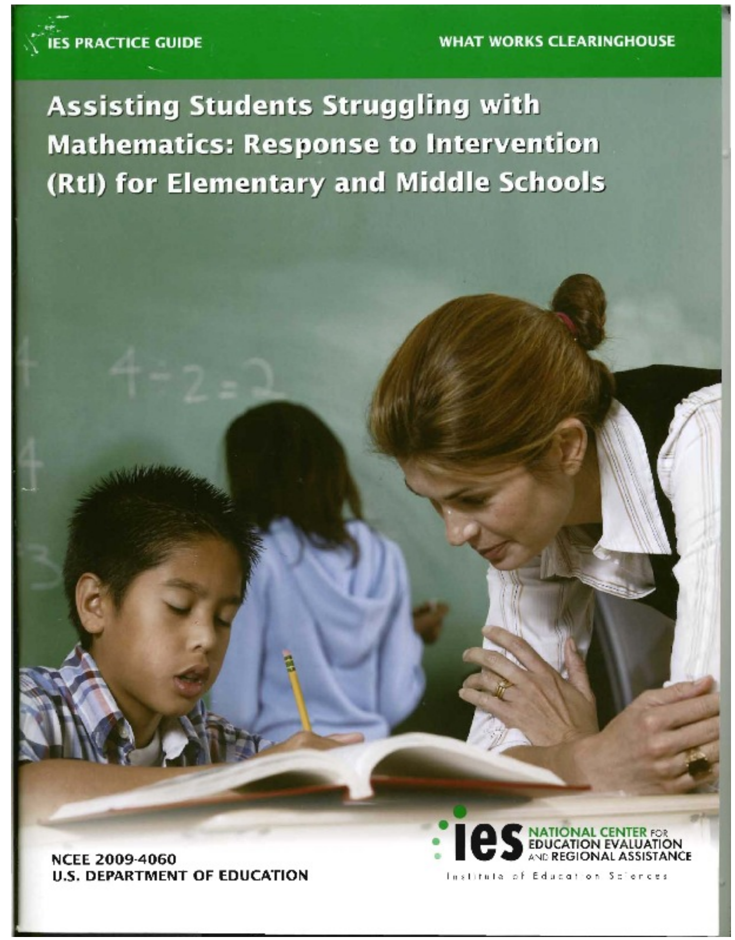
NCTM Resources

- Math of Tomorrow (MOTO Project)
- Pre-conference institute: New Orleans, April 9
- Summer Institutes

Recommendations for Identifying and Supporting Students Struggling in Mathematics

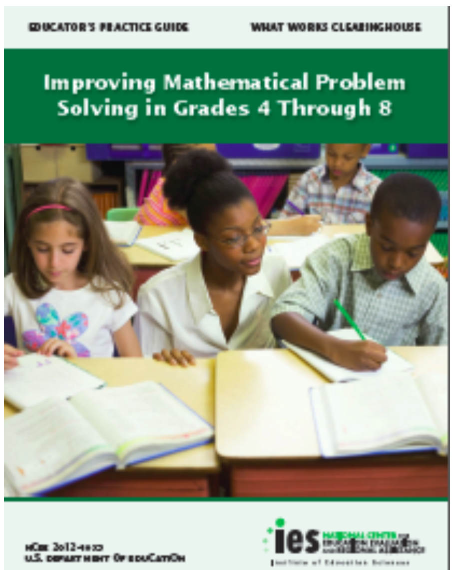
- Recommendations are based on **strong** and **moderate** levels of evidence resulting from comprehensive reviews of current research literature

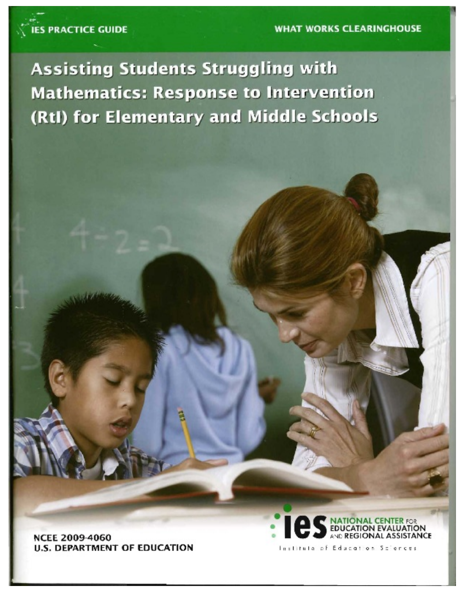
http://ies. ed.gov/ncee/wwc/publications/practiceguides/

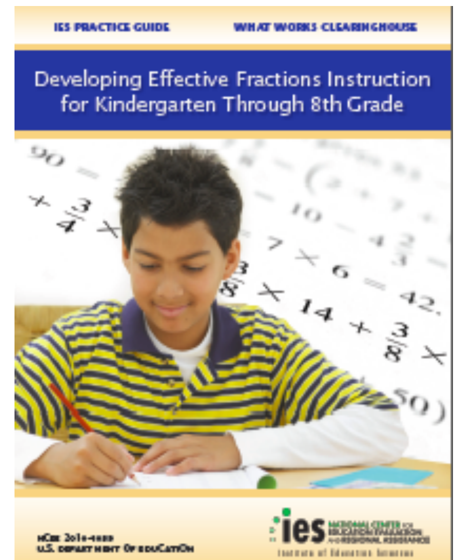


Recommendations for Identifying and Supporting Students Struggling in Mathematics

- Based on **strong** and **moderate** levels of evidence resulting from comprehensive reviews of current research







Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC:* National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/.